EVENT,CAUSE, SPACE-TIMEAND QUANTUM MEMORY REGISTER-
AN AUGMENTATION-ARRONDISSEMENT MODEL

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ABSTRACT: *We study a consolidated system of event; cause and n Qubit register which makes computation with n Qubits. Model extensively dilates upon systemic properties and analyses the systemic behaviour of the equations together with other concomitant properties. Inclusion of event and cause ,we feel enhances the "Quantum ness" of the system holistically and brings out a relevance in the Quantum Computation on par with the classical system, in so far as the analysis is concerned. Additional VARIABLES OF Space Time provide bastion for the quantum space time studies.*

INTRODUCTION:

EVENT AND ITS VINDICATION:

There definitely is a sense of compunction, contrition, hesitation, regret, remorse, hesitation and reservation to the **acknowledgement of** the fact that there is a personal **relation t**o what **happens to** oneself. Louis de Broglie said that the events have already happened and it **shall disclose** to the people **based on** their level of consciousness. So there is destiny to start with! Say I am undergoing some seemingly insurmountable problem, which has hurt my sensibilities, susceptibilities and sentimentalities that I refuse to accept that that event was waiting for me to happen. In fact this is the statement of stoic philosophy which is referred to almost as bookish or abstract. Wound is there; it **had to happen** to me. So I was wounded. Stoics tell us that the wound **existed b**efore me; I was born **to embody** it. It is the question of consummation, consolidation, concretization, consubstantiation, that of this, that **creates an** "event" in us; thus you have **become a quasi cause for** this wound. For instance, my feeling to **become an** actor made me to behave with such perfectionism everywhere, that people's expectations rose and when I did not come up to them I **fell;** thus the 'wound' was waiting for me and "I' was waiting for the wound! One fellow professor used to say like you are searching for ides, ideas also searching for you. Thus the wound **possesses in itself** a nature which is "impersonal and preindividual" in character, beyond general and particular, the collective and the private. It is the question **of becoming** universalistic and holistic in your outlook. Unless this fate had not befallen you, the "**grand design**" would not have taken place in its entire entirety. It had to happen. And the concomitant ramifications and pernicious or positive **implications.** Everything is in order **because the** fate befell you. It is not as if the wound had to get something that is best from me or that I am a chosen by God to face the event. As said earlier 'the grand design" would have **been altered**. And **it cannot alter**. You got to play your part and go; there is just no other way. The legacy must go on. You shall be torch bearer and you shall hand over the torch to somebody. This is the name of the game in totalistic and holistic way.

When it comes to ethics, I would say it makes no sense if any obstreperous, obstreperous, ululations, serenading, tintinnabulations are made for the event has happened to me. It means to say that you are unworthy of the fate that has befallen you. To feel that what happened to you was unwarranted and not autonomous, telling the world that you are aggressively iconoclastic, veritably resentful, and volitionally resentient, is choosing the cast of allegation aspersions and accusations at the Grand Design. What is immoral is to invoke the name of god, because some event has **happened to** you. Cursing him is immoral. Realize that it is all "grand design" and you are playing **a part.** Resignation, renunciation, revocation is only one form of resentience. Willing the event is primarily **to release** the eternal truth; in fact you cannot release an event despite the fact everyone tries all ways and means they pray god; they prostrate for others destitution, poverty, penury, misery. But **releasing a**n event is something like an "action at a distance" which only super natural power can do.

Here we are face to face with volitional intuition and repetitive transmutation. Like a premeditated skirmisher, **one quarrel** with one self, with others, with god, and finally the accuser **leaves** this world in despair. Now look at this sentence which was quoted by I think Bousquet "if there is a **failure of** will", "I will **substitute a** longing for death" for that shall be apotheosis, a perpetual and progressive glorification of the will.

EVENT AND SINGULARITIES IN QUANTUM SYSTEMS:

What is an event? Or for that matter an ideal event? An event **is a** singularity or rather a set of singularities or set of singular points **characterizing a** mathematical curve, a physical state of affairs, a psychological person or a moral person. Singularities are turning points and points of inflection: they are bottle necks, foyers and centers; they are points of fusion; condensation and boiling; points of tears and joy; sickness and health; hope and anxiety; they are so to say “sensitive" points; such singularities should not be confused or confounded, aggravated or exacerbated with personality of a system expressing itself; or the individuality and idiosyncrasies of a system which is designated with a proposition. They should also **not be fused** with the generalizational concept or universalistic axiomatic predications and postulation alcovishness, or the dipsomaniac flageolet dirge of a concept. Possible a concept could be signified by a figurative representation or a schematic configuration. "Singularity is essentially, pre individual, and has no personalized bias in it, or for that matter a prejudice or pre circumspection of a conceptual scheme. It is in this sense **we can define a** "singularity" as being neither affirmative nor non affirmative. It can be positive or negative; it can **create or destroy**. On the other hand it must be noted that singularity is different both in its thematic discursive from the run of the mill day to day musings and mundane drooling. They are in that sense "extra-ordinary".

Each singularity is a **source and resource**, the origin, reason and raison d’être of a mathematical series, it could be any series any type, and that is interpolated or extrapolated to the structural location of the **destination of** another singularity. This according to this standpoint, there are different. It can be positive or negative; it can create or destroy. On the other hand it must be noted that singularity is different both in its thematic discursive from the run of the mill day to day musings and mundane drooling. There are in that sense "extra-ordinary".

This according to the widely held standpoint, there are different, multifarious, myriad, series IN A structure. In the eventuality of the fact that we conduct an unbiased and prudent examination of the series belonging to different "singularities" we can come to indubitable **conclusions t**hat the "singularity" of one system is different from the "other system" in the subterranean realm and ceratoid dualism of comparison and contrast

EPR experiment derived that there exists a communications between two particles. We go a further step to say that there **exists a channel** of communication however slovenly, inept, clumpy, between the two singularities. It is also possible the communication exchange could be one of belligerence, cantankerousness, tempestuousness, astutely truculent, with ensorcelled frenzy. That does not matter. All we are telling is that singularities communicate with each other.

Now, how do find **the reaction** of systems to these singularities. You do the same thing a boss does for you. "Problematize" the events and see how you behave. I will resort to "pressure tactics”. “intimidation of deriding report", or “cut in the increment" to make you undergo trials, travails and tribulations. I am happy to see if you improve your work; but may or may not be sad if you succumb to it and hang yourself! We do the same thing with systems. systems show conducive response, felicitous reciprocation or behave erratically with inner roil, eponymous radicalism without and with blitzy conviction say like a solipsist nature of bellicose and blustering particles, or for that matter coruscation, trepidiational **motion in** fluid flows, or seemingly perfidious incendiaries in gormandizing fellow elementary particles, abnormal ebullitions, surcharges calumniations and unwarranted(you think so but the system does not!) unrighteous fulminations.

So the point that is made here is “like we problematize the "events" to understand the human behaviour we have to "problematize" the events of systems to understand their behaviour.

This statement is made in connection to the fact that there shall be **creation or destruction** of particles or complete obliteration of the system (blackhole evaporation) or obfuscation of results. Some systems are like “inside traders" they will not put signature at all! How do you find they did it! Anyway, there are possibilities of a CIA finding out as they recently did! So we can do the same thing with systems to. This is accentuation, corroboration, fortification, .fomentatory notes to explain the various coefficients we have used in the model as also the dissipations called for

In the Bank example we have clarified that various systems are individually conservative, and their conservativeness extends holisticallytoo.that one law is universal does not mean there is complete adjudication of **nonexistence of** totality or global or holistic figure. Total always exists and “individual” systems always exist, if we do not bring Kant in to picture! For the time being let us not! Equations would become more eneuretic and frenzied...

Various, myriad, series in a structure. In the eventuality of the fact that we conduct an unbiased and prudent examination of the series belonging to different "singularities" we can come to indubitable conclusions that the "singularity" of one system is different from the "other system" in the subterranean realm and ceratoid dualism of comparison and contrast.

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CONSERVATION LAWS:

Conservation laws bears ample testimony ,infallible observatory, and impeccable demonstration to the fact that the essential predications, character constitutions, ontological consonances remain unchanged with evolution despite the system’s astute truculence, serenading whimsicality,assymetric disposition or on the other hand anachronistic dispensation ,eponymous radicality,entropic entrepotishness or the subdued ,relationally contributive, diverse parametrisizational,conducive reciprocity to environment, unconventional behaviour,eneuretic nonlinear frenetic ness ,ensorcelled frenzy, abnormal ebulliations,surcharged fulminations , or the inner roil. And that holds well with the evolution with time. We present a model of the generalizational conservation of the theories. A theory of all the conservation theories. That all conservation laws hold and there is no relationship between them is bête noir. We shall on this premise build a 36 storey model that deliberates on various issues, structural, dependent, thematic and discursive,

Note THAT The classification is executed on systemic properties and parameters. And everything that is known to us measurable. We do not know”intangible”.Nor we accept or acknowledge that. All laws of conservation must holds. Hence the holistic laws must hold. Towards that end, interrelationships must exist. All science like law wants evidence and here we shall provide one under the premise that for all conservations laws to hold each must be interrelated to the other, lest the very conception is a fricative contretemps. And we live in “Measurement” world.

QUANTUM REGISTER:

Devices that **harness and explore** the fundamental axiomatic predications of Physics has wide ranging amplitidunial **ramification** with its essence of locus and focus on information processing that outperforms their classical counterparts, and for unconditionally secure communication. However, in particular, implementations **based on** condensed-matter systems face the challenge of short coherence times. Carbon materials, particularly diamond, however, are suitable **for hosting** robust solid-state quantum registers**, owing to** their spin-free lattice and weak spin–orbit **coupling**. Studies with the structurally notched criticism and schizoid fragments of manifestations of historical perspective of diamond hosting quantum register have borne ample testimony and, and at differential and determinate levels have articulated the generalized significations and manifestations of quantum logic elements can **be realized** by exploring long-range magnetic dipolar coupling between individually addressable single electron spins **associated with** separate colour centres in diamond. The strong distance dependence of this coupling was used to characterize the separation of single qubits (98±3 Å) with accuracy close to the value of the crystal-lattice spacing. Coherent **control over** electron spins, **conditional** dynamics, selective readout as well as switchable **interaction** should rip open glittering façade for a prosperous and scintillating irreducible affirmation of open the way towards a viable room-temperature solid-state quantum register. As both electron spins are optically **addressable,** this solid-state quantum device **operating at** ambient conditions **provides a** degree of **control** that is at present available only for a few systems at low temperature (See for instance P. Neumann, R. Kolesov, B. Naydenov, J. Bec F. Rempp, M. Steiner’ V. Jacques,, G. Balasubramanian,M, M. L. Markham,, D. J. Twitchen,, S. Pezzagna,, J. Meijer, J. Twamley, F. Jelezko & J. Wrachtrup)

CAUSE AND EVENT:

MODULE NUMBERED ONE

NOTATION :

G_{13} : CATEGORY ONE OF CAUSE

G_{14} : CATEGORY TWO OF CAUSE

G_{15} : CATEGORY THREE OF CAUSE

T_{13} : CATEGORY ONE OF EVENT

T_{14} : CATEGORY TWO OF EVENT

T_{15} :CATEGORY THREE OFEVENT

FIRST TWO CATEGORIES OF QUBITS COMPUTATION:

MODULE NUMBERED TWO:

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G_{16} : CATEGORY ONE OF FIRST SET OF QUBITS

