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Some Common Fixed Point Theorems Using Faintly Compatible Maps in Fuzzy Metric Space with Integral Type Inequality

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Abstract **-**In present paper we proved Some Common Fixed Point theorems for four self-mappings by using the general contractive condition given by Malhotra et al. [6] and improvises the result by replacing the occasionally weakly compatible (owc) mappings by the faintly compatible pair of mapping in Fuzzy Metric Space with Integral Type Inequality

Keywords **–**Fuzzy Metric Space, Common Fixed Point, Property (E.A), Sub Sequentially Continuity, faintly Compatible maps.

Mathematics Subject Classification: 52H25, 47H10.

INTRODUCTION

Many renowned mathematicians worked with fixed point & gave many results .In 1998,Jungck et al. [4] introduced the notion of weakly compatible mappings and showed that compatible mappings are weakly compatible but not conversely. Al-Thagafi et al. [2] introduced the concept of occasionally weakly compatible (owc) mappings which is more general than the concept of weakly compatible mappings. Aamri et al. [1] generalized the concepts of non-compatibility by defining the notion of (E.A) property in metric space.

Pant et al. [7] introduced the concept of conditional compatible maps.Bisht et al. [3] criticize the concept of occasionally weakly compatible (owc) as follows "Under contractive conditions the existence of a common fixed point and occasional weak compatibility are equivalent conditions, andconsequently, proving existence of fixed points by assuming occasional weak compatibility is equivalent to proving the existence of fixed points by assuming the existence of fixed points". Therefore use of occasional weak compatibility is a redundancy for fixed point theoremsunder contractive conditions to removes this redundancy we used faintly compatiblemapping in our paper which is weaker than weak compatibility or semi compatibility. Faintly compatible maps introduced by Bishtet al. [3] is an improvement of conditionally compatible maps .Using these concepts Wadhwa et al. [8,9] proved some common fixed point theorems. In this paper we prove some common fixed point for four mappings using the concept of faintly compatible pair of mappings in fuzzy metric spaces with Integral Type Inequality.

Preliminary Notes

Definition 2.1: A mapping $*: [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous t-norm if * is satisfying the following conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0, 1]$;
(iv) $a * b \geq c * d$ whenever $a \leq c$ and $b \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.2: A 3- tupple$(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set ,* is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions, for all $x,y,z \in X$ and $s, t > 0$;

(i) $M(x,y,t) > 0$
(ii) $M(x,y,t) = 1$ for all $t>0$ if and only if $x=y$
(iii) $M(x,y,t) = M(y,x,t)$
(iv) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$
(v) $M(x,y,.): \times(0,\infty) \to (0,1]$ is continuous.

Then M is called a fuzzy metric on X. Then $M(x,y,t)$ denotes the degree of nearness between x and y and $t > 0$.

Definition 2.3[5]: A fuzzy metric space (X, M, *) is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.4[5]: Let A and B be mappings from fuzzy metric space (X, M,∗) into itself. The maps A and B are said to be compatible if, for all t >0, $\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$for some x ∈ X.

Definition 2.5 [8]A pair of self-maps (A, B) on afuzzy metric Space (X, M,∗) is said to be
Conditionally compatible:iff whenever the set of sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n$ is non-empty , there exists a sequence $\{z_n\}$ in X Such that $\lim_{n\to\infty} Az_n = \lim_{n\to\infty} Bz_n = t$, for some t ∈ X and $\lim_{n\to\infty} M(ABz_n, BAz_n, t) = 1$, for all t > 0.
Faintly compatible:iff (A, B) is conditionally compatible and A and B commute on a non-empty subset of the set of coincidence points, whenever the set of coincidence points is nonempty.
Satisfy the property (E.A.): if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = t$ for some t ∈X.
Sub sequentially continuous:iff there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x \in X$ and satisfy $\lim_{n\to\infty} ABx_n = Ax$, $\lim_{n\to\infty} BAx_n = Bx$.
Semi-compatible: if $\lim_{n\to\infty} M(ABx_n, Bx_n, t) = 1$, whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$, for some x ∈X.

Definition 2.6 [5]: Two self mappings A and B of a fuzzy metric space (X, M,∗) is said to be non-compatible if there exists at list one sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z in X but neither $\lim_{n\to\infty} M(ABx_n, BAx_n, t) \neq 1$ or the limit does not exists.
Definition 2.7[5]: Let (X, M,∗) be fuzzy metric space. Let A and B be self-maps on X. Then a point x in X is called a coincidence point of A and B iff Ax=Bx. In this case, w=Ax=Bx is called a point of coincidence of A and B.

Definition 2.8 [5]: A pair of self-mappings (A, B) of a fuzzy metric Space (X, M,∗) is said to be weakly compatible if they commute at their Coincidence points i.e Ax=Bx for some x in X, then ABx=BAx.
Definition 2.9 [6]:A pair of self-mappings (A, B) of a fuzzy metric Space (X, M,∗) is said to be occasionally weakly compatible(owc) iff there is a point x in X which is a coincidence point of A and B at which A and B commute.

