 

# H-Function and a problem related to a String

S. N. Singh
Head, Department of Mathematics Jamtara College
Jamtara (Jharkhand)
email : singhsn@live.in

Raj Mehta
Deptt . of Mathematics,
Guru Ramdas Khalsa Institute of Science & Technology,
Barela. Distt. Jabalpur (M.P.) 483001, INDIA
email : raj.2512@rediffmail.com, raj.251264@gmail.com

**Abstract**
The aim of this paper is to obtain the solution of a problem related to a String with the help of H–function of one variable.

**Keywords:** Motion, Velocity, Transverse displacement

*1. Introduction:*

The H-function of one variable [3, p.10] is defined as:

$$H^{m,n}_{p,q}\left[x\left|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right.\right] = (1/2\pi i)\int_L \theta(s)\, x^s\, ds \tag{1.1}$$

where $i = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\ \prod_{j=1}^{n}\Gamma(1 - a_j + \alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1 - b_j + \beta_j s)\ \prod_{j=n+1}^{p}\Gamma(a_j - \alpha_j s)}$$

where

$$\sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j \equiv M > 0,$$

and $|\arg x| < \frac{1}{2} M\pi$.

In this paper, we shall make application of following modified form of the integral [2, p.372]:

$$\int_0^{\pi}(\sin x)^{\omega-1}\sin nx\, dx = \frac{\pi \sin \frac{1}{2} n\pi\, \Gamma(\omega)}{2^{\omega-1}\,\Gamma\{\frac{1}{2}(\omega+n+1)\}\,\Gamma\{\frac{1}{2}(\omega-n+1)\}},$$

$Re(\omega) > 0.$ (1.2)

*2. Integral:*

The integral to be established here is

$$\int_0^{\pi}(\sin x)^{\omega-1}\sin nx\, H^{m,l}_{p,q}\left[z(\sin x)^{\lambda}\left|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right.\right]dx$$

$$= 2^{1-\omega}\pi\sin\tfrac{1}{2}n\pi\, H^{m,l+1}_{p+1,q+2}\left[z\,2^{-\lambda}\left|\begin{matrix}(1-\omega,\lambda),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},(1/2-\omega/2\pm n/2,\lambda/2)\end{matrix}\right.\right], \tag{2.1}$$

where

$$\sum_{j=1}^{l}\alpha_j - \sum_{j=l+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j \equiv M > 0,$$

$|\arg z| < \frac{1}{2} M\pi$, $\lambda \ge 0$ and $Re(\omega) > 0$.

*Proof:*

Replace the H- function by its equivalent contour integral as given in (1.1), change the order of integration, evaluate the inner integral with the help of (1.2) and finally interpret it with (1.1), to get (2.1).

*3. Problem related to String:*

In this section, we consider a string, stretched between the point 0 and $\pi$ on the x-axis and initially at rest, is released from the position y = f(x). Air resistance opposes its motion, which is proportional to the velocity at each point. Let the unit of time be chosen so that the equation of motion becomes

$$y_{tt}(x, t) = y_{xx}(x, t) - 2\beta\, y_t(x, t), \tag{3.1}$$

where $\beta$ is a positive constant. Assuming that $0 < \beta < 1$, solution of (3.1) is given by [1, p.119]:

$$y(x, t) = \exp(-\beta t) \sum_{n=1}^{\infty} b_n [\cos \alpha_n t + (\beta/\alpha_n) \sin \alpha_n t] \sin nx, \tag{3.2}$$

where

$$\alpha_n = \sqrt{(n^2 - \beta^2)}$$

$$b_n = (2/\pi) \int_0^{\pi} f(x) \sin nx \, dx, \; n = 1, 2, \ldots.. \tag{3.3}$$

for the transverse displacement.

Now choose

$$f(x) = (\sin x)^{\omega - 1} H^{m, l}_{p, q}\left[z (\sin x)^{\lambda} \left| \begin{matrix} (a_j, \alpha_j)_{1, p} \\ (b_j, \beta_j)_{1, q} \end{matrix} \right.\right] \tag{3.4}$$

*4. Solution of the Problem:*

Combining (3.4) and (3.3) and making the use of the integral (2.1), we derive

$$b_n = 2^{2-\omega} \sin \tfrac{1}{2} n\pi \, H^{m, l+1}_{p+1, q+2}\left[z\, 2^{-\lambda} \left| \begin{matrix} (1-\omega, \lambda), (aj, \alpha j)_{1, p} \\ (bj, \beta j)_{1, q}, (1/2 - \omega/2 \pm n/2, \lambda/2) \end{matrix} \right.\right], \tag{4.1}$$

Putting the value of $b_n$ from (4.1) in (3.2), we get the following required solution of the problem:

$$y(x, t) = 2^{2-\omega} \exp(-\beta t) \sum_{n=1}^{\infty} \left\{ \sin \tfrac{1}{2} n\pi [\cos \alpha_n t + (\beta/\alpha_n) \sin \alpha_n t] \sin nx, \times \right.$$

$$\left. \times \; H^{m, l+1}_{p+1, q+2}\left[z\, 2^{-\lambda} \left| \begin{matrix} (1-\omega, \lambda), (aj, \alpha j)_{1, p} \\ (bj, \beta j)_{1, q}, (1/2 - \omega/2 \pm n/2, \lambda/2) \end{matrix} \right.\right], \right\} \tag{4.2}$$

Provided the condition stated with (2.1) are satisfied.

***References***

1. Churchill, R.V.: Fourier Series and Boundary Value Problems, McGraw–Hill, New York (1988).
2. Gradshteyn, I. S. and Ryzhik, I. M.: Tables of Integrals, Series and Products ,Academic Press, Inc. New York, 1980.
3. Srivastava, H. M., Gupta, K. C. and Goyal, S. P.: The H-function of one and two variables with applications, South Assian Publishers, New Delhi, 1982.