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# Composition of Soft Set Relations and Construction of Transitive Closure

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**Abstract**

In [3] concepts of soft set relations, partition, composition and function are discussed. In this paper, we present composition of relations in soft set context and give their matrix representation. Finally, the concepts of reflexive, symmetric and transitive closure are presented and show that construction of transitive closure in soft set satisfies Warshall's Algorithm.

## 1 Preliminaries and basic definitions

The origin of soft set theory could be traced to the work of Pawlak [6] in 1993 titled *Hard and Soft Set* in Proceeding of the International EWorkshop on rough sets and knowledge discovery at Banff. His notion of soft sets is a unified view of classical, rough and fuzzy sets. This motivated D. Molodtsov's work [2] in 1999 titled *soft set theory: first result*. Therein, the basic notions of the theory of soft sets and some of its possible applications were presented. For positive motivation, the work discusses some problems of the future with regards to the theory.

Let $U$ be a universal set and let $E$ be a set of parameters (each parameter could be a word or a sentence). Let $P(U)$ denotes the power set of $U$. In [2] and [5], a pair $(F, E)$ is called a *soft set* over a given universal set $U$, if and only if $F$ is a mapping of a set of parameters $E$, into the power set of $U$. That is, $F : E \longrightarrow P(U)$. Clearly, a soft set over $U$ is a parameterized family of subsets of a given universe $U$. Also, for any $e \in E$, $F(e)$ is considered as the set of $e -$ approximate element of the soft set $(F, E)$.

Example 1

Let $U = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}\}$ be the set of Cars under consideration, $E$ be a set of parameters.

$E = \{ e_1 =$ expensive, $e_2 =$ beautiful, $e_3 =$ manual gear, $e_4 =$ cheap, $e_5 =$ automatic gear, $e_6 =$ in good repair, $e_7 =$ in bad repair$\}$.

The soft set $(F, E)$ describes the attractiveness of the cars. For more examples, see [1], [2], [3], [4] and [5].

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**Definition 1:** A soft set $(F, E)$ over a universe $U$ is said to be *null* soft set denoted by $\tilde{\emptyset}$, if $\forall e \in E, F(e) = \emptyset$.

**Definition 2:** A soft set $(F, A)$ over a universe $U$ is called *absolute* soft set denoted by $\widehat{(F, A)}$, if $\forall e \in E$, $F(e) = U$.

 

**Definition 3:** Let $E = \{e_1, e_2, e_3, \ldots, e_n\}$ be a set of parameters. The not-set of $E$ denoted by $\neg E$ is defined as $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \ldots, \neg e_n\}$.

**Definition 4:** The complement of a soft set $(F, E)$ denoted by $(F, E)^c$ is defined as $(F, E)^c = (F^c, \neg E)$.

Where: $F^c : \neg E \rightarrow P(U)$ is a mapping given by $F^c(\alpha) = U - F(\neg\alpha), \forall \alpha \in \neg E$

$F^c$ is called the soft complement function of $F$.

Clearly, (i) $(F^c)^c = F$ and (ii) $((F, E)^c)^c = (F, E)$

**Definition 5:** For any two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$, we say that $(F, A)$ is a soft subset of $(G, B)$ if

(i) $A \subset B$, and

(ii) $\forall e \in A$, $F(e)$ and $G(e)$ are identical approximations.

We write $(F, A) \tilde{\subset} (G, B)$.

$(F, A)$ is said to be a soft super set of $(G, B)$, if $(G, B)$ is a subset of $(F, A)$ and it is denoted by $(F, A) \tilde{\supset} (G, B)$.

**Definition 6:** Two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$ are said to be soft equal if $(F, A)$ is a soft subset of $(G, B)$ and $(G, B)$ is a soft subset of $(F, A)$. It is denoted by $(F, A) = (G, B)$

**Definition 7:** If $(F, A)$ and $(G, B)$ are two soft sets then "$(F, A)\ AND\ (G, B)$" denoted by $(F, A) \wedge (G, B)$ is defined as $(F, A) \wedge (G, B) = (H, A \times B)$, where $(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall (\alpha, \beta) \in A \times B$.