***Lemma 1*.[6]:** Let (X, M,∗) be a fuzzy metric space. If there exists a number q ∈ (0, 1) M(x, y, qt) ≥ M(x, y, t) for all x, y ∈ X & t >0 then x=y.
MAIN RESULTS

Theorem 3.1 Let (X, M, ∗) be a fuzzy metric space with continuous t-norm P, S, Q, T be mappings from X into itself .If there exists q∈(0,1) such that

$$\int_0^{M(Px,Qy,qt)} \emptyset(t)\,dt \geq \int_0^{Min\{M(Sx,Ty,t),M(Sx,Px,t),M(Qy,Ty,t),M(Px,Ty,t),M(Qy,Sx,t)\}} \emptyset(t)dt$$

(1)

For all x, y ∈ X and for all t >0. If pairs (P, S) and (Q, T) satisfies E.A. property with sub sequentially continuous faintly compatible map then P, S, Q, T have a unique common fixed point in X.

Proof :(P, S) and (Q, T) satisfy E.A property which implies that there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Sx_n = t_1$ for some t_1 ∈X also $\lim_{n\to\infty} Qx_n = \lim_{n\to\infty} Tx_n = t_2$ for some t_2 ∈X. Since pairs (P, S) and (Q, T) are faintly compatible therefore conditionally compatibility of (P, S) and (Q, T) implies that there exist sequences $\{z_n\}$ and $\{z_{n'}\}$ in X satisfying $\lim_{n\to\infty} Pz_n = \lim_{n\to\infty} Sz_n = u$ for some u ∈X, such that $M(PSz_n, SPz_n, t) = 1$, also $\lim_{n\to\infty} Qz_{n'} = \lim_{n\to\infty} Tz_{n'} = v$ for some v ∈X, such that $M(QTz_n', TQz_n', t) = 1$.As the pairs (P, S) and (Q, T) are sub sequentially continuous, we get $\lim_{n\to\infty} PSz_n = Pu$, $\lim_{n\to\infty} SPz_n = Su$ and so Pu = Su ,also $\lim_{n\to\infty} QTz_{n'} = Qv$, $\lim_{n\to\infty} TQz_{n'} = Tv$ and so Qv = Tv. Since pairs (P, S) and (Q, T) are faintly compatible, we get PSu = SPu& So PPu = PSu =SPu = SSu also QTv = TQv& So QQv=QTv=TQv=TTv.

Now we show that Pu=Qv.

Let x=u and y=v in equation (1) we have

$$\int_0^{M(Pu,Qv,qt)} \emptyset(t)\,dt \geq \int_0^{Min\{M(Su,Tv,t),M(Su,Pu,t),M(Qv,Tv,t),M(Pu,Tv,t),M(Qv,Su,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{Min\{M(Su,Tv,t),1,M(Qv,Tv,t),M(Pu,Tv,t),M(Qv,Su,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{Min\{M(Pu,Tv,t),1,1,M(Pu,Tv,t),M(Tv,Pu,t)\}} \emptyset(t)dt$$

Hence from the lemma it is clear that Pu=Qv
Now we have to show that PPu=Pu
Let x=Pu and y=v in equation (1)
We get

$$\int_0^{M(PPu,Pu,qt)} \emptyset(t)\,dt \geq \int_0^{Min\{M(SPu,Tv,t),M(SPu,PPu,t),M(Qv,Tv,t),M(PPu,Tv,t),M(Qv,SPu,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{Min\{M(PPu,Qv,t),1,1,M(PPu,Qv,t),M(\{Pu,PPu,t)\}} \emptyset(t)dt$$

Now by the lemma it is clear that PPu=Pu
Now we have to show that Pu=QQv
Let x=u and y=Qv in equation (1)
We get

$$\int_0^{M(Pu,QQv,qt)} \emptyset(t)\,dt \geq \int_0^{Min\{M(Su,TQv,t),M(Su,Pu,t),M(QQv,TQv,t),M(Pu,TQv,t),M(QQv,Su,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{Min\{M(Su,TQv,t),M(Su,Pu,t),M(QQv,TQv,t),M(Pu,TQv,t),M(QQv,Su,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{Min\{M(Pu,QQv,t),1,1,M(Pu,QQv,t),M(QQv,Pu,t)\}} \emptyset(t)dt$$

Now by the lemma it is clear that Pu=QQv
Now we have
PPu=SPu=Pu
Pu=QQv=QPu
And
Pu=QQv=TQv=TPu
Since Qv=Pu,
Hence we have P(Pu)=S(Pu)=Q(Pu)=T(Pu)
Let Pu=w
P (w) =S (w) =Q (w) =T (w)
Where w is a common fixed point of P, S, Q, and T
Hence the uniqueness of the fixed point holds from equation (1).
Hence Proved.