G_{17} : CATEGORY TWO OF FIRST SET OF QUBITS

G_{18} : CATEGORY THREE OF FIRST SET OF QUBITS

T_{16} :CATEGORY ONE OF SECOND SET OF QUBITS

T_{17} : CATEGORY TWO OF SECOND SET OF QUBITS

T_{18} : CATEGORY THREE OF SECOND SET OF QUBITS

THIRD SET OF QUBITS AND FOURTH SET OF QUBITS:

MODULE NUMBERED THREE:

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G_{20} : CATEGORY ONE OF THIRD SET OF QUBITS

G_{21} :CATEGORY TWO OF THIRD SET OF QUBITS

G_{22} : CATEGORY THREE OF THIRD SET OF QUBITS

T_{20} : CATEGORY ONE OF FOURTH SET OF QUBITS

T_{21} :CATEGORY TWO OF FOURTH SET OF QUBITS

T_{22} : CATEGORY THREE OF FOURTH SET OF QUBITS

FIFTH SET OF QUBITS AND SIXTH SET OF QUBITS

: MODULE NUMBERED FOUR:

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G_{24} : CATEGORY ONE OF FIFTH SET OF QUBITS

G_{25} : CATEGORY TWO OF FIFTH SET OF QUBITS

G_{26} : CATEGORY THREE OF FIFTH SET OF QUBITS

T_{24} :CATEGORY ONE OF SIXTH SET OF QUBITS

T_{25} :CATEGORY TWO OF SIXTH SET OF QUBITS

T_{26} : CATEGORY THREE OF SIXTH SET OF QUBITS

SEVENTH SET OF QUBITS AND EIGHTH SET OF QUBITS:

MODULE NUMBERED FIVE:

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G_{28} : CATEGORY ONE OF SEVENTH SET OF QUBITS

G_{29} : CATEGORY TWO OFSEVENTH SET OF QUBITS

G_{30} :CATEGORY THREE OF SEVENTH SET OF QUBITS

T_{28} :CATEGORY ONE OF EIGHTH SET OF QUBITS

T_{29} :CATEGORY TWO OF EIGHTH SET OF QUBITS

T_{30} :CATEGORY THREE OF EIGHTH SET OF QUBITS

(n-1)TH SET OF QUBITS AND nTH SET OF QUBITS :

MODULE NUMBERED SIX:

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G_{32} : CATEGORY ONE OF(n-1)TH SET OF QUBITS

G_{33} : CATEGORY TWO OF(n-1)TH SET OF QUBITS

G_{34} : CATEGORY THREE OF (N-1)TH SET OF QUBITS

T_{32} : CATEGORY ONE OF n TH SET OF QUBITS

T_{33} : CATEGORY TWO OF n TH SET OF QUBITS

T_{34} : CATEGORY THREE OF n TH SET OF QUBITS

GLOSSARY OF MODULE NUMBERED SEVEN

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G_{36} : CATEGORY ONE OF TIME

G_{37} : CATEGORY TWO OF TIME

G_{38} : CATEGORY THREE OF TIME

T_{36} : CATEGORY ONE OF SPACE

T_{37} : CATEGORY TWO OF SPACE

T_{38} : CATEGORY THREE OF SPACE

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$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$
$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}: (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}$,
$(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$,
$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$, $(a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$
$(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$, $(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

are Dissipation coefficients

CAUSE AND EVENT: 1

MODULE NUMBERED ONE

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13}$$ 2

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14}$$ 3

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15}$$ 4

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13}$$ 5

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14}$$ 6

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15}$$ 7

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor 8

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

FIRST TWO CATEGORIES OF QUBITS COMPUTATION: 9

MODULE NUMBERED TWO:

The differential system of this model is now (Module numbered two)

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16}$ 10

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17}$ 11

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18}$ 12

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16}$ 13

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17}$ 14

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18}$ 15

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor 16

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor 17

THIRD SET OF QUBITS AND FOURTH SET OF QUBITS: 18

MODULE NUMBERED THREE

The differential system of this model is now (Module numbered three)

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20}$ 19

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21}$ 20

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22}$ 21

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20}$ 22

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21}$ 23

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22}$ 24

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor 25

FIFTH SET OF QUBITS AND SIXTH SET OF QUBITS : MODULE NUMBERED FOUR 26

The differential system of this model is now (Module numbered Four)

$\frac{dG_{24}}{dt} = (a_{24})^{(4)} G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) \right] G_{24}$ 27

$\frac{dG_{25}}{dt} = (a_{25})^{(4)} G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t) \right] G_{25}$ 28

$\frac{dG_{26}}{dt} = (a_{26})^{(4)} G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t) \right] G_{26}$ 29

$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) \right] T_{24}$ 30

$\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27}), t\big) \right] T_{25}$ 31

$\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27}), t\big) \right] T_{26}$ 32

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor** 33

$-(b''_{24})^{(4)}\big((G_{27}), t\big) =$ **First detritions factor** 34

SEVENTH SET OF QUBITS AND EIGHTH SET OF QUBITS: 35

MODULE NUMBERED FIVE

The differential system of this model is now (Module number five)

$\frac{dG_{28}}{dt} = (a_{28})^{(5)} G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right] G_{28}$ 36

$\frac{dG_{29}}{dt} = (a_{29})^{(5)} G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t) \right] G_{29}$ 37

$\frac{dG_{30}}{dt} = (a_{30})^{(5)} G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t) \right] G_{30}$ 38

$\frac{dT_{28}}{dt} = (b_{28})^{(5)} T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) \right] T_{28}$ 39

$\frac{dT_{29}}{dt} = (b_{29})^{(5)} T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\big((G_{31}), t\big) \right] T_{29}$ 40

$\frac{dT_{30}}{dt} = (b_{30})^{(5)} T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\big((G_{31}), t\big) \right] T_{30}$ 41

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor** 42

$-(b''_{28})^{(5)}\big((G_{31}), t\big) =$ **First detritions factor** 43

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n-1)TH SET OF QUBITS AND nTH SET OF QUBITS : 45

MODULE NUMBERED SIX:

The differential system of this model is now (Module numbered Six)

$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)} (T_{33}, t) \right] G_{32}$ 46

$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}, t) \right] G_{33}$ 47

$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}, t) \right] G_{34}$ 48

$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)} \left((G_{35}), t \right) \right] T_{32}$ 49

$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)} \left((G_{35}), t \right) \right] T_{33}$ 50

$\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)} \left((G_{35}), t \right) \right] T_{34}$ 51

$+(a''_{32})^{(6)} (T_{33}, t) =$ **First augmentation factor** 52

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GOVERNING EQUATIONS:

The differential system of this model is now (SEVENTH MODULE)

$\frac{dG_{36}}{dt} = (a_{36})^{(7)} G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)} (T_{37}, t) \right] G_{36}$ 54

$\frac{dG_{37}}{dt} = (a_{37})^{(7)} G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}, t) \right] G_{37}$ 55

$\frac{dG_{38}}{dt} = (a_{38})^{(7)} G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}, t) \right] G_{38}$ 56

$\frac{dT_{36}}{dt} = (b_{36})^{(7)} T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)} \left((G_{39}), t \right) \right] T_{36}$ 57

$\frac{dT_{37}}{dt} = (b_{37})^{(7)} T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)} \left((G_{39}), t \right) \right] T_{37}$ 58

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$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\big((G_{39}),t\big)\right]T_{38} \tag{60}$$

$$+(a''_{36})^{(7)}(T_{37},t) = \textbf{ First augmentation factor} \tag{61}$$

$$-(b''_{36})^{(7)}\big((G_{39}),t\big) = \textbf{ First detritions factor} \tag{62}$$

FIRST MODULE CONCATENATION:

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{matrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7)}(T_{37},t)} \end{matrix}\right]G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{matrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7)}(T_{37},t)} \end{matrix}\right]G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{matrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7)}(T_{37},t)} \end{matrix}\right]G_{15}$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{38})^{(7)}(T_{37},t)}$ ARESEVENTHAUGMENTATION COEFFICIENTS

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{matrix} (b'_{13})^{(1)}\boxed{-(b''_{16})^{(1)}(G,t)}\ \boxed{-(b''_{36})^{(7,)}(G_{39},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \\ \boxed{-(b''_{36})^{(7,)}(G_{39},t)} \end{matrix}\right]T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{matrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,)}(G_{39},t)} \end{matrix}\right]T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[\begin{matrix} (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \; \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,)}(G_{39},t)} \end{matrix} \right] T_{15}$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,)}(G_{39},t)}$ ARE SEVENTH DETRITION COEFFICIENTS

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{matrix} (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \end{matrix} \right] T_{15}$$ 63

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 64

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and 3

SECONDMODULE CONCATENATION: 65

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{matrix} (a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,)}(T_{37},t)} \end{matrix} \right] G_{16}$$ 66

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{matrix} (a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,)}(T_{37},t)} \end{matrix} \right] G_{17}$$ 67

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{matrix} (a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,)}(T_{37},t)} \end{matrix} \right] G_{18}$$ 68

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3 69

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3 70

$\boxed{+(a''_{36})^{(7,7,)}(T_{37},t)}\boxed{+(a''_{37})^{(7,7,)}(T_{37},t)}\boxed{+(a''_{38})^{(7,7,)}(T_{37},t)}$ARE SEVENTH DETRITION COEFFICIENTS 71

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[\begin{matrix}(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}\\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{36})^{(7,7)}(G_{39},t)}\end{matrix}\right]T_{16} \tag{72}$$

$$\frac{dT_{17}}{dt}=(b_{17})^{(2)}T_{16}-\left[\begin{matrix}(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}\\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{37})^{(7,7)}(G_{39},t)}\end{matrix}\right]T_{17} \tag{73}$$

$$\frac{dT_{18}}{dt}=(b_{18})^{(2)}T_{17}-\left[\begin{matrix}(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}\\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{38})^{(7,7)}(G_{39},t)}\end{matrix}\right]T_{18} \tag{74}$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3 75

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detritions coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detritions coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detritions coefficients for category 1,2 and 3

$$\boxed{-(b''_{36})^{(7,7)}(G_{39},t)}\boxed{-(b''_{36})^{(7,7)}(G_{39},t)}\boxed{-(b''_{36})^{(7,7)}(G_{39},t)}are\ seventh\ detrition\ coefficients$$

THIRD MODULE CONCATENATION:

$$\frac{dG_{20}}{dt}= \tag{76}$$

$$(a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7.7.7.)}(T_{37},t)} \end{bmatrix} G_{20}$$

77

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7.7.7.)}(T_{37},t)} \end{bmatrix} G_{21}$$

78

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7.7.7.)}(T_{37},t)} \end{bmatrix} G_{22}$$

79

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

80

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7.7.7.)}(T_{37},t)}\boxed{+(a''_{37})^{(7.7.7.)}(T_{37},t)}\boxed{+(a''_{38})^{(7.7.7.)}(T_{37},t)}$ are seventh augmentation coefficient 81

82

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{36})^{(7,7,7)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} \end{bmatrix} T_{20}$$

83

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} \end{bmatrix} T_{21}$$

84

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} \end{bmatrix} T_{22}$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detritions coefficients for category 1, 2 and 3* 85

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detritions coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detritions coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detritions coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detritions coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$are seventh detritions coefficients 86

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FOURTH MODULE CONCATENATION:

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\left[\begin{array}{c}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}\end{array}\right]G_{24} \tag{87}$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\left[\begin{array}{c}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,)}(T_{37},t)}\end{array}\right]G_{25} \tag{88}$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\left[\begin{array}{c}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,)}(T_{37},t)}\end{array}\right]G_{26} \tag{89}$$