**Definition 8:** If $(F, A)$ and $(G, B)$ are two soft sets then "$(F, A)\ OR\ (G, B)$" denoted by $(F, A) \vee (G, B)$ is defined by $(F, A) \vee (G, B) = (P, A \times B)$.

Where, $P(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall (\alpha, \beta) \in A \times B$

## 2 Soft set relations

Let $(F, A)$ and $(G, B)$ be two soft sets over $U$, then the Cartesian product of $(F, A)$ and $(G, B)$ is define as $(F, A) \times (G, B) = (H, A \times B)$, where $A \times B \rightarrow P(U \times U)$ and $H(a, b) = F(a) \times G(b)$ where $(a, b) \in A \times B$. i.e. $H(a, b) = \{(h_i, h_j)$; where $h_i \in F(a)$ and $h_j \in G(b)\}$ [3].

The Cartesian product of three or more nonempty soft sets can be defined by generalizing the definition of the Cartesian product of two soft sets. The Cartesian $(F_1, A) \times (F_2, A) \times \ldots \times (F_n, A)$ of the nonempty soft sets $(F_1, A), (F_2, A), \ldots, (F_n, A)$ is the soft sets of all ordered n-tuples $(h_1, h_2, \ldots, h_n)$ where $h_i \in F_i(a)$.

Example 2

Let soft sets $(F, A)$ and $(G, B)$ describe the "cost of the houses" and "attractiveness of houses" respectively.

Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}\}$

$A = \{Very\ costly, Costly, Cheap\}$

$B = \{Beautiful, in\ the\ green\ surrounding, Cheap\}$.

Let $a_1$, $a_2$, $a_3$, $b_1$ and $b_2$ respectively stand for very costly, costly, cheap, beautiful and in the green surrounding.

Suppose $(F, A)$ and $(G, B)$ defined as following:

$F(a_1) = \{h_2, h_4, h_7, h_8\}$
$F(a_2) = \{h_1, h_3, h_5\}$
$F(a_3) = \{h_1, h_6, h_9\}$

$G(b_1) = \{h_2, h_3, h_7\}$
$G(b_2) = \{h_5, h_6, h_8\}$
Now, $(F, A) \times (G, B) = (H, A \times B)$ where a typical element will look like

$H(a_1, b_1) = \{h_2, h_4, h_7, h_8\} \times \{h_2, h_3, h_7\}$

$= (h_2, h_2), (h_2, h_3), (h_2, h_7), (h_4, h_2), (h_4, h_3), (h_4, h_7), (h_7, h_2), (h_7, h_3), (h_7, h_7), (h_8, h_2),$

$(h_8, h_3), (h_8, h_7)$.

Relation in soft set is defined in terms of ordered pairs.

**Definition 9:** Let $(F, A)$ and $(G, B)$ be two soft sets over $U$, then a relation from $(F, A)$ to $(G, B)$ is a soft subset of $(F, A) \times (G, B)$. A relation from $(F, A)$ to $(G, B)$ is of the form $(H_1, S)$ where $S \subset A \times B$ and $H_1(a, b) = H(a, b)\ \forall\, a, b \in S$. Any subset of $(F, A) \times (F, A)$ is called a relation on $(F, A)$. In an equivalent way, we can define the relation $R$ on the soft set $(F, A)$ in the parameterized form as follows.

If $(F, A) = \{F(a), F(b), \ldots\}$, then $F(a) R\, F(b)$ if and only if $F(a) \times F(b) \in R$.

**Definition 10:** Let $R$ be a soft set relation from $(F, A)$ to $(G, B)$, then the domain of $R$ denoted as $dom\ R$, is defined as the soft set $(D, A_1)$ where

$A_1 = \{a \in A: H(a, b) \in R \text{ for some } b \in B\}$ and $D(a_1) = F(a_1), \forall a_1 \in A$.