Theorem 3.2 Let (X, M,∗) be a complete fuzzy metric space and let P, S, Q, T are mappings from X into itself such that

$$\int_0^{M(Px,Qy,qt)} \emptyset(t)\,dt \geq \int_0^{\phi\{M(Sx,Ty,t),M(Sx,Px,t),M(Qy,Ty,t),M(Px,Ty,t),M(Qy,Sx,t)\}} \emptyset(t)dt$$

(2)

For all x, y ∈ X and $\emptyset:[0,1]^5 \to [0,1]$ such that $\emptyset(t,1,1,t,t) > t$ for all 0 < t <1.

If pairs (P, S) and (Q, T) satisfies E.A. property with sub sequentially continuous faintly compatible map then P, S, Q, T have a unique common fixed point in X.

Proof *:*(P, S) and (Q, T) satisfy E.A property which implies that there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Sx_n = t_1$ for some $t_1 \in X$ also $\lim_{n\to\infty} Qx_n = \lim_{n\to\infty} Tx_n = t_2$ for some $t_2 \in X$. Since pairs (P, S) and (Q, T) are faintly compatible therefore conditionally compatibility of (P, S) and (Q, T) implies that there exist sequences $\{z_n\}$ and $\{z_{n'}\}$ in X satisfying $\lim_{n\to\infty} Pz_n = \lim_{n\to\infty} Sz_n = u$ for some u ∈X, such that M $(PSz_n, SPz_n, t) = 1$, also $\lim_{n\to\infty} Qz_{n'} = \lim_{n\to\infty} Tz_{n'} = v$ for some v ∈X, such that M $(QTz_n', TQz_n', t) = 1$.As the pairs (P, S) and (Q, T) are sub sequentially continuous, we get $\lim_{n\to\infty} PSz_n = Pu$, $\lim_{n\to\infty} SPz_n = Su$ and so Pu = Su ,also $\lim_{n\to\infty} QTz_{n'} = Qv$, $\lim_{n\to\infty} TQz_{n'} = Tv$ and so Qv = Tv. Since pairs (P, S) and (Q, T)

are faintly compatible, we get PSu = SPu& So PPu = PSu =SPu = SSu also QTv = TQv& So QQv=QTv=TQv=TTv.

Now we show that Pu=Qv.
Let x=u and y=v in equation (2) we have

$$\int_0^{M(Pu,Qv,qt)} \emptyset(t)\,dt \geq \int_0^{\phi\{M(Su,Tv,t),M(Su,Pu,t),M(Qv,Tv,t),M(Pu,Tv,t),M(Qv,Su,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{\phi\{M(Su,Tv,t),1,M(Qv,Tv,t),M(Pu,Tv,t),M(Qv,Su,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{\phi\{M(Pu,Tv,t),1,1,M(Pu,Tv,t),M(Pu,Tv,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{M(Pu,Qv,t)} \emptyset(t)dt$$

Hence from the lemma it is clear that Pu=Qv
Nowwe show thatPPu=Pu
Let x=Pu and y=v in equation (2)
We get

$$\int_0^{M(PPu,Pu,qt)} \emptyset(t)\,dt \geq \int_0^{\phi\{M(SPu,Tv,t),M(SPu,PPu,t),M(Qv,Tv,t),M(PPu,Tv,t),M(Qv,SPu,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{\phi\{M(PPu,Qv,t),1,1,M(PPu,Qv,t),M(\{Pu,PPu,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{M(PPu,Pu,t)} \emptyset(t)dt$$

Now by the lemma it is clear that PPu=Pu
Now we have to show that Pu=QQv
Let x=u and y=Qv in equation (2)
We get

$$\int_0^{M(Pu,QQv,qt)} \emptyset(t)\,dt \geq \int_0^{\phi\{M(Su,TQv,t),M(Su,Pu,t),M(QQv,TQv,t),M(Pu,TQv,t),M(QQv,Su,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{\phi\{M(Su,TQv,t),M(Su,Pu,t),M(QQv,TQv,t),M(Pu,TQv,t),M(QQv,Su,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{\phi\{M(Pu,QQv,t),1,1,M(Pu,QQv,t),M(QQv,Pu,t)\}} \emptyset(t)dt$$

$$\geq \int_0^{M(Pu,QQv,t)} \emptyset(t)dt$$

Now by the lemma it is clear that Pu=QQv
Now we have
PPu=SPu=Pu
Pu=QQv=QPu
and
Pu=QQv=TQv=TPu
Since Qv=Pu,
Hence we have P(Pu)=S(Pu)=Q(Pu)=T(Pu)
Let Pu=w
P (w) =S (w) =Q (w) =T (w)
Where w is a common fixed point of P, S, Q, and T
Hence the uniqueness of the fixed point holds from equation (2)
Hence Proved.

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