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$Where\ \boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ $are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

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$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ $are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ $are\ third\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}\boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}\boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}$ARE SEVENTH augmentation coefficients

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$$\frac{dT_{24}}{dt} = (b_{24})^{(4)} T_{25} - \left[\begin{matrix} (b'_{24})^{(4)} \boxed{-(b''_{24})^{(4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,,,)}(G_{39},t)} \end{matrix} \right] T_{24}$$

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$$\frac{dT_{25}}{dt} = (b_{25})^{(4)} T_{24} - \left[\begin{matrix} (b'_{25})^{(4)} \boxed{-(b''_{25})^{(4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,77,,)}(G_{39},t)} \end{matrix} \right] T_{25}$$

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$$\frac{dT_{26}}{dt} = (b_{26})^{(4)} T_{25} - \left[\begin{matrix} (b'_{26})^{(4)} \boxed{-(b''_{26})^{(4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,,7,,)}(G_{39},t)} \end{matrix} \right] T_{26}$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3 96

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$ *are fourth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,,)}(G_{39},t)} \boxed{-(b''_{37})^{(7,7,7,7,7,,)}(G_{39},t)} \boxed{-(b''_{38})^{(7,7,7,7,7,,)}(G_{39},t)}$ *ARE* SEVENTH DETRITION

COEFFICIENTS

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FIFTH MODULE CONCATENATION: 98

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$$\frac{dG_{28}}{dt} = (a_{28})^{(5)} G_{29} - \left[\begin{matrix} (a'_{28})^{(5)} \boxed{+(a''_{28})^{(5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,,7,,7,7)}(T_{37},t)} \end{matrix} \right] G_{28}$$

100

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)} G_{28} - \left[\begin{matrix} (a'_{29})^{(5)} \boxed{+(a''_{29})^{(5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,,,7,,7,7)}(T_{37},t)} \end{matrix} \right] G_{29}$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,,7,,7,7)}(T_{37},t)} \end{array}\right] G_{30}$$ 101

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1,2$ *and* 3 102

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1,2$ *ar*

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1,2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

103

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{c} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,)}(G_{23},t)}\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,,7,7,7,)}(G_{38},t)} \end{array}\right] T_{28}$$ 104

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{c} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,)}(G_{38},t)} \end{array}\right] T_{29}$$ 105

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{c} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,)}(G_{38},t)} \end{array}\right] T_{30}$$ 106

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* $1,2$ *and* 3 107

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

SIXTH MODULE CONCATENATION 108

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,)}(T_{37},t)} \end{array}\right] G_{32} \quad 109$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,,7,7,7,)}(T_{37},t)} \end{array}\right] G_{33} \quad 110$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,)}(T_{37},t)} \end{array}\right] G_{34} \quad 111$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3 112

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *an*

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *an*

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,,77,7,,7,)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,,7,7,7,)}(T_{37},t)}$ ARE SVENTH AUGMENTATION COEFFICIENTS

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$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{c} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,,7,7,7)}(G_{39},t)} \end{array}\right] T_{32} \quad 114$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{array}{c} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ -(b''_{37})^{(7,7,7,,7,7,7)}(G_{39},t) \end{array}\right] T_{33} \quad 115$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{array}{c} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,,7,7,7)}(G_{39},t)} \end{array}\right] T_{34} \quad 116$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3 117

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ ARE SEVENTH DETRITION COEFFICIENTS

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SEVENTH MODULE CONCATENATION: *119*

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t) + (a''_{16})^{(7)}(T_{17},t) + (a''_{20})^{(7)}(T_{21},t) + (a''_{24})^{(7)}(T_{23},t)G_{36} + (a''_{28})^{(7)}(T_{29},t) + (a''_{32})^{(7)}(T_{33},t) + (a''_{13})^{(7)}(T_{14},t)\right] G_{36} \quad 120$$

121

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t) + (a''_{14})^{(7)}(T_{14},t) + (a''_{21})^{(7)}(T_{21},t) + (a''_{17})^{(7)}(T_{17},t) + (a''_{25})^{(7)}(T_{25},t) + (a''_{33})^{(7)}(T_{33},t) + (a''_{29})^{(7)}(T_{29},t)\right] G_{37} \quad 122$$

Type equation here.

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t) + (a''_{15})^{(7)}(T_{14},t) + (a''_{22})^{(7)}(T_{21},t) + (a''_{18})^{(7)}(T_{17},t) + (a''_{26})^{(7)}(T_{25},t) + (a''_{34})^{(7)}(T_{33},t) + (a''_{30})^{(7)}(T_{29},t)\right] G_{38} \quad 123$$

124

125

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t) - (b''_{16})^{(7)}((G_{17}),t) - (b''_{13})^{(7)}((G_{14}),t) - \right]T_{36} \quad 126$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 127$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 128$$

129

130

131

132

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor** 134

(A) $(a_i)^{(2)},(a'_i)^{(2)},(a''_i)^{(2)},(b_i)^{(2)},(b'_i)^{(2)},(b''_i)^{(2)} > 0, \quad i,j = 16,17,18$ 135

(B) The functions $(a''_i)^{(2)},(b''_i)^{(2)}$ are positive continuous increasing and bounded. 136

<u>Definition of</u> $(\mathrm{p_i})^{(2)}, \ (\mathrm{r_i})^{(2)}$: 137

$$(a''_i)^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16} \right)^{(2)} \tag{138}$$

$$(b''_i)^{(2)}(G_{19},t) \le \ (r_i)^{(2)} \le (b'_i)^{(2)} \le (\hat{B}_{16})^{(2)} \tag{139}$$

(C) $\lim_{T_2\to\infty}(a''_i)^{(2)}\,(T_{17},t) = (p_i)^{(2)}$ 140

$\lim_{G\to\infty}(b''_i)^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)}$ 141

<u>Definition of</u> $(\hat{A}_{16})^{(2)},(\hat{B}_{16})^{(2)}$: 142

Where $\boxed{(\hat{A}_{16})^{(2)},(\hat{B}_{16})^{(2)},(p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition: 143

$$|(a''_i)^{(2)}(T'_{17},t) - (a''_i)^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T'_{17}|e^{-(\hat{M}_{16})^{(2)}t} \tag{144}$$

$$|(b''_i)^{(2)}((G_{19})',t) - (b''_i)^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \tag{145}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(2)}(T'_{17},t)$ and$(a''_i)^{(2)}(T_{17},t)$. (T'_{17},t) And (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)},(\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a''_i)^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a''_i)^{(2)}(T_{17},t)$, the SECOND augmentation coefficient would be absolutely continuous. 146

<u>Definition of</u> $(\hat{M}_{16})^{(2)},(\hat{k}_{16})^{(2)}$: 147

(D) $(\hat{M}_{16})^{(2)},(\hat{k}_{16})^{(2)},$ are positive constants 148

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

<u>Definition of</u> $(\hat{P}_{13})^{(2)},(\hat{Q}_{13})^{(2)}$: 149

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)},(\hat{k}_{16})^{(2)},(\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)},(a'_i)^{(2)},(b_i)^{(2)},(b'_i)^{(2)},(p_i)^{(2)}, \ (r_i)^{(2)},i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{\mathrm{M}}_{16})^{(2)}}[\ (\mathrm{a_i})^{(2)} + (\mathrm{a'_i})^{(2)} + \ (\hat{\mathrm{A}}_{16})^{(2)} + \ (\hat{\mathrm{P}}_{16})^{(2)}\ (\hat{\mathrm{k}}_{16})^{(2)}] < 1 \tag{150}$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b'_i)^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \tag{151}$$

Where we suppose 152

(E) $(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 153

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 154

$\mathrm{lim}_{\mathrm{G}\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$ 155

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$: 156

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 157

$|(a_i'')^{(3)}(T'_{21},t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T'_{21}|e^{-(\hat{M}_{20})^{(3)}t}$ 158

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$ 159

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T'_{21},t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T'_{21},t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the THIRD augmentation coefficient, would be absolutely continuous. 160

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 161

(F) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants 162

$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ 163

satisfy the inequalities 164

$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$ 165

$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$ 166

167

Where we suppose 168

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 169

(H) The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

170

(I) $lim_{T_2 \to \infty}(a_i'')^{(4)}\,(T_{25},t) = (p_i)^{(4)}$
$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}),t\big) = \ (r_i)^{(4)}$

<u>Definition of</u> $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 171

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) And (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 4$ then the function $(a_i'')^{(4)}(T_{25},t)$, the FOURTH **augmentation coefficient WOULD** be absolutely continuous. 172

173

<u>Defi174nition of</u> $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 174

$(\hat{M}_{24})176^{175(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

<u>Definition of</u> $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 175

(L) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}$*and* $(\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}, i = 24,25,26,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ \ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose 176

) $(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30$ 177

(N) The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

<u>Definition of</u> $(p_i)^{(5)},\ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le \ (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

178

(O) $lim_{T_2\to\infty}(a_i'')^{(5)}\,(T_{29},t) = (p_i)^{(5)}$
$\lim_{G\to\infty}(b_i'')^{(5)}\,(G_{31},t) = \ (r_i)^{(5)}$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)},\ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 179

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$ then the function $(a_i'')^{(5)}(T_{29},t)$, theFIFTH **augmentation coefficient** attributable would be absolutely continuous. 180

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 181

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 182

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)},\ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose 183

$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$ 184

(R) The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)},\ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le \ (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

185

(S) $lim_{T_2\to\infty}(a_i'')^{(6)}\,(T_{33},t) = (p_i)^{(6)}$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = (r_i)^{(6)}$$

<u>Definition of</u> $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition: 186

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 6$ then the function $(a_i'')^{(6)}(T_{33},t)$, the SIXTH **augmentation coefficient** would be absolutely continuous. 187

<u>Definition of</u> $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 188

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

<u>Definition of</u> $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 189

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with
$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants
$(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

190

<u>Theorem 1:</u> if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions 191

<u>Definition of</u> $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t} , \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t} , \boxed{T_i(0) = T_i^0 > 0}$$

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193

<u>Definition of</u> $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t} , G_i(0) = G_i^0 > 0$$

$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \quad , \quad T_i(0) = T_i^0 > 0$

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$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$

Definition of $G_i(0)\,, T_i(0)$: 196

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$

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Definition of $G_i(0)\,, T_i(0)$: 199

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,, T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 200

$G_i(0) = G_i^0\,,\ T_i(0) = T_i^0\,,\ G_i^0 \leq (\hat{P}_{13})^{(1)}\,, T_i^0 \leq (\hat{Q}_{13})^{(1)},$ 201

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 202

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 203

By 204

$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{13}\left(s_{(13)}\right)\right] ds_{(13)}$

$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}\left(s_{(13)}\right) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{14}\left(s_{(13)}\right)\right] ds_{(13)}$ 205

$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}\left(s_{(13)}\right) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}\left(T_{14}\left(s_{(13)}\right), s_{(13)}\right)\right) G_{15}\left(s_{(13)}\right)\right] ds_{(13)}$ 206

$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}\left(s_{(13)}\right) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}\left(G\left(s_{(13)}\right), s_{(13)}\right)\right) T_{13}\left(s_{(13)}\right)\right] ds_{(13)}$ 207

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \Big[(b_{14})^{(1)} T_{13}\big(s_{(13)}\big) - \Big((b'_{14})^{(1)} - (b''_{14})^{(1)}\big(G\big(s_{(13)}\big), s_{(13)}\big)\Big) T_{14}\big(s_{(13)}\big)\Big]\, ds_{(13)}$$ 208