The range of $R$ denoted by $ran\ R$, is defined as the soft set $(T, B_1)$, where $B_1 \subset B$ and $B_1 = \{b \epsilon B: H(a, b) \in R \text{ for some } a \in A\}$ and $T(b_1) = G(b_1) \forall b_1 \in B$, where $ran\ R = T$.

**Definition 11**: Let $(F, A)$ be any soft set. Then $(F, A) \times (F, A)$ and $\emptyset$ are soft subsets of $(F, A) \times (F, A)$ and hence are relation on $(F, A)$ called *universal* relation and *empty* relation respectively. Thus for any relation $R$ on $(F, A)$ we have $\emptyset \subseteq R \subseteq (F, A) \times (F, A)$

**Definition 12**: The identity relation $R$ on any soft set $(F, A)$ is defined as follows $F(a) R F(b)$ iff $F(a) = F(b)$.

 

## 3 Composition of soft set relation

**Definition 13**: Let $(F,A),(G,B)$ and $(H,C)$ be three soft sets. Let $R$ be a soft set relation from $(F,A)$ to $(G,B)$ and $S$ be another soft set relation from $(G,B)$ to $(H,C)$, then the composition of $R$ and $S$ is a new soft set relation from $(F,A)$ to $(H,C)$ expressed as $SoR$ and is defined as follows;

If $F(a)$ is in $(F,A)$ and $H(c)$ is in $(H,C)$ then $F(a)\ SoR\ H(c)$ iff there is some $G(b)$ in $(G,B)$ such that $F(a)\ R\ G(b)$ and $G(b)\ R\ H(c)$.

Example 3

Let $A=\{a_1,a_2,a_3\}$, $B=\{b_1,b_2\}$, $C=\{c_1,c_2\}$ and $U=\{h_1,h_2,h_3,h_4,h_5\}$. Let $R$ and $S$ be soft set relation defined respectively from $(F,A)$ to $(G,B)$ and $(G,B)$ to $(H,C)$ as

$R=\{F(a_1)\times G(b_1),F(a_2)\times G(b_2),F(a_3)\times G(b_2)\}$ and

$S=\{G(b_1)\times H(c_1),G(b_2)\times H(c_2)\}$

Then $SoR=\{F(a_1)\times H(c_1),F(a_2)\times H(c_2),F(a_3)\times H(c_2)\}$. In general $SoR\neq RoS$.

### 3.2 Types of soft set relation

**Definition 14:** Let $R$ be a relation on $(F,A)$, then

(i) $R$ is reflexive if $H_1(a,a)\in R,\forall a\in A$.

(ii) $R$ is symmetric if $H_1(a,b)\in R\leftrightarrow H_1(b,a)\in R,\forall(a,b)\in A\times A$.

(iii) $R$ is anti-symmetric if whenever $H_1(a,b)\in R$ and $H_1(b,a)\in R$ then $a=b,\quad \forall a,b\in A\times A$

(iv) $R$ is transitive if $H_1(a,b)\in R,H_1(b,c)\in R\rightarrow H_1(a,c)\in R,\forall a,b,c\in A$.

(v) $R$ is an equivalence relation if it is reflexive, symmetry and transitive.

**Definition 15**: The inverse of a soft set relation $R$ denoted by $R^{-1}$ is defined by $R^{-1}=\{F(b)\times F(a): F(a)RF(b)$

**Theorem** Let $R$ be soft set relation from $(F,A)$ to $(G,B)$ and $S$ be a soft set relation from $(G,B)$ to $(H,C)$. Then $(SoR)^{-1}=R^{-1}oS^{-1}$.

**Proof:** See [3]

## 4. Composition of soft set relation using matrices

In this section, we give matrix representation of composition of soft set relations. If the resultant matrices from the relations are compatible, the representation is straightforward otherwise an adjustment is made to the matrices before the composition.

