

An Inventory Control Model for Fixed Deterioration and Logarithmic Demand Rates

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Abstract
This paper proposes an inventory control model for fixed deterioration and Logarithmic demand rat for the optimal stock of commodities to meet the future demand which may either arise at a constant rate or may vary with time. The analytical development is provided to obtain the optimal solution to minimize the total cost per time unit of an inventory control system. Numerical analysis has been presented to accredit the validity of the mentioned model. Effect of change in the values of different parameters on the decision variable and objectives function has been studied.

Keywords: Inventory Control, Fixed Deterioration, Logarithmic Demand rate, Commodities.

Introduction Deterioration means damage, spoilage, dryness, vaporization, etc. The products like fresh food (meat, fish, fruits, and vegetables), photographic films, batteries, human blood, photographic films, etc having a maximum usable lifetime are known as perishable products and the products like alcohol, gasoline, radioactive substances, lubricants, glues, paints, chemical ingredients, etc having no shelf-life at all are known as decaying products. No obsolescence/deterioration refers to inventories that their shelf life can be indefinite and hence they would fall under no obsolescence/deterioration category. In this paper the deteriorating inventories are considered. There are many classifications for the deterioration. The deteriorating properties of inventory are classified by Ghare and Schrader [3] into three categories: 1-direct spoilage, e.g., vegetable, fruit and fresh food etc.; 2-physical depletion, e.g., gasoline and alcohol etc.;3-deterioration such as radiation change, negative spoiling and loss of efficacy in inventory, e.g., electronic components and medicine. . From another point of view, deterioration and the logarithmic relationship of ordering cost to investment discussed is not only an interesting special case but also a practical one.

Recently, researchers have examined the classical inventory model with infinite and finite production rates. Inventory model for single period with deterministic demand have been considered in the past. Ghare and Schrader (1963) were the first to address the inventory problem under constant demand and constant deterioration. Covert and Philip (1973) relaxed the assumption of constant deterioration rate by considering a two parameter Weibull distribution and assuming that the average carrying cost can be estimated as half of the replenishment size. Shah (1976) extended this model to allow for backlogging. Dave and Patel (1981) were the first to depart from the restrictive assumption of constant demand over an infinite planning horizon. Sachan (1984) extended the model of Dave and Patel (1981) to allow for shortage after correcting some of its approximation errors. Bahari Kashani (1989) relaxed the assumption of equal replenishment cycle when backorders are not allowed. Prior to the work of Sachan, Hollier and Mak (1983) developed two mathematical models for items that are deteriorating at a constant rate in an exponentially decaling market. Sharma and Kumar (2001) presented an inventory model for exponentially decaying inventory with known demand. In this model, demand is a function of selling price and rate of deterioration is a function of time. Nita H. Shah and Ankit S. Acharya (2008) presented a time dependent deteriorating order level inventory model for exponentially declining demand.

In the proposed study, an inventory model has been developed for fixed deterioration and logarithmic demand rate. The objective is to minimize the total cost per time unit of an inventory system.