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \Big[(b_{15})^{(1)} T_{14}\big(s_{(13)}\big) - \Big((b'_{15})^{(1)} - (b''_{15})^{(1)}\big(G\big(s_{(13)}\big), s_{(13)}\big)\Big) T_{15}\big(s_{(13)}\big)\Big]\, ds_{(13)}$$ 209

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

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Proof: 211

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \leq (\ \hat{P}_{16}\ )^{(2)} \ , T_i^0 \leq (\ \hat{Q}_{16}\ )^{(2)},$$ 212

$$0 \leq G_i(t) - G_i^0 \leq (\ \hat{P}_{16}\ )^{(2)} e^{(\ \hat{M}_{16}\ )^{(2)} t}$$ 213

$$0 \leq T_i(t) - T_i^0 \leq (\ \hat{Q}_{16}\ )^{(2)} e^{(\ \hat{M}_{16}\ )^{(2)} t}$$ 214

By 215

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \Big[(a_{16})^{(2)} G_{17}\big(s_{(16)}\big) - \Big((a'_{16})^{(2)} + a''_{16})^{(2)}\big(T_{17}\big(s_{(16)}\big), s_{(16)}\big)\Big) G_{16}\big(s_{(16)}\big)\Big]\, ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \Big[(a_{17})^{(2)} G_{16}\big(s_{(16)}\big) - \Big((a'_{17})^{(2)} + (a''_{17})^{(2)}\big(T_{17}\big(s_{(16)}\big), s_{(17)}\big)\Big) G_{17}\big(s_{(16)}\big)\Big]\, ds_{(16)}$$ 216

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \Big[(a_{18})^{(2)} G_{17}\big(s_{(16)}\big) - \Big((a'_{18})^{(2)} + (a''_{18})^{(2)}\big(T_{17}\big(s_{(16)}\big), s_{(16)}\big)\Big) G_{18}\big(s_{(16)}\big)\Big]\, ds_{(16)}$$ 217

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \Big[(b_{16})^{(2)} T_{17}\big(s_{(16)}\big) - \Big((b'_{16})^{(2)} - (b''_{16})^{(2)}\big(G\big(s_{(16)}\big), s_{(16)}\big)\Big) T_{16}\big(s_{(16)}\big)\Big]\, ds_{(16)}$$ 218

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \Big[(b_{17})^{(2)} T_{16}\big(s_{(16)}\big) - \Big((b'_{17})^{(2)} - (b''_{17})^{(2)}\big(G\big(s_{(16)}\big), s_{(16)}\big)\Big) T_{17}\big(s_{(16)}\big)\Big]\, ds_{(16)}$$ 219

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \Big[(b_{18})^{(2)} T_{17}\big(s_{(16)}\big) - \Big((b'_{18})^{(2)} - (b''_{18})^{(2)}\big(G\big(s_{(16)}\big), s_{(16)}\big)\Big) T_{18}\big(s_{(16)}\big)\Big]\, ds_{(16)}$$ 220

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 221

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \leq (\ \hat{P}_{20}\ )^{(3)} \ , T_i^0 \leq (\ \hat{Q}_{20}\ )^{(3)},$$ 222

$$0 \leq G_i(t) - G_i^0 \leq (\ \hat{P}_{20}\ )^{(3)} e^{(\ \hat{M}_{20}\ )^{(3)} t}$$ 223

$$0 \leq T_i(t) - T_i^0 \leq (\ \hat{Q}_{20}\ )^{(3)} e^{(\ \hat{M}_{20}\ )^{(3)} t}$$ 224

By 225

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \Big[(a_{20})^{(3)} G_{21}\big(s_{(20)}\big) - \Big((a'_{20})^{(3)} + a''_{20})^{(3)}\big(T_{21}\big(s_{(20)}\big), s_{(20)}\big)\Big) G_{20}\big(s_{(20)}\big)\Big]\, ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \Big[(a_{21})^{(3)} G_{20}\big(s_{(20)}\big) - \Big((a'_{21})^{(3)} + (a''_{21})^{(3)}\big(T_{21}\big(s_{(20)}\big), s_{(20)}\big)\Big) G_{21}\big(s_{(20)}\big)\Big]\, ds_{(20)}$$ 226

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}\left(s_{(20)}\right) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}\left(T_{21}\left(s_{(20)}\right), s_{(20)}\right)\right) G_{22}\left(s_{(20)}\right)\right] ds_{(20)}$$ 227

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}\left(s_{(20)}\right) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}\left(G\left(s_{(20)}\right), s_{(20)}\right)\right) T_{20}\left(s_{(20)}\right)\right] ds_{(20)}$$ 228

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}\left(s_{(20)}\right) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}\left(G\left(s_{(20)}\right), s_{(20)}\right)\right) T_{21}\left(s_{(20)}\right)\right] ds_{(20)}$$ 229

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}\left(s_{(20)}\right) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}\left(G\left(s_{(20)}\right), s_{(20)}\right)\right) T_{22}\left(s_{(20)}\right)\right] ds_{(20)}$$ 230

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 231

$$G_i(0) = G_i^0 ,\ T_i(0) = T_i^0 ,\ G_i^0 \leq (\hat{P}_{24})^{(4)} ,\ T_i^0 \leq (\hat{Q}_{24})^{(4)},$$ 232

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$ 233

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$ 234

By 235

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[(a_{24})^{(4)} G_{25}\left(s_{(24)}\right) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}\left(T_{25}\left(s_{(24)}\right), s_{(24)}\right)\right) G_{24}\left(s_{(24)}\right)\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^{0} + \int_0^t \left[(a_{25})^{(4)} G_{24}\left(s_{(24)}\right) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}\left(T_{25}\left(s_{(24)}\right), s_{(24)}\right)\right) G_{25}\left(s_{(24)}\right)\right] ds_{(24)}$$ 236

$$\bar{G}_{26}(t) = G_{26}^{0} + \int_0^t \left[(a_{26})^{(4)} G_{25}\left(s_{(24)}\right) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}\left(T_{25}\left(s_{(24)}\right), s_{(24)}\right)\right) G_{26}\left(s_{(24)}\right)\right] ds_{(24)}$$ 237

$$\bar{T}_{24}(t) = T_{24}^{0} + \int_0^t \left[(b_{24})^{(4)} T_{25}\left(s_{(24)}\right) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}\left(G\left(s_{(24)}\right), s_{(24)}\right)\right) T_{24}\left(s_{(24)}\right)\right] ds_{(24)}$$ 238

$$\bar{T}_{25}(t) = T_{25}^{0} + \int_0^t \left[(b_{25})^{(4)} T_{24}\left(s_{(24)}\right) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}\left(G\left(s_{(24)}\right), s_{(24)}\right)\right) T_{25}\left(s_{(24)}\right)\right] ds_{(24)}$$ 239

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^{0} + \int_0^t \left[(b_{26})^{(4)} T_{25}\left(s_{(24)}\right) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}\left(G\left(s_{(24)}\right), s_{(24)}\right)\right) T_{26}\left(s_{(24)}\right)\right] ds_{(24)}$$ 240

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 241

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$$G_i(0) = G_i^0 ,\ T_i(0) = T_i^0 ,\ G_i^0 \leq (\hat{P}_{28})^{(5)} ,\ T_i^0 \leq (\hat{Q}_{28})^{(5)},$$ 243

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$ 244

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$ 245

By 246

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}\left(s_{(28)}\right) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}\left(T_{29}\left(s_{(28)}\right), s_{(28)}\right)\right) G_{28}\left(s_{(28)}\right)\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}\left(s_{(28)}\right) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}\left(T_{29}\left(s_{(28)}\right), s_{(28)}\right)\right) G_{29}\left(s_{(28)}\right)\right] ds_{(28)}$$ 247

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}\left(s_{(28)}\right) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}\left(T_{29}\left(s_{(28)}\right), s_{(28)}\right)\right) G_{30}\left(s_{(28)}\right)\right] ds_{(28)}$$ 248

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}\left(s_{(28)}\right) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}\left(G\left(s_{(28)}\right), s_{(28)}\right)\right) T_{28}\left(s_{(28)}\right)\right] ds_{(28)}$$ 249

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}\left(s_{(28)}\right) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}\left(G\left(s_{(28)}\right), s_{(28)}\right)\right) T_{29}\left(s_{(28)}\right)\right] ds_{(28)}$$ 250

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}\left(s_{(28)}\right) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}\left(G\left(s_{(28)}\right), s_{(28)}\right)\right) T_{30}\left(s_{(28)}\right)\right] ds_{(28)}$$ 251

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

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Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)},\ T_i^0 \le (\hat{Q}_{32})^{(6)},$$ 253

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$ 254

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$ 255

By 256

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}\left(s_{(32)}\right) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}\left(T_{33}\left(s_{(32)}\right), s_{(32)}\right)\right) G_{32}\left(s_{(32)}\right)\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}\left(s_{(32)}\right) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}\left(T_{33}\left(s_{(32)}\right), s_{(32)}\right)\right) G_{33}\left(s_{(32)}\right)\right] ds_{(32)}$$ 257

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}\left(s_{(32)}\right) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}\left(T_{33}\left(s_{(32)}\right), s_{(32)}\right)\right) G_{34}\left(s_{(32)}\right)\right] ds_{(32)}$$ 258

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}\left(s_{(32)}\right) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}\left(G\left(s_{(32)}\right), s_{(32)}\right)\right) T_{32}\left(s_{(32)}\right)\right] ds_{(32)}$$ 259

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}\left(s_{(32)}\right) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}\left(G\left(s_{(32)}\right), s_{(32)}\right)\right) T_{33}\left(s_{(32)}\right)\right] ds_{(32)}$$ 260

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}\left(s_{(32)}\right) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}\left(G\left(s_{(32)}\right), s_{(32)}\right)\right) T_{34}\left(s_{(32)}\right)\right] ds_{(32)}$$ 261

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

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(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 263

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 264

$$(G_{13}(t) - G_{13}^0)e^{-(\hat{M}_{13})^{(1)}t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right)e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$ 265

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 266

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)}\left(G_{17}^0 + (\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right] ds_{(16)} = \left(1 + (a_{16})^{(2)}t\right)G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t} - 1\right) \tag{267}$$

From which it follows that 268

$$(G_{16}(t) - G_{16}^0)e^{-(\hat{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right)e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$ 269

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 270

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[(a_{20})^{(3)}\left(G_{21}^0 + (\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)}t\right)G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t} - 1\right)$$

From which it follows that 271

$$(G_{20}(t) - G_{20}^0)e^{-(\hat{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right)e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$ 272

(b) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 273

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}\left(G_{25}^0 + (\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)}t\right)G_{25}^0 + \frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t} - 1\right)$$

From which it follows that 274

$$(G_{24}(t) - G_{24}^0)e^{-(\hat{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right)e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

(c) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 275

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)}\left(G_{29}^0 + (\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^0+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 276

$$(G_{28}(t)-G_{28}^0)e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{P}_{28})^{(5)}+G_{29}^0\right)e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^0}{G_{29}^0}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

(d) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 277

$$G_{32}(t)\leq G_{32}^0+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^0+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^0+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 278

$$(G_{32}(t)-G_{32}^0)e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)}+G_{33}^0\right)e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^0}{G_{33}^0}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for G_{25} , G_{26}, T_{24}, T_{25}, T_{26}

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280

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 281

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

282

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)}$$ 283

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)}$$ 284

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying GLOBAL EQUATIONS into itself 285