Example 3

Let $A=\{a_1,a_2,a_3\}$, $B=\{b_1,b_2,b_3\}$, $C=\{c_1,c_2\}$ and $U=\{h_1,h_2,h_3,h_4,h_5\}$. Let $R$ and $S$ be soft set relation defined respectively from $(F,A)$ to $(G,B)$ and $(G,B)$ to $(H,C)$ as

$R=\{F(a_1)\times G(b_1), F(a_1)\times G(b_3), F(a_2)\times G(b_1), F(a_2)\times G(b_2)\}$ and

$S=\{G(b_1)\times H(c_1), G(b_2)\times H(c_2), G(b_3)\times H(c_2)\}$

By definition 13, $SoR=\{F(a_1)\times H(c_1), F(a_1)\times H(c_2), F(a_2)\times H(c_1), F(a_2)\times H(c_2)\}$

The matrices representation of $R$, $S$ and $SoR$ respectively are,

$$M_R=\begin{array}{c} \\ a_1\\ a_2\\ a_3\end{array}\begin{array}{c} b_1\quad b_2\quad b_3\\ \begin{bmatrix}1&0&1\\1&1&0\\0&0&0\end{bmatrix}\end{array},\qquad M_S=\begin{array}{c} \\ b_1\\ b_2\\ b_3\end{array}\begin{array}{c} c_1\quad c_2\\ \begin{bmatrix}1&0\\0&1\\0&1\end{bmatrix}\end{array}\quad\text{and}\quad SoR=\begin{array}{c} \\ a_1\\ a_2\\ a_3\end{array}\begin{array}{c} c_1\quad c_2\\ \begin{bmatrix}1&1\\1&1\\0&0\end{bmatrix}\end{array}$$

Multiplying the two matrices, $M_R$ and $M_S$ we obtain the matrix $M=M_RM_S=\begin{array}{c} \\ a_1\\ a_2\\ a_3\end{array}\begin{array}{c} c_1\quad c_2\\ \begin{bmatrix}1&1\\1&1\\0&0\end{bmatrix}\end{array}$

$M = SoR$

The non-zero entries in these matrices indicate the elements that are related. Thus, $M=M_RM_S$ and $RoS$ have the same nonzero entries.

Hence,
$SoR=M=M_RM_S=\{F(a_1)\times H(c_1), F(a_1)\times H(c_2), F(a_2)\times H(c_1), F(a_2)\times H(c_2)\}$.

Example 4

Let $A=\{a_1,a_2,a_3\}$, $B=\{b_1,b_2,b_3\}$, $C=\{c_1,c_2\}$ and $U=\{h_1,h_2,h_3,h_4,h_5\}$. Let $R$ and $S$ be soft set relation defined respectively from $(F,A)$ to $(G,B)$ and $(G,B)$ to $(H,C)$ as

$R=\{F(a_1)\times G(b_1), F(a_2)\times G(b_2), F(a_3)\times G(b_1)\}$ and

$S=\{G(b_1)\times H(c_2), G(b_2)\times H(c_1), G(b_3)\times H(c_1)\}$

By definition 13, $SoR=\{F(a_1)\times H(c_2), F(a_2)\times H(c_1), F(a_3)\times H(c_2)\}\ldots\{1\}$

Matrix representation of $R$ and $S$ respectively are,

$b_1\quad b_2\qquad\qquad c_1\quad c_2$

$$M_R = \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad M_S = \begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

Clearly, $M_R$ is of order $3 \times 2$ and $M_S$ is of order $3 \times 2$ which are not compatible to multiplication. Therefore, an adjustment is made to make the two compatible by creating a third column fill with zeros in $M_R$.