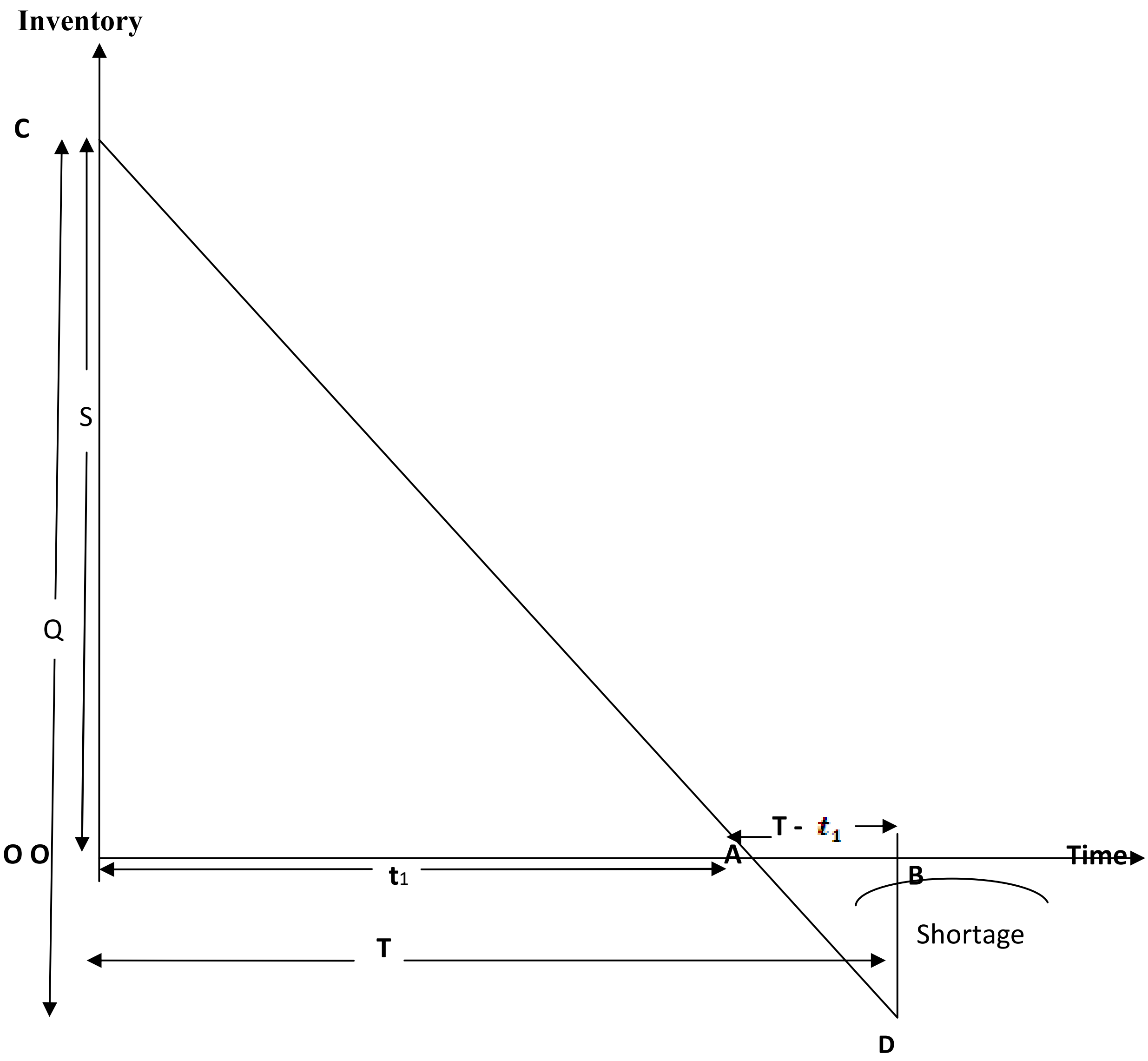
1. Assumptions and Notations

The inventory model is developed on the basis of the following assumption and notations:

(1) Replenishment size is constant and the replenishment rate is infinite.
(2) The lead time is zero.
(3) T is the fixed length of each production cycle.
(4) A log (1+t) is the demand rate at time t.
(5) θ is the constant rate of deterioration.
(6) C_1 is the inventory holding cost per unit time.
(7) C_2 is the shortage cost per unit time.
(8) C is the cost of each deteriorated unit.
(9) D is the total amount of deteriorated unit.
(10) $q(t)$ is the on hand inventory at any time t.
(11) Q is the total amount of inventory produced at the beginning of each period.
(12) S (> 0) is the initial inventory after fulfilling back order.
(13) K is the total average cost.

2. A MATHEMATICAL FORMULATION

We consider Q as total amount of inventory at the beginning of each period and S as the initial inventory after fulfilling backorders. Inventory level gradually decreases during time $(0, t_1)$, $(t_1 < T)$ due to the reasons of market demand and deterioration. It ultimately falls to zero at time $t = t_1$. Shortages occur during time period (t_1, T) which are fully backlogged. This model has been shown graphically in Figure 1. The differential equation to which the on hand inventory q(t) satisfying in two different parts of the cycle time T are given by:



$$\frac{dq(t)}{dt} + \theta q(t) = -\log(1+t) \qquad 0 \le t \le t_1 \ldots\ldots\ldots\ldots (1)$$

$$\frac{dq(t)}{dt} = -\log(1+t) \qquad t_1 < t \le T \ldots\ldots\ldots\ldots (2)$$

Equation (1) is linear differential equations. Its solution is given by:

$$q(t)e^{\theta t} = -\int log(1+t)\, e^{\theta t} dt + c$$

Where c is the constant of integration

$$q(t)e^{\theta t} = \left(\frac{(1+2\theta)}{6}t^3 + \frac{(3\theta-2)}{24}t^4 - \frac{t^2}{2}\right) + c$$

$$q(t) = \left(\frac{1+2\theta)}{6}t^3 + \frac{(3\theta-2)}{24}t^4 - \frac{t^2}{2}\right)e^{-\theta t} + ce^{-\theta t}$$

Applying the boundary condition at t=0, q (t) =s it reduces to

S = c

Therefore,

$$q(t) = \left(\frac{1-2\theta)}{6}t^3 + \frac{(3\theta-2)}{24}t^4 - \frac{t^2}{2}\right)e^{-\theta t} + Se^{-\theta t} \ldots\ldots(3)$$