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 286

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote 287

Definition of $\tilde{G}, \tilde{T}$:

$$\left(\tilde{G}, \tilde{T} \right) = \mathcal{A}^{(1)}(G, T)$$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)} \left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} |(a''_{13})^{(1)} \left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)} \left(T_{14}^{(2)}, s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, \mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}} \left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$ 288

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$. 289

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 290

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)} (T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$ 291

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1$, *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 292

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 293

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 294

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to 295

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 297

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + ((\hat{P}_{16})^{(2)} + G_j^0)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)}$$ 298

299

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{Q}_{16})^{(2)} + T_j^0)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying 300

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 301

$$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 302

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 303

$$|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)}\,|G_{17}^{(1)} - G_{17}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}|G_{16}^{(1)} - G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$

$G_{16}^{(2)} |(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} \} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 304

From the hypotheses it follows

305

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| \mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}} \leq \frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) \mathrm{d}\left(\left((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 306

Remark 1: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ respectively of $\mathbb{R}_+$. 307

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 308

From 19 to 24 it results

$$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0 \mathrm{e}^{\left[-\int_0^{\mathrm{t}} \{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\} \mathrm{d}s_{(16)}\right]} \geq 0$$

$$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0 \mathrm{e}^{(-(b_i')^{(2)} \mathrm{t})} > 0 \quad \text{for } \mathrm{t} > 0$$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1, \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 309

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} \mathrm{G}_{17}$ and by integrating

$$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 / (a'_{18})^{(2)}$$

310

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 311

Remark 5: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}((G_{19})(\mathrm{t}), \mathrm{t})) = (b'_{17})^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 312

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $\mathrm{t} > \mathrm{t}_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(\mathrm{t}), \mathrm{t}) < \varepsilon_2, \mathrm{T}_{16}\,(\mathrm{t}) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 313

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$ 314

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 316

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

317
$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)}$$

318
$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i into itself 319

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 320

$$d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}\right), \left((G_{23})^{(2)}, (T_{23})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 321

<u>Definition of</u> $\widetilde{G_{23}}, \widetilde{T_{23}} : \left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 322

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)} - G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} \, ds_{(20)} +$$

$$\int_0^t \{(a_{20}')^{(3)} \left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

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$$(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}'')^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right| \; e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

324
$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a_{20}')^{(3)} + (\widehat{A}_{20})^{(3)} +\right.$$
$$\left.(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}; (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}$ *and* $(\hat{Q}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 325

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(3)}$ and $(b''_i)^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 326

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(3)} - (a''_i)^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(3)}t)} > 0 \quad \text{for } t > 0$$

Definition of $((\hat{M}_{20})^{(3)})_1, ((\hat{M}_{20})^{(3)})_2$ *and* $((\hat{M}_{20})^{(3)})_3$: 327

Remark 3: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\hat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq ((\hat{M}_{20})^{(3)})_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq ((\hat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)}((\hat{M}_{20})^{(3)})_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq ((\hat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)}((\hat{M}_{20})^{(3)})_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 328

Remark 5: If T_{20} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(3)}((G_{23})(t), t)) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$. 329

Definition of $(m)^{(3)}$ and ε_3 : 330

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b''_i)^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 331

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $\lim_{t\to\infty}(b''_{22})^{(3)}((G_{23})(t), t) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 333

$(\hat{\mathrm{P}}_{24})^{(4)}$ and $(\hat{\mathrm{Q}}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)}$$ 334

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)}$$ 335

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying IN to itself 336

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 337

$$d\left(\left((G_{27})^{(1)},(T_{27})^{(1)}\right),\left((G_{27})^{(2)},(T_{27})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)} - G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)} +$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)} - G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)} - G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

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$$\left|(G_{27})^{(1)} - (G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t} \leq$$ 339

$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} +\right.$$

$$\left.(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(\left((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 340

$(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ and $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 341

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)}t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 342

Remark 3: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)}G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a_{25}')^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a_{26}')^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 4: If $G_{24}\ is$ bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 343

Remark 5: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 344

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 345

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\ \ t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded. The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

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It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 347

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + ((\hat{P}_{28})^{(5)} + G_j^0)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)}$$ 348

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{Q}_{28})^{(5)} + T_j^0)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)}$$ 349

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i into itself 350

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 351

$$d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}\right), \left((G_{31})^{(2)}, (T_{31})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, \mathrm{t}]$

From the hypotheses it follows

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$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)}t} \leq$$ 353

$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} +\right.$$

$$\left.(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as 354 not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 355

From GLOBAL EQUATIONS it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)}t\right)} > 0$ for t > 0

Definition of $\left((\hat{M}_{28})^{(5)}\right)_1, \left((\hat{M}_{28})^{(5)}\right)_2\ and\ \left((\hat{M}_{28})^{(5)}\right)_3$: 356

Remark 3: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\hat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\hat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\hat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\hat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\hat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\hat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 4: If $G_{28}\ is$ bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 357

Remark 5: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 358

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$ 359

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 360

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1-e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded. The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for G_{33}, $G_{34}, T_{32}, T_{33}, T_{34}$

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It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 362

$(\hat{\mathrm{P}}_{32})^{(6)}$ and $(\hat{\mathrm{Q}}_{32})^{(6)}$ large to have

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$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + ((\hat{P}_{32})^{(6)} + G_j^0)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{32})^{(6)}$$

364

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{Q}_{32})^{(6)} + T_j^0)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \le (\hat{Q}_{32})^{(6)}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i into itself 365

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 366

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}((G_{35}),(T_{35}))$

It results

$$|\tilde{G}_{32}^{(1)} - \tilde{G}_i^{(2)}| \le \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}(T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} |(a''_{32})^{(6)}(T_{33}^{(1)}, s_{(32)}) - (a''_{32})^{(6)}(T_{33}^{(2)}, s_{(32)})|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0, \mathrm{t}]$ 367

From the hypotheses it follows

368

$$|(G_{35})^{(1)} - (G_{35})^{(2)}| e^{-(\widehat{M}_{32})^{(6)} t} \le$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} +\right.$$
$$\left.(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 369

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32{,}33{,}34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 370

From 69 to 32 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b'_i)^{(6)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\hat{M}_{32})^{(6)}\right)_1, \left((\hat{M}_{32})^{(6)}\right)_2\ and\ \left((\hat{M}_{32})^{(6)}\right)_3$: 371

Remark 3: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\hat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\hat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$$G_{33} \leq \left((\hat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\hat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\hat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\hat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 4: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 372

Remark 5: If T_{32} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(6)}\,((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 373

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t\ > t_6$

$$(b_{33})^{(6)} - (b''_i)^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

374

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 375

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded. The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right)=(b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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Behavior of the solutions 377

_If we denote and define

Definition of $(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$:

(a) $\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)}\le-(a'_{13})^{(1)}+(a'_{14})^{(1)}-(a''_{13})^{(1)}(T_{14},t)+(a''_{14})^{(1)}(T_{14},t)\le-(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)}\le-(b'_{13})^{(1)}+(b'_{14})^{(1)}-(b''_{13})^{(1)}(G,t)-(b''_{14})^{(1)}(G,t)\le-(\tau_1)^{(1)}$$

Definition of $(\nu_1)^{(1)},(\nu_2)^{(1)},(u_1)^{(1)},(u_2)^{(1)},\nu^{(1)},u^{(1)}$: 378

(b) By $(\nu_1)^{(1)}>0,(\nu_2)^{(1)}<0$ and respectively $(u_1)^{(1)}>0,(u_2)^{(1)}<0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2+(\sigma_1)^{(1)}\nu^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_1)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(\bar\nu_1)^{(1)},,(\bar\nu_2)^{(1)},(\bar u_1)^{(1)},(\bar u_2)^{(1)}$: 379

By $(\bar\nu_1)^{(1)}>0,(\bar\nu_2)^{(1)}<0$ and respectively $(\bar u_1)^{(1)}>0,(\bar u_2)^{(1)}<0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2+(\sigma_2)^{(1)}\nu^{(1)}-(a_{13})^{(1)}=0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2+(\tau_2)^{(1)}u^{(1)}-(b_{13})^{(1)}=0$

Definition of $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)},(\nu_0)^{(1)}$:- 380

(c) If we define $(m_1)^{(1)},(m_2)^{(1)},(\mu_1)^{(1)},(\mu_2)^{(1)}$ by

$$(m_2)^{(1)}=(\nu_0)^{(1)},(m_1)^{(1)}=(\nu_1)^{(1)},\ if\ (\nu_0)^{(1)}<(\nu_1)^{(1)}$$

$$(m_2)^{(1)}=(\nu_1)^{(1)},(m_1)^{(1)}=(\bar\nu_1)^{(1)},if\ (\nu_1)^{(1)}<(\nu_0)^{(1)}<(\bar\nu_1)^{(1)},$$

and $\boxed{(\nu_0)^{(1)}=\frac{G_{13}^0}{G_{14}^0}}$

$$(m_2)^{(1)}=(\nu_1)^{(1)},(m_1)^{(1)}=(\nu_0)^{(1)},\ if\ (\bar\nu_1)^{(1)}<(\nu_0)^{(1)}$$

and analogously 381

$$(\mu_2)^{(1)}=(u_0)^{(1)},(\mu_1)^{(1)}=(u_1)^{(1)},\ if\ (u_0)^{(1)}<(u_1)^{(1)}$$

$$(\mu_2)^{(1)}=(u_1)^{(1)},(\mu_1)^{(1)}=(\bar u_1)^{(1)},if\ (u_1)^{(1)}<(u_0)^{(1)}<(\bar u_1)^{(1)},$$

and $\boxed{(u_0)^{(1)}=\frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)}=(u_1)^{(1)},(\mu_1)^{(1)}=(u_0)^{(1)},if\ (\bar u_1)^{(1)}<(u_0)^{(1)}$ where $(u_1)^{(1)},(\bar u_1)^{(1)}$ 382

are defined respectively

Then the solution satisfies the inequalities 383

$$G_{13}^0e^{((S_1)^{(1)}-(p_{13})^{(1)})t}\le G_{13}(t)\le G_{13}^0e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{\left((S_1)^{(1)}-(p_{13})^{(1)}\right)t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\Big(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}\left((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)}\right)} \left[e^{\left((S_1)^{(1)}-(p_{13})^{(1)}\right)t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \leq G_{15}(t) \leq$$ 384

$$\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}\left((S_1)^{(1)}-(a'_{15})^{(1)}\right)} [e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\Big)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{\left((R_1)^{(1)}+(r_{13})^{(1)}\right)t}}$$ 385

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{\left((R_1)^{(1)}+(r_{13})^{(1)}\right)t}$$ 386

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}\left((R_1)^{(1)}-(b'_{15})^{(1)}\right)} \left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$$ 387

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}\left((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)}\right)} \left[e^{\left((R_1)^{(1)}+(r_{13})^{(1)}\right)t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 388

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

Behavior of the solutions 389

_If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 390

(d) $\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \leq -(\sigma_1)^{(2)}$ 391

$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right) - (b''_{17})^{(2)}\left((G_{19}), t\right) \leq -(\tau_1)^{(2)}$ 392

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 393

By $(\nu_1)^{(2)} > 0, (\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots 394

(e) of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 395

and $(b_{14})^{(2)}\left(u^{(2)}\right)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 396

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 397

By $(\bar{\nu}_1)^{(2)} > 0, (\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the 398

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 399 and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 400

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 401

(f) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 402

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}, \ \boldsymbol{if} \ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ 403

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if} \ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)},$ 404