Let $M_R = \begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline a_1 & 1 & 0 & 0 \\ a_2 & 0 & 1 & 0 \\ a_3 & 1 & 0 & 0 \end{array}$ and $M_S = \begin{array}{c|cc} & c_1 & c_2 \\ \hline b_1 & 0 & 1 \\ b_2 & 1 & 0 \\ b_3 & 1 & 0 \end{array}$

$M_R$ is now of order $3 \times 3$ while the order of $M_S$ remain unchanged, thus compatible to matrix multiplication. Therefore,

Let $M_R . M_S = \begin{array}{c|ccc} & c_1 & c_2 & c_3 \\ \hline a_1 & 1 & 0 & 0 \\ a_2 & 0 & 1 & 0 \\ a_3 & 1 & 0 & 0 \end{array} \begin{array}{c|cc} & c_1 & c_2 \\ \hline b_1 & 0 & 1 \\ b_2 & 1 & 0 \\ b_3 & 1 & 0 \end{array} = \begin{array}{c|cc} & c_1 & c_2 \\ \hline a_1 & 0 & 1 \\ a_2 & 1 & 0 \\ a_3 & 0 & 1 \end{array}$

From the matrices result, we have

$SoR = \{F(a_1) \times H(c_2), F(a_2) \times H(c_1), F(a_3) \times H(c_2)\}$ …{2}

Since {1} and {2} yield the same result, hence the adjustment is valid.

Remark 1: Composition of soft set relations is not commutative in general just like matrix multiplication.

## 5 Closure of a relation

Suppose that $R$ is a relation on a soft set $(F, A)$, $R$ may or may not have some property $P$, such as reflexivity, symmetry, or transitivity. If there is a relation $S$ with property $P$ containing $R$ such that $S$ is a sub soft set of every relation with property $P$ containing $R$, then $S$ is called the closure of $R$ with respect to $P$. The closure of a relation with respect to a property may or may not exist.

### 5.1 Construction of closures of a relation

**Reflexive Closure**

Reflexive closure of $R$ can be formed by adding to $R$ all pairs of the form $(F(a), F(a))$ with $F(a) \in (F, A)$, not already in $R$. The addition of these pairs produces a new relation that is reflexive, contains $R$, and is contained within any reflexive relation containing $R$.

 

**Definition 17:** The reflexive closure of $R$ equals $R \cup \Delta$, where $\Delta = \{(F(a), F(a)): F(a) \in (F, A)\}$ is the diagonal relation on $(F, A)$.

Example 5

The relation $R = \{F(e_1) \times F(e_1),\ F(a_1) \times F(a_2),\ F(a_2) \times F(a_1),\ F(a_3) \times F(a_2)$ on the soft set $(F, A)$ with $A = \{e_1, e_2, e_3\}$ is not reflexive. To make this reflexive relation containing R is simply done by adding $\{F(e_2) \times F(e_2)\}$ and $\{F(e_3) \times F(e_3)\}$ to $R$, since these are the only pairs of the form $(F(a), F(a))$ that are not in $R$. Clearly, this new relation contains R. Furthermore, any reflexive relation that contains $R$ must also contains $\{F(e_2) \times F(e_2)\}$ and $\{F(e_3) \times F(e_3)\}$. Because this relation contains $R$, is reflexive, and is contained with every reflexive relation that contain $R$, it is called the reflexive closure of $R$.

**Symmetry Closure**

The symmetry closure of a relation $R$ is constructed by adding all ordered pairs of the form $(F(b),\ F(a))$, where $(F(a),\ F(b))$ is in the relation, that are not already present in $R$. Adding these pairs produces a relation that is symmetric relation that contains $R$.

**Definition 18:** The symmetric closure of a relation is obtained by taking the union of relation with its inverse i.e. $R \cup R^{-1}$ where $R^{-1} = \{(F(b),\ F(a)): (F(a),\ F(b) \in R\}$.

Example 6

Let R be a relation such that

$R = \{F(e_1) \times F(e_1),\ F(a_1) \times F(a_2), F(a_2) \times F(a_2)\ F(a_2) \times F(a_3),\ F(a_3) \times F(a_1),\ F(a_3) \times F(a_2)\}$

on the soft set $(F, A)$ with $A = \{e_1, e_2, e_3\}$ is not symmetric. To make $R$ symmetric, we need to add $F(a_2) \times F(a_1)$ and $F(a_1) \times F(a_3)$, since these are the only pair of the form $(F(b), F(a))$ with $(F(a), F(b)) \epsilon R$ that are not in $R$. These new relation is symmetric and contains $R$. Furthermore, any symmetric relation that contains $R$ must contain this new relations, since a symmetric relations that contains $R$ must contain $F(a_2) \times F(a_1)$ and $F(a_1) \times F(a_3)$. Consequently, this new relation is the symmetric closure of $R$.