Solution of equation (2) is given by

$$q(t) = (1+t)\log(1+t) - t + c$$

Where c is the constant of integration

Applying boundary condition at $t = t_1$, q(t) = 0, we get

$$0 = (1+t_1)\log(1+t_1) - t_1 + c$$

$$or \qquad c = t_1 - (1+t_1)\log(1+t_1)$$

$$\therefore \quad q(t) = (1+t)\log(1+t) - (1+t_1)\log(1+t_1) - (t-t_1) \ \ldots.(4)$$

Now since at $t = t_1$, q (t) = 0,

Apply the this condition to equation (3) it reduces to

$$0 = \left(\frac{1-2\theta)}{6}t_1^3 + \frac{(3\theta-2)}{24}t_1^4 - \frac{t_1^2}{2}\right)e^{-\theta t_1} + Se^{-\theta t_1}$$

$$S = \left(\frac{2\theta-1)}{6}t_1^3 + \frac{(2-3\theta)}{24}t_1^4 + \frac{t_1^2}{2}\right)\ldots\ldots\ldots(5)$$

Amount of the unit deteriorated is given by

$$D = S - \int_0^{t_1}\log(1+t)\,dt$$

$$= S - [(1+t_1)\log(1+t_1) - t_1]$$

$$D = \left[\left\{\frac{(2\theta-1)}{6}t_1^3 + \frac{(2-3\theta)}{24}t_1^4 + \frac{t_1^2}{2}\right\} - \{(1+t_1)\log(1+t_1) - t_1\}\right] \text{..........} (6)$$

Expression for total average inventory during $(0, t_1)$

$$I_1(t_1) = \frac{1}{T}\int_0^{t_1} q(t)dt$$

$$I_1(t_1) = \frac{1}{T}\int_0^{t_1}\left[\left(\frac{1-2\theta)}{6}t^3 + \frac{(3\theta-2)}{24}t^4 - \frac{t^2}{2}\right)e^{-\theta t} + Se^{-\theta t}\right]dt$$

$$= \frac{1}{T}\left[\left(\frac{1-2\theta)}{6}\frac{t_1^4}{4} + \frac{(3\theta-2)}{24\times 5}t_1^5 - \frac{t_1^3}{6} + St_1\right) - \theta\left(\frac{St_1^2}{2} + \frac{t_1^4}{8} + \frac{t_1^5}{30} - \frac{t_1^6}{72}\right)\right]$$

Using the value of S it reduces to

$$I_1(t_1) = \frac{1}{T}\left[\left\{\left(\frac{1-2\theta)}{24}t_1^4 + \frac{(3\theta-2)}{120}t_1^5 - \frac{t_1^3}{6} + \left(\frac{2\theta-1)}{6}t_1^4 + \frac{(2-3\theta)}{24}t_1^5 + \frac{t_1^3}{2}\right)\right) - \theta\left(\left(\frac{(2\theta-1)}{12}t_1^5 + \frac{(2-3\theta)}{48}t_1^6 + \frac{t_1^4}{4}\right) + \frac{t_1^4}{8} + \frac{t_1^5}{30} - \frac{t_1^6}{72}\right)\right\}\right]$$

$$I_1(t_1) = \frac{1}{T}\left[\left\{\left(\frac{(2\theta-1)}{8}t_1^4 + \frac{(2-3\theta)}{30}t_1^5 + \frac{t_1^3}{3}\right) + \theta\left(-\frac{3t_1^4}{8} + \frac{t_1^5}{20} - \frac{t_1^6}{32}\right)\right\}\right]$$

Now the expression for the average number of units in shortage during the interval (t_1, T) is given by

$$I_2(t_1) = \frac{1}{T}\int_{t_1}^{T} q(t)dt$$

$$I_2(t_1) = \frac{1}{T}\left[\int_{t_1}^{T}\{(1+t)\log(1+t) - (1+t_1)\log(1+t_1) - t + t_1\}dt\right]$$

$$= \frac{1}{T}\left[\left\{\frac{(1+t)^2}{2}\log(1+t) - \frac{(1+t)^2}{4}\right\} - t(1+t_1)\log(1+t_1) - \frac{t^2}{2} + tt_1\right]_{t_1}^{T}$$

$$= \frac{1}{T}\left[\left\{\frac{(1+T)^2}{2}\log(1+T) - \frac{(1+T)^2}{4}\right\} - T(1+t_1)\log(1+t_1) - \frac{T^2}{2} + Tt_1 - \left\{\frac{(1+t_1)^2}{2}\log(1+t_1) - \frac{(1+t_1)^2}{4}\right\} - t_1(1+t_1)\log(1+t_1) - \frac{t_1^2}{2} + t_1^2\right]$$

$$= \frac{1}{T}\left[\left\{\frac{(1+T)^2}{2}\left(\log(1+T)-\frac{1}{2}\right)\right\} + (1+t_1)\log(1+t_1)\left\{t_1 - \frac{(1+t_1)}{2} - T\right\} + \frac{(1+t_1)^2}{4} + \frac{t_1^2}{2} - \frac{T^2}{2} + Tt_1\right]$$