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}, \ \boldsymbol{if} \ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 405

and analogously 406

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \ \boldsymbol{if} \ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if} \ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \ \boldsymbol{if} \ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 407

Then the solution satisfies the inequalities 408

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined 409

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 410$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}\left((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)}\right)}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \leq G_{18}(t) \leq \right. \quad 411$$
$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}\left((S_1)^{(2)} - (a_{18}')^{(2)}\right)}\left[e^{(S_1)^{(2)}t} - e^{-(a_{18}')^{(2)}t}\right] + G_{18}^0 e^{-(a_{18}')^{(2)}t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 412$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 413$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}\left((R_1)^{(2)} - (b_{18}')^{(2)}\right)}\left[e^{(R_1)^{(2)}t} - e^{-(b_{18}')^{(2)}t}\right] + T_{18}^0 e^{-(b_{18}')^{(2)}t} \leq T_{18}(t) \leq \quad 414$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}\left((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)}\right)}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 415

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16}')^{(2)}$ 416

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b_{16}')^{(2)}$ 417

$(R_2)^{(2)} = (b_{18}')^{(2)} - (r_{18})^{(2)}$

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Behavior of the solutions 419

If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \leq -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 420

(b) By $(\nu_1)^{(3)} > 0, (\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0, (\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 421

(c) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}, \ \boldsymbol{if} \ (\nu_0)^{(3)} < (\nu_1)^{(3)}$$

$$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}, \boldsymbol{if} \ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)},$$

and $\boxed{(\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}, \ \boldsymbol{if} \ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$$

and analogously 422

$$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \ \boldsymbol{if} \ (u_0)^{(3)} < (u_1)^{(3)}$$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if} \ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \ \boldsymbol{if} \ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$$

Then the solution satisfies the inequalities

$$G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined 423

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$$ 424

$$\left(\frac{(a_{22})^{(3)}G_{20}^{0}}{(m_1)^{(3)}\left((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)}\right)}\left[e^{\left((S_1)^{(3)}-(p_{20})^{(3)}\right)t}-e^{-(S_2)^{(3)}t}\right]+G_{22}^{0}e^{-(S_2)^{(3)}t}\leq G_{22}(t)\leq\right.$$ 425

$$\left.\frac{(a_{22})^{(3)}G_{20}^{0}}{(m_2)^{(3)}\left((S_1)^{(3)}-(a'_{22})^{(3)}\right)}\left[e^{(S_1)^{(3)}t}-e^{-(a'_{22})^{(3)}t}\right]+\ G_{22}^{0}e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t}\leq T_{20}(t)\leq T_{20}^{0}e^{\left((R_1)^{(3)}+(r_{20})^{(3)}\right)t}}$$ 426

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t}\leq T_{20}(t)\leq\frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{\left((R_1)^{(3)}+(r_{20})^{(3)}\right)t}$$ 427

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}\left((R_1)^{(3)}-(b'_{22})^{(3)}\right)}\left[e^{(R_1)^{(3)}t}-e^{-(b'_{22})^{(3)}t}\right]+T_{22}^{0}e^{-(b'_{22})^{(3)}t}\leq T_{22}(t)\leq$$ 428

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}\left((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)}\right)}\left[e^{\left((R_1)^{(3)}+(r_{20})^{(3)}\right)t}-e^{-(R_2)^{(3)}t}\right]+T_{22}^{0}e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)},(S_2)^{(3)},(R_1)^{(3)},(R_2)^{(3)}$:- 429

Where $(S_1)^{(3)}=(a_{20})^{(3)}(m_2)^{(3)}-(a'_{20})^{(3)}$

$(S_2)^{(3)}=(a_{22})^{(3)}-(p_{22})^{(3)}$

$(R_1)^{(3)}=(b_{20})^{(3)}(\mu_2)^{(3)}-(b'_{20})^{(3)}$

$(R_2)^{(3)}=(b'_{22})^{(3)}-(r_{22})^{(3)}$

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Behavior of the solutions 432

If we denote and define

Definition of $(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$:

(d) $(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)}\leq-(a'_{24})^{(4)}+(a'_{25})^{(4)}-(a''_{24})^{(4)}(T_{25},t)+(a''_{25})^{(4)}(T_{25},t)\leq-(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)}\leq-(b'_{24})^{(4)}+(b'_{25})^{(4)}-(b''_{24})^{(4)}\left((G_{27}),t\right)-(b''_{25})^{(4)}\left((G_{27}),t\right)\leq-(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)},(\nu_2)^{(4)},(u_1)^{(4)},(u_2)^{(4)},\nu^{(4)},u^{(4)}$: 433

(e) By $(\nu_1)^{(4)}>0,(\nu_2)^{(4)}<0$ and respectively $(u_1)^{(4)}>0,(u_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2+(\sigma_1)^{(4)}\nu^{(4)}-(a_{24})^{(4)}=0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2+(\tau_1)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$ and

Definition of $(\bar{\nu}_1)^{(4)},,(\bar{\nu}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 434

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By $(\bar{\nu}_1)^{(4)}>0,(\bar{\nu}_2)^{(4)}<0$ and respectively $(\bar{u}_1)^{(4)}>0,(\bar{u}_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2+(\sigma_2)^{(4)}\nu^{(4)}-(a_{24})^{(4)}=0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2+(\tau_2)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$

Definition of $(m_1)^{(4)},(m_2)^{(4)},(\mu_1)^{(4)},(\mu_2)^{(4)},(\nu_0)^{(4)}$:- 436

(f) If we define $(m_1)^{(4)},(m_2)^{(4)},(\mu_1)^{(4)},(\mu_2)^{(4)}$ by

$$(m_2)^{(4)}=(\nu_0)^{(4)},(m_1)^{(4)}=(\nu_1)^{(4)},\ \boldsymbol{if}\ (\nu_0)^{(4)}<(\nu_1)^{(4)}$$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, $\boldsymbol{if}$ $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, $\boldsymbol{if}$ $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously 437

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}$ $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, $\boldsymbol{if}$ $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, $\boldsymbol{if}$ $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$ are defined by 59 and 64 respectively

Then the solution satisfies the inequalities 439

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$$G_{24}^0 e^{\left((S_1)^{(4)}-(p_{24})^{(4)}\right)t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$ 441

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where $(p_i)^{(4)}$ is defined 443

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$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{\left((S_1)^{(4)}-(p_{24})^{(4)}\right)t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$ 446

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$$\left(\frac{(a_{26})^{(4)}G_{24}^0}{(m_1)^{(4)}\left((S_1)^{(4)}-(p_{24})^{(4)}-(S_2)^{(4)}\right)}\left[e^{\left((S_1)^{(4)}-(p_{24})^{(4)}\right)t}-e^{-(S_2)^{(4)}t}\right]+G_{26}^0 e^{-(S_2)^{(4)}t} \leq G_{26}(t) \leq \frac{(a_{26})^{(4)}G_{24}^0}{(m_2)^{(4)}\left((S_1)^{(4)}-(a_{26}')^{(4)}\right)}\left[e^{(S_1)^{(4)}t}-e^{-(a_{26}')^{(4)}t}\right]+ G_{26}^0 e^{-(a_{26}')^{(4)}t}\right)$$ 448

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{\left((R_1)^{(4)}+(r_{24})^{(4)}\right)t}}$$ 449

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{\left((R_1)^{(4)}+(r_{24})^{(4)}\right)t}$$ 450

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}\left((R_1)^{(4)}-(b_{26}')^{(4)}\right)}\left[e^{(R_1)^{(4)}t}-e^{-(b_{26}')^{(4)}t}\right]+T_{26}^0 e^{-(b_{26}')^{(4)}t} \leq T_{26}(t) \leq$$ 451

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}\left((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)}\right)}\left[e^{\left((R_1)^{(4)}+(r_{24})^{(4)}\right)t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 452

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a_{24}')^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b_{24}')^{(4)}$

$(R_2)^{(4)} = (b_{26}')^{(4)} - (r_{26})^{(4)}$ 453

Behavior of the solutions 454

If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \leq -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \leq -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \leq -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 455

(h) By $(\nu_1)^{(5)} > 0, (\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 456

By $(\bar{\nu}_1)^{(5)} > 0, (\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

(i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \ \boldsymbol{if} \ (\nu_0)^{(5)} < (\nu_1)^{(5)}$$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if} \ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$$

and $\boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \ \boldsymbol{if} \ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$$

and analogously 457

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \ \boldsymbol{if} \ (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if} \ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if} \ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined respectively

Then the solution satisfies the inequalities 458

$$G_{28}^0 e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t}$$ 459

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$$\left(\frac{(a_{30})^{(5)}G_{28}^0}{(m_1)^{(5)}\left((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)}\right)}\left[e^{\left((S_1)^{(5)}-(p_{28})^{(5)}\right)t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^0e^{-(S_2)^{(5)}t}\leq G_{30}(t)\leq \frac{(a_{30})^{(5)}G_{28}^0}{(m_2)^{(5)}\left((S_1)^{(5)}-(a_{30}')^{(5)}\right)}\left[e^{(S_1)^{(5)}t}-e^{-(a_{30}')^{(5)}t}\right]+G_{30}^0e^{-(a_{30}')^{(5)}t}\right)$$ 461

$$\boxed{T_{28}^0e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq T_{28}^0e^{\left((R_1)^{(5)}+(r_{28})^{(5)}\right)t}}$$ 462

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^0e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq \frac{1}{(\mu_2)^{(5)}}T_{28}^0e^{\left((R_1)^{(5)}+(r_{28})^{(5)}\right)t}$$ 463

$$\frac{(b_{30})^{(5)}T_{28}^0}{(\mu_1)^{(5)}\left((R_1)^{(5)}-(b_{30}')^{(5)}\right)}\left[e^{(R_1)^{(5)}t}-e^{-(b_{30}')^{(5)}t}\right]+T_{30}^0e^{-(b_{30}')^{(5)}t}\leq T_{30}(t)\leq$$ 464

$$\frac{(a_{30})^{(5)}T_{28}^0}{(\mu_2)^{(5)}\left((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)}\right)}\left[e^{\left((R_1)^{(5)}+(r_{28})^{(5)}\right)t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^0e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 465

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a_{28}')^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b_{28}')^{(5)}$

$(R_2)^{(5)}=(b_{30}')^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions 466

If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq -(a_{32}')^{(6)}+(a_{33}')^{(6)}-(a_{32}'')^{(6)}(T_{33},t)+(a_{33}'')^{(6)}(T_{33},t)\leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq -(b_{32}')^{(6)}+(b_{33}')^{(6)}-(b_{32}'')^{(6)}\left((G_{35}),t\right)-(b_{33}'')^{(6)}\left((G_{35}),t\right)\leq -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)},(\nu_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},\nu^{(6)},u^{(6)}$: 467

(k) By $(\nu_1)^{(6)}>0,(\nu_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0,(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2+(\sigma_1)^{(6)}\nu^{(6)}-(a_{32})^{(6)}=0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{\nu}_1)^{(6)},,(\bar{\nu}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 468

By $(\bar{\nu}_1)^{(6)}>0,(\bar{\nu}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0,(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2+(\sigma_2)^{(6)}\nu^{(6)}-(a_{32})^{(6)}=0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(\nu_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(\nu_0)^{(6)},(m_1)^{(6)}=(\nu_1)^{(6)},\ \boldsymbol{if}\ (\nu_0)^{(6)}<(\nu_1)^{(6)}$