**Transitive closure**

The construction of transitive closure of a relation is complicated than that of reflexive or symmetric closure. The transitive closure of a relation can be found by adding new ordered pairs that must be present and then repeating this process until no new ordered pairs are needed.

$R^*$ is said to be transitive closure of $R$ if it satisfies the following conditions (i) $R^*$ is transitive (ii) $R \subseteq R^*$ (iii) $R^*$ is the smallest transitive relation containing $R$.

**Definition 19**: Let $R$ be a relation on a soft set $(F, A)$. We define $R^* = \bigcup_{i=1}^{\infty} R$

 

**Properties of closures**

Let $R$ be a relation on a soft set $(F, A)$ with $n$ elements. Then

(i) transitive$(R) = R \cup R^2 \cup \ldots \cup R^n$,

(ii) $M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee \ldots \vee M_R^{[n]}$, where $M_R$ is the zero-one matrix of the relation $R$ (iii) $M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2}$ and $M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2}$, where $R_1$ and $R_2$ are relations on $(F, A)$ with zero-one matrices $M_{R_1}$ and $M_{R_2}$.

Example 7

Suppose $R$ is a relation on $(F, A)$ with $A = \{e_1, e_2, e_3\}$ , where $R = \{F(e_1) \times F(e_2), F(e_2) \times F(e_3), F(e_3) \times F(e_3)\}$, the zero-one matrix for $R$ is given by

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus $M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]}$ since $n = 3$.

Now $R^2 = M_{R^2} = M_R . M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$R^3 = R^2 . R = M_{R^2} . M_R = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{R^*} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Reading from the zero-one matrix, we see that

$R^* = \{F(e_1) \times F(e_2), F(e_1) \times F(e_3), F(e_2) \times F(e_3), F(e_3) \times F(e_3)\}$ is the $transitive(R)$.

**Warshall's Algorithm**

Warshall's algorithm is an efficient method for computing the transitive closure of a relation. The following illustrate the algorithm for computing the transitive closure of a relation.

Step I: let $R$ be a given relation

Calculate $R_1 = R \cup (RoR)$

 

If $R_1 = R$, then stop $\Longrightarrow R$ transitive

Step II:let $R_1 \neq R$

Calculate $R_2 = R_1 \cup (R_1 o R_1)$

If $R_2 = R_1$, stop $\Longrightarrow R_1$ is the transitive closure of $R$.

...

In general,

Calculate $R_{i+1} = R_i \cup (R_i o R_i)$

If $R_{i+1} = R_i$ stop, $\Longrightarrow R_i$ is the transitive closure of $R_{i+1}$

Continue otherwise.

This algorithms holds for constructing transitive closure of a soft set relation using zero-one matrix. For example, we use this algorithm on example 7, where $R = \{F(e_1) \times F(e_2), F(e_2) \times F(e_3), F(e_3) \times F(e_3)\}$.

The zero-one matrix for $R$ is $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$M_R . M_R = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{R_1} = M_{R_1} \cup (M_R o M_R) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Longrightarrow M_{R_1} \neq M_R$, hence we calculate $M_{R_2}$.

$$M_{R_2} = M_{R_1} \cup (M_{R_1} o M_{R_1}) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Longrightarrow M_{R_2} = M_{R_1}$

Therefore, $M_{R_1}$ is the transitive closure of $M_R$ which is the same as in example 7.

## 6 Conclusions

Soft set has potential applications in several directions, such as smoothness of functions, game theory, operations research, Riemann-integral, measurement theory[1] and [2]. Transitive closure is applied in modeling, networking and in operations research hence developing transitive closure in soft set context is a fruitful exercise.

## Reference

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