Finally the expression for total average cost per unit time is given by

$$K = \frac{C_1}{T}\left[\left\{\left(\frac{(2\theta-1)}{8}t_1^4 + \frac{(2-3\theta)}{30}t_1^5\right) + \theta\left(-\frac{3t_1^4}{8} + \frac{t_1^5}{20} - \frac{t_1^6}{32}\right)\right\}\right] - \frac{C_2}{T}\left[\left[\left\{\frac{(1+T)^2}{2}\left(\log(1+T)-\frac{1}{2}\right)\right\} + (1+t_1)\log(1+t_1)\left\{\frac{(t_1-1)}{2} - T\right\} + \frac{(1+t_1)^2}{4} + \frac{t_1^2}{2} - \frac{T^2}{2} + Tt_1\right]\right] + C\left[\left\{\frac{(2\theta-1)}{6}t_1^3 + \frac{(2-3\theta)}{24}t_1^4 + \frac{t_1^2}{2}\right\} - 2\{(1+t_1)\log(1+t_1) - t_1\}\right] \ldots\ldots\ldots\ldots(9)$$

Condition for optimality is

$$\frac{dK}{dt_1} = 0$$

This gives

$$= \frac{C_1}{T}\left[\left\{\left(\frac{(2\theta-1)}{2}t_1^3 + \frac{(2-3\theta)}{6}t_1^4\right) + \theta\left(+\frac{t_1^4}{4} - \frac{3t_1^5}{16} - \frac{3t_1^3}{2}\right)\right\}\right] - \frac{C_2}{T}\left[t_1\log(1+t_1) + \frac{(1+t_1)}{2} - T\log(1+t_1) - T + \frac{(1+t_1)}{2} + t_1 + T\right] + C\left[\left\{\frac{(2\theta-1)}{2}t_1^2 + \frac{(2-3\theta)}{6}t_1^3 + t_1 - \log(1+t_1) - 1 + 1\right\} \ldots\ldots\ldots(10)\right]$$

Equation (10) is a non linear equation in t_1. In general it is easy to solve analytically. It can be solved by computational numerical method with the help of computer. In this way we can find the optimal value of t_1(say t_1^*). The minimum value t_1^* gives the minimum cost of the system, and other parameter of the system in question.

Optimal amount of the initial inventory after fulfilling backorder S denoted by S^* is given by

$$S^* = \left[\left(\frac{(2\theta-1)}{6}t_1^{*3} + \frac{(2-3\theta)}{24}t_1^{*4} + \frac{t_1^{*2}}{2}\right)\right]\ldots\ldots\ldots\ldots\ldots\ (11)$$

Optimal amount of the unit deteriorated D denoted by D^* is given by

$$D^* = \left[\left\{\frac{(2\theta-1)}{6}t_1^{*3} + \frac{(2-3\theta)}{24}t_1^{*4} + \frac{t_1^{*2}}{2}\right\} - a\{(1+t_1^*)\log(1+t_1^*) - t_1^*\}\right]\ldots\ldots(12)$$

Thus minimum value of the total cost K denoted by K^* is given by

$$K^* = \frac{C_1}{T}\left[\left\{\left(\frac{(2\theta-1)}{8}t_1^{*4} + \frac{(2-3\theta)}{30}t_1^{*5}\right) + \theta\left(-\frac{3t_1^{*4}}{8} + \frac{t_1^{*5}}{20} - \frac{t_1^{*6}}{32}\right)\right\}\right]$$
$$- \frac{C_2}{T}\left[\left[\left\{\frac{(1+T)^2}{2}\left(\log(1+T) - \frac{1}{2}\right)\right\} + (1+t_1^*)\log(1+t_1^*)\left\{\frac{(t_1^*-1)}{2} - T\right\}\right.\right.$$
$$\left.\left. + \frac{(1+t_1^*)^2}{4} + \frac{t_1^{*2}}{2} - \frac{T^2}{2} + Tt_1^*\right]\right]$$
$$+ C\left[\left\{\frac{(2\theta-1)}{6}t_1^{*3} + \frac{(2-3\theta)}{24}t_1^{*4} + \frac{t_1^{*2}}{2}\right\} - 2\{(1+t_1^*)\log(1+t_1^*) - t_1^*\}\right]$$

2.4 CONCLUSIONS

In this chapter an inventory model with logarithmic demand pattern and fixed rate of deterioration has been developed and discussed under certain assumption. Cost minimization technique is used to obtain the optimal value of stock, time and total cost. Deterministic cases of demand are considered by allowing shortage. Approximate expression for initial inventory, total number of deteriorated units total minimum average cost is obtained.

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