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$(m_2)^{(6)}=(\nu_1)^{(6)},(m_1)^{(6)}=(\bar{\nu}_6)^{(6)}\,,\boldsymbol{if}\;(\nu_1)^{(6)}<(\nu_0)^{(6)}<(\bar{\nu}_1)^{(6)},$

and $\boxed{(\nu_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$

$(\,m_2)^{(6)}=(\nu_1)^{(6)},(m_1)^{(6)}=(\nu_0)^{(6)},\;\boldsymbol{if}\;(\bar{\nu}_1)^{(6)}<(\nu_0)^{(6)}$

and analogously 471

$(\mu_2)^{(6)}=(u_0)^{(6)},(\mu_1)^{(6)}=(u_1)^{(6)},\;\boldsymbol{if}\;(u_0)^{(6)}<(u_1)^{(6)}$

$(\mu_2)^{(6)}=(u_1)^{(6)},(\mu_1)^{(6)}=(\bar{u}_1)^{(6)}\,,\boldsymbol{if}\;(u_1)^{(6)}<(u_0)^{(6)}<(\bar{u}_1)^{(6)},$

and $\boxed{(u_0)^{(6)}=\frac{T_{32}^0}{T_{33}^0}}$

$(\,\mu_2)^{(6)}=(u_1)^{(6)},(\mu_1)^{(6)}=(u_0)^{(6)},\boldsymbol{if}\;(\bar{u}_1)^{(6)}<(u_0)^{(6)}$ where $(u_1)^{(6)},(\bar{u}_1)^{(6)}$ are defined respectively

Then the solution satisfies the inequalities 472

$$G_{32}^0e^{((S_1)^{(6)}-(p_{32})^{(6)})t}\le G_{32}(t)\le G_{32}^0e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined

$$\frac{1}{(m_1)^{(6)}}G_{32}^0e^{((S_1)^{(6)}-(p_{32})^{(6)})t}\le G_{33}(t)\le\frac{1}{(m_2)^{(6)}}G_{32}^0e^{(S_1)^{(6)}t}$$ 473

$$\left(\frac{(a_{34})^{(6)}G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)}-(p_{32})^{(6)}-(S_2)^{(6)})}\left[e^{((S_1)^{(6)}-(p_{32})^{(6)})t}-e^{-(S_2)^{(6)}t}\right]+G_{34}^0e^{-(S_2)^{(6)}t}\le G_{34}(t)\le\frac{(a_{34})^{(6)}G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)}-(a_{34}')^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a_{34}')^{(6)}t}\right]+\;G_{34}^0e^{-(a_{34}')^{(6)}t}\right)$$ 474

$$\boxed{T_{32}^0e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^0e^{((R_1)^{(6)}+(r_{32})^{(6)})t}}$$ 475

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^0e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^0e^{((R_1)^{(6)}+(r_{32})^{(6)})t}$$ 476

$$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b_{34}')^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b_{34}')^{(6)}t}\right]+T_{34}^0e^{-(b_{34}')^{(6)}t}\le T_{34}(t)\le$$ 477

$$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^0e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 478

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a_{32}')^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b_{32}')^{(6)}$

$(R_2)^{(6)}=(b_{34}')^{(6)}\;-(r_{34})^{(6)}$

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Proof : From GLOBAL EQUATIONS we obtain 480

$$\frac{d\nu^{(1)}}{dt}=(a_{13})^{(1)}-\left((a_{13}')^{(1)}-(a_{14}')^{(1)}+(a_{13}'')^{(1)}(T_{14},t)\right)-\;(a_{14}'')^{(1)}(T_{14},t)\nu^{(1)}-(a_{14})^{(1)}\nu^{(1)}$$

Definition of $\nu^{(1)}$:- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$ 481

It follows

$$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \leq \frac{d\nu^{(1)}}{dt} \leq -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$:-

(a) For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \geq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 482

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 483

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

(c) If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 484

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}$, $\boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in GLOBAL E486QUATIONS we get easily the result stated in the theorem.

Particular case : 485

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

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we obtain 487

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 488

It follows 489

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 490

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

(d) For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right) t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right) t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 491

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right) t\right]}}{1 + (\bar{\mathrm{C}})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right) t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 492

(e) If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 493

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right) t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right) t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

(f) If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 494

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

<u>Definition of</u> $\nu^{(2)}(t)$:- 495

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 496

<u>Definition of</u> $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

. 497

<u>Particular case :</u> 498

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

499

From GLOBAL EQUATIONS we obtain 500

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

<u>Definition of</u> $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 501

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

502

From which one obtains

(a) For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 503

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

<u>Definition of</u> $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

(b) If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 504

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

(c) If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 505

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

<u>Definition of</u> $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

<u>Definition of</u> $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

<u>Particular case :</u>

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

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: From GLOBAL EQUATIONS we obtain 507

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$ 508

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

(d) For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 509

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

(e) If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 510

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

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(f) If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}$, $\boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** . 513

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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From GLOBAL EQUATIONS we obtain 515

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

(g) For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right) t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right) t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 516

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right) t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right) t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

(h) If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 517

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right) t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right) t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right) t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right) t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

(i) If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 518

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

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And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

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we obtain 521

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right) - (a''_{33})^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

(j) For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 522

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}} \qquad 523$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

(k) If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 524

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

(l) If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 525

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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We can prove the following 528

<u>Theorem 3</u>: If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined, then the system

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If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions 530.

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 531

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 532

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 533

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 534

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied , then the system

If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions 535

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined are satisfied , then the system

If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions 536

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined are satisfied , then the system

If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions 537

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined satisfied , then the system

If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions 538

$(a_{32}')^{(6)}(a_{33}')^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a_{32}')^{(6)}(a_{33}')^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a_{33}')^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b_{32}')^{(6)}(b_{33}')^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

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$(b_{32}')^{(6)}(b_{33}')^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b_{32}')^{(6)}(r_{33})^{(6)} - (b_{33}')^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined are satisfied , then the system

$(a_{13})^{(1)}G_{14} - [(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14})]G_{13} = 0$ 540

$(a_{14})^{(1)}G_{13} - [(a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14})]G_{14} = 0$ 541

$(a_{15})^{(1)}G_{14} - [(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14})]G_{15} = 0$ 542

$(b_{13})^{(1)}T_{14} - [(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G)]T_{13} = 0$ 543

$(b_{14})^{(1)}T_{13} - [(b_{14}')^{(1)} - (b_{14}'')^{(1)}(G)]T_{14} = 0$ 544

$(b_{15})^{(1)}T_{14} - [(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G)]T_{15} = 0$ 545

has a unique positive solution , which is an equilibrium solution for the system 546

$(a_{16})^{(2)}G_{17} - [(a_{16}')^{(2)} + (a_{16}'')^{(2)}(T_{17})]G_{16} = 0$ 547

$(a_{17})^{(2)}G_{16} - [(a_{17}')^{(2)} + (a_{17}'')^{(2)}(T_{17})]G_{17} = 0$ 548

$(a_{18})^{(2)}G_{17} - [(a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17})]G_{18} = 0$ 549

$(b_{16})^{(2)}T_{17} - [(b_{16}')^{(2)} - (b_{16}'')^{(2)}(G_{19})]T_{16} = 0$ 550

$(b_{17})^{(2)}T_{16} - [(b_{17}')^{(2)} - (b_{17}'')^{(2)}(G_{19})]T_{17} = 0$ 551

$(b_{18})^{(2)}T_{17} - [(b_{18}')^{(2)} - (b_{18}'')^{(2)}(G_{19})]T_{18} = 0$ 552

has a unique positive solution , which is an equilibrium solution for 553

$(a_{20})^{(3)}G_{21} - [(a_{20}')^{(3)} + (a_{20}'')^{(3)}(T_{21})]G_{20} = 0$ 554

$(a_{21})^{(3)}G_{20} - [(a_{21}')^{(3)} + (a_{21}'')^{(3)}(T_{21})]G_{21} = 0$ 555

$(a_{22})^{(3)}G_{21} - [(a_{22}')^{(3)} + (a_{22}'')^{(3)}(T_{21})]G_{22} = 0$ 556

$(b_{20})^{(3)}T_{21} - [(b_{20}')^{(3)} - (b_{20}'')^{(3)}(G_{23})]T_{20} = 0$ 557

$(b_{21})^{(3)}T_{20} - [(b_{21}')^{(3)} - (b_{21}'')^{(3)}(G_{23})]T_{21} = 0$ 558

$(b_{22})^{(3)}T_{21} - [(b_{22}')^{(3)} - (b_{22}'')^{(3)}(G_{23})]T_{22} = 0$ 559

has a unique positive solution , which is an equilibrium solution 560

$(a_{24})^{(4)}G_{25} - [(a_{24}')^{(4)} + (a_{24}'')^{(4)}(T_{25})]G_{24} = 0$ 561

$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0$ 563

$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0$ 564

$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0$ 565

$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0$ 566

$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0$ 567

has a unique positive solution , which is an equilibrium solution for the system 568

$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0$ 569

$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0$ 570

$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0$ 571

$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0$ 572

$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0$ 573

$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0$ 574

has a unique positive solution , which is an equilibrium solution for the system 575

$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0$ 576

$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0$ 577

$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0$ 578

$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0$ 579

$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0$ 580

$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0$ 584

has a unique positive solution , which is an equilibrium solution for the system 582

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(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) +$

$(a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

585

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$ 586

587

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

588

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

589

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

560

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Definition and uniqueness of T^*_{14} :- 561

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 562

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(\mathrm{T}^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 563

Definition and uniqueness of T^*_{21} :- 564

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

565

Definition and uniqueness of T^*_{25} :- 566

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 567

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 568

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

(e) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if 569

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi(G^*) = 0$

(f) By the same argument, the equations 92,93 admit solutions G_{16}, G_{17} if 570

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G^*_{14} such that $\varphi((G_{19})^*) = 0$ 571

(g) By the same argument, the concatenated equations admit solutions G_{20}, G_{21} if 572

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

573

(h) By the same argument, the equations of modules admit solutions G_{24}, G_{25} if 574

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

(i) By the same argument, the equations (modules) admit solutions G_{28}, G_{29} if 575

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

(j) By the same argument, the equations (modules) admit solutions G_{32}, G_{33} if 578

579

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

580

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

581

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of 89 to 94 582

G^*_{14} given by $\varphi(G^*) = 0$, T^*_{14} given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)} G^*_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)} G^*_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T^*_{14})]}$$

$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G^*)]}$, $T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G^*)]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 583

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(\mathrm{T}_{17}^*) = 0$ and 584

$\mathrm{G}_{16}^* = \frac{(\mathrm{a}_{16})^{(2)} \mathrm{G}_{17}^*}{[(\mathrm{a}_{16}')^{(2)} + (\mathrm{a}_{16}'')^{(2)}(\mathrm{T}_{17}^*)]}$, $\mathrm{G}_{18}^* = \frac{(\mathrm{a}_{18})^{(2)} \mathrm{G}_{17}^*}{[(\mathrm{a}_{18}')^{(2)} + (\mathrm{a}_{18}'')^{(2)}(\mathrm{T}_{17}^*)]}$ 585

$\mathrm{T}_{16}^* = \frac{(\mathrm{b}_{16})^{(2)} \mathrm{T}_{17}^*}{[(\mathrm{b}_{16}')^{(2)} - (\mathrm{b}_{16}'')^{(2)}((G_{19})^*)]}$, $\mathrm{T}_{18}^* = \frac{(\mathrm{b}_{18})^{(2)} \mathrm{T}_{17}^*}{[(\mathrm{b}_{18}')^{(2)} - (\mathrm{b}_{18}'')^{(2)}((G_{19})^*)]}$ 586

Obviously, these values represent an equilibrium solution 587

Finally we obtain the unique solution 588

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a_{20}')^{(3)} + (a_{20}'')^{(3)}(T_{21}^*)]}$, $G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a_{22}')^{(3)} + (a_{22}'')^{(3)}(T_{21}^*)]}$

$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b_{20}')^{(3)} - (b_{20}'')^{(3)}(G_{23}{}^*)]}$, $T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b_{22}')^{(3)} - (b_{22}'')^{(3)}(G_{23}{}^*)]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 589

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a_{24}')^{(4)} + (a_{24}'')^{(4)}(T_{25}^*)]}$, $G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a_{26}')^{(4)} + (a_{26}'')^{(4)}(T_{25}^*)]}$

$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b_{24}')^{(4)} - (b_{24}'')^{(4)}((G_{27})^*)]}$, $T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b_{26}')^{(4)} - (b_{26}'')^{(4)}((G_{27})^*)]}$ 590

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 591

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a_{28}')^{(5)} + (a_{28}'')^{(5)}(T_{29}^*)]}$, $G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a_{30}')^{(5)} + (a_{30}'')^{(5)}(T_{29}^*)]}$

$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b_{28}')^{(5)} - (b_{28}'')^{(5)}((G_{31})^*)]}$, $T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b_{30}')^{(5)} - (b_{30}'')^{(5)}((G_{31})^*)]}$ 592

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 593

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a_{32}')^{(6)} + (a_{32}'')^{(6)}(T_{33}^*)]}$, $G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a_{34}')^{(6)} + (a_{34}'')^{(6)}(T_{33}^*)]}$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]}$$ 594

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS 595

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(1)}$ *and* $(b''_i)^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof:_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$ 596

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}} (T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b''_i)^{(1)}}{\partial G_j} (G^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 597

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14}$$ 598

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14}$$ 599

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14}$$ 600

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right)$$ 601

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right)$$ 602

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^* \mathbb{G}_j\right)$$ 603

If the conditions of the previous theorem are satisfied and if the functions $(\mathrm{a}''_i)^{(2)}$ and $(\mathrm{b}''_i)^{(2)}$ Belong to $\mathrm{C}^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable 604

Denote 605

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$\mathrm{G}_i = \mathrm{G}_i^* + \mathbb{G}_i \qquad , \mathrm{T}_i = \mathrm{T}_i^* + \mathbb{T}_i$$ 606

$$\frac{\partial (a''_{17})^{(2)}}{\partial \mathrm{T}_{17}} (\mathrm{T}_{17}^*) = (q_{17})^{(2)} \quad , \quad \frac{\partial (b''_i)^{(2)}}{\partial \mathrm{G}_j} ((G_{19})^*) = s_{ij}$$ 607

taking into account equations (global)and neglecting the terms of power 2, we obtain 608

$$\frac{\mathrm{d}\mathbb{G}_{16}}{\mathrm{dt}} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)} \mathrm{G}_{16}^* \mathbb{T}_{17}$$ 609

$$\frac{\mathrm{d}\mathbb{G}_{17}}{\mathrm{dt}} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)} \mathrm{G}_{17}^* \mathbb{T}_{17}$$ 610

$$\frac{\mathrm{d}\mathbb{G}_{18}}{\mathrm{dt}} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)} \mathrm{G}_{18}^* \mathbb{T}_{17}$$ 611

$$\frac{\mathrm{d}\mathbb{T}_{16}}{\mathrm{dt}} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)} \mathrm{T}_{16}^* \mathbb{G}_j\right)$$ 612

$$\frac{\mathrm{d}\mathbb{T}_{17}}{\mathrm{dt}} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right)$$ 613

$$\frac{\mathrm{d}\mathbb{T}_{18}}{\mathrm{dt}} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right)$$ 614

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\,\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl 615

_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial (b''_i)^{(3)}}{\partial G_j}\left(\,(G_{23})^*\,\right) = s_{ij}$$

616

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 617

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21}$$ 618

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21}$$ 619

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21}$$ 6120

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right)$$ 621

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right)$$ 622

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right)$$ 623

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(4)}$ *and* $(b''_i)^{(4)}$ Belong to $C^{(4)}(\,\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl 624

_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 625

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial (b''_i)^{(4)}}{\partial G_j}\left((G_{27})^*\ \right) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 626

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25}$$ 627

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25}$$ 628

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25}$$ 629

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right)$$ 630

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right)$$ 631

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right)$$ 632

633

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(5)}$ and $(b''_i)^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 634

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a''_{29})^{(5)}}{\partial T_{29}}(T_{29}^{*}) = (q_{29})^{(5)} \quad , \frac{\partial (b''_i)^{(5)}}{\partial G_j}\left((G_{31})^{*} \right) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 635

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29}$$ 636

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^{*}\mathbb{T}_{29}$$ 637

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^{*}\mathbb{T}_{29}$$ 638

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^{*}\mathbb{G}_j\right)$$ 639

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^{*}\mathbb{G}_j\right)$$ 640

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^{*}\mathbb{G}_j\right)$$ 641

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(6)}$ and $(b''_i)^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable 642

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 643

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a''_{33})^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial (b''_i)^{(6)}}{\partial G_j}\left((G_{35})^{*} \right) = s_{ij}$$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain 644

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33}$$ 645

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33}$$ 646

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33}$$ 647

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right)$$ 648

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right)$$ 649

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right)$$ 650

651

The characteristic equation of this system is 652

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$ 653

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\Big(\big((\lambda)^{(2)}\big)^2+\big(\,(a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\big)\,(\lambda)^{(2)}\Big)\,(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\big((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\big)\,\big((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\big)$$

$$\Big(\big((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\big)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\Big)\}=0$$

$$+$$

$$\big((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\big)\{\big((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\big)$$

$$\Big[\Big(\big((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\big)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\Big)\Big]$$

$$\Big(\big((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\big)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\Big)$$

$$+\Big(\big((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\big)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\Big)$$

$$\Big(\big((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\big)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\Big)$$

$$\Big(\big((\lambda)^{(3)}\big)^2+\big(\,(a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\big)\,(\lambda)^{(3)}\Big)$$

$$\Big(\big((\lambda)^{(3)}\big)^2+\big(\,(b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\big)\,(\lambda)^{(3)}\Big)$$

$$+\Big(\big((\lambda)^{(3)}\big)^2+\big(\,(a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\big)\,(\lambda)^{(3)}\Big)\,(q_{22})^{(3)}G_{22}$$

$$+\big((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\big)\,\big((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\big)$$

$$\Big(\big((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\big)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\Big)\}=0$$

$$+$$

$$\big((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\big)\{\big((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\big)$$

$$\Big[\Big(\big((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\big)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\Big)\Big]$$

$$\Big(\big((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\big)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\Big)$$

$$+\Big(\big((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\big)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\Big)$$

$$\Big(\big((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\big)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\Big)$$

$$\Big(\big((\lambda)^{(4)}\big)^2+\big(\,(a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\big)\,(\lambda)^{(4)}\Big)$$

$$\Big(\big((\lambda)^{(4)}\big)^2+\big(\,(b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\big)\,(\lambda)^{(4)}\Big)$$

$$+\Big(\big((\lambda)^{(4)}\big)^2+\big(\,(a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\big)\,(\lambda)^{(4)}\Big)\,(q_{26})^{(4)}G_{26}$$

$$+((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}) \, ((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25} + (a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24})$$

$$(((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})s_{(25),(26)}T^*_{25} + (b_{25})^{(4)}s_{(24),(26)}T^*_{24})\} = 0$$

$$+$$

$$((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)})\{((\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)})$$

$$[(((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)})(q_{29})^{(5)}G^*_{29} + (a_{29})^{(5)}(q_{28})^{(5)}G^*_{28})]$$

$$(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(29)}T^*_{29} + (b_{29})^{(5)}s_{(28),(29)}T^*_{29})$$

$$+(((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)})(q_{28})^{(5)}G^*_{28} + (a_{28})^{(5)}(q_{29})^{(5)}G^*_{29})$$

$$(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(28)}T^*_{29} + (b_{29})^{(5)}s_{(28),(28)}T^*_{28})$$

$$(((\lambda)^{(5)})^2 + ((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}) \, (\lambda)^{(5)})$$

$$(((\lambda)^{(5)})^2 + ((b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)}) \, (\lambda)^{(5)})$$

$$+(((\lambda)^{(5)})^2 + ((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}) \, (\lambda)^{(5)}) \, (q_{30})^{(5)}G_{30}$$

$$+((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}) \, ((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29} + (a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28})$$

$$(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(30)}T^*_{29} + (b_{29})^{(5)}s_{(28),(30)}T^*_{28})\} = 0$$

$$+$$

$$((\lambda)^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)})\{((\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)})$$

$$[(((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)})(q_{33})^{(6)}G^*_{33} + (a_{33})^{(6)}(q_{32})^{(6)}G^*_{32})]$$

$$(((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(33)}T^*_{33} + (b_{33})^{(6)}s_{(32),(33)}T^*_{33})$$

$$+(((\lambda)^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)})(q_{32})^{(6)}G^*_{32} + (a_{32})^{(6)}(q_{33})^{(6)}G^*_{33})$$

$$(((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(32)}T^*_{33} + (b_{33})^{(6)}s_{(32),(32)}T^*_{32})$$

$$(((\lambda)^{(6)})^2 + ((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)}) \, (\lambda)^{(6)})$$

$$(((\lambda)^{(6)})^2 + ((b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)}) \, (\lambda)^{(6)})$$

$$+(((\lambda)^{(6)})^2 + ((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)}) \, (\lambda)^{(6)}) \, (q_{34})^{(6)}G_{34}$$

$$+((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)}) \, ((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33} + (a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32})$$

$$\left(\left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)}\right) s_{(33),(34)} T^*_{33} + (b_{33})^{(6)} s_{(32),(34)} T^*_{32}\right)\} = 0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature 's L:etters,Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, Deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidiation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

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(20)^ [2] Cockcroft-Walton experiment

(21)^ *a b c* Conversions used: 1956 International (Steam) Table (IT) values where one calorie ≡ 4.1868 J and one BTU ≡ 1055.05585262 J. Weapons designers' conversion value of one gram TNT

≡ 1000 calories used.

(22)^ Assuming the dam is generating at its peak capacity of 6,809 MW.

(23)^ Assuming a 90/10 alloy of Pt/Ir by weight, a C_p of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average C_p of 25.8, 5.134 moles of metal, and 132 $J.K^{-1}$ for the prototype. A variation of ±1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are ±2 micrograms.

(24)^ [3] Article on Earth rotation energy. Divided by c^2.

(25)^ ***a b*** Earth's gravitational self-energy is 4.6×10^{-10} that of Earth's total mass, or 2.7 trillion metric tons. Citation: *The Apache Point Observatory Lunar Laser-Ranging Operation (APOLLO)*, T. W. Murphy, Jr. *et al.* University of Washington, Dept. of Physics (132 kB PDF, here.).

(26)^ There is usually more than one possible way to define a field energy, because any field can be made to couple to gravity in many different ways. By general scaling arguments, the correct answer at everyday distances, which are long compared to the quantum gravity scale, should be *minimal coupling*, which means that no powers of the curvature tensor appear. Any non-minimal couplings, along with other higher order terms, are presumably only determined by a theory of quantum gravity, and within string theory, they only start to contribute to experiments at the string scale.

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