

E- CONVEX FUNCTIONS

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Abstract

Youness introduced the concepts of E – convex sets and E – convex functions and studied their properties. Following this in this paper we further characterize E- convex functions.

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1. Introduction and Preliminaries

Youness introduced the concepts of E – convex sets and E – convex functions in 1999 which have applications in various branches of Mathematical Sciences. In this paper we further discuss some basic properties of E- convex functions.

Definition 1.1: Let $E : R^n \rightarrow R^n$ be a map . A set $M \subseteq R^n$ is said to be E – convex if $\lambda E(x) + (1 - \lambda)E(y) \in M$ for $x, y \in M$ and $0 \leq \lambda \leq 1$ [6]

$E(M) \subseteq M$ whenever M is E-convex.

Definition 1.2: Let $E : R^n \rightarrow R^n$ be a map and let $M \subseteq R^n$ be convex. A function $f: R^n \rightarrow R$ is said to be E – convex on M if

$$f(\lambda E(x) + (1 - \lambda)E(y)) \leq \lambda f(E(x)) + (1 - \lambda) f(E(y)) \text{ for } x, y \in M \text{ and } 0 \leq \lambda \leq 1 \text{ [6]}$$

Definition 1.3:A function f on R^n is said to be positively homogeneous if for every x $f(\lambda x) = \lambda f(x), 0 < \lambda < \infty$. [1]

It can be proved that if f is positively homogeneous then $f(0) = 0$

Lemma 1.4: Let $f : R^n \rightarrow R, E : R^n \rightarrow R^n$ and $M \subseteq R^n$ be an E – convex set such that $E(M)$ is a convex set in R^n . Then f is E- convex if and only if $f(\alpha_1 E(x_1) + \alpha_2 E(x_2) + \dots + \alpha_m E(x_m)) \leq \sum_{i=1}^m \alpha_i f(E(x_i))$ for every integer $m \geq 2$ and all $x_i \in M, \alpha_i \geq 0, i = 1, 2, \dots, m$ with $\sum_{i=1}^m \alpha_i = 1$ [7]

2 Properties

Theorem 2.1: Let $f : R^n \rightarrow R$, be any E convex function and $\alpha \in (-\infty, \infty)$, the sets (i) $\{x : f(E(x)) < \alpha\}$ and (ii) $\{x : f(E(x)) \leq \alpha\}$ are E – convex then It has equivalence property.

Reflexive Property : Let $M = \{x \in R^n : f(E(x)) < \alpha\}$. Let $x \in M, 0 \leq \lambda \leq 1$.

Then $f(E(x)) < \alpha$. Since f is E – convex, using Definition 1.2

$$\text{we have } f(\lambda E(x) + (1 - \lambda)E(x)) \leq \lambda f(E(x)) + (1 - \lambda)f(E(x))$$

$$< \lambda \alpha + (1 - \lambda)\alpha$$

$$= \alpha$$

Symmetry Property :

$$\text{we have } f(\lambda E(x) + (1 - \lambda)E(y)) \leq \lambda f(E(x)) + (1 - \lambda)f(E(y))$$

$$< \lambda \alpha + (1 - \lambda)\alpha$$

$$= \alpha$$

$$\begin{aligned} f(\lambda E(y) + (1 - \lambda)E(x)) &\leq \lambda f(E(y)) + (1 - \lambda)f(E(x)) \\ &< \lambda \alpha + (1 - \lambda)\alpha \\ &= \alpha \end{aligned}$$

Transitive Property : $f(\lambda E(x) + (1 - \lambda)E(y)) < \alpha$

$$f(\lambda E(y) + (1 - \lambda)E(z)) < \alpha$$

then $f(\lambda E(x) + (1 - \lambda)E(z)) < \alpha$

The proof for (ii) is analog.

Lemma 1.5: Let $f : R^n \rightarrow R$, $E : R^n \rightarrow R^n$ and $M \subseteq R^n$ be an E -convex set such that $E(M)$ is a convex set in R^n . Then f is E -convex if and only if $f(\alpha_1 E(x_1) . \alpha_2 E(x_2) \dots \alpha_m E(x_m)) \leq \prod_{i=1}^m \alpha_i f(E(x_i))$ for every integer $m \geq 2$ and all $x_i \in M$, $\alpha_i \geq 0$, $i = 1, 2, \dots, m$ with $\prod_{i=1}^m \alpha_i = 1$

Lemma 1.6: Let $E : R^n \rightarrow R^n$ be linear. Let C_1, C_2, \dots, C_m be the E -convex subsets of R^n . Then $\lambda_1 C_1 . \lambda_2 C_2 \dots . \lambda_m C_m$ is an E -convex set where $\lambda_i \in R$ for $i = 1, 2, 3, \dots, m$.

Theorem 2.2 : Let f be a positively homogeneous E -convex function on $M \subseteq R^n$ such that $E(M)$ is convex in R^n . Then for $\lambda_1 > 0, \lambda_2 > 0, \dots, \lambda_m > 0$ we have

$$f(\lambda_1 E(x_1) . \lambda_2 E(x_2) \dots \lambda_m E(x_m)) \leq \prod_{i=1}^m \lambda_i f(E(x_i))$$

Proof: Let $\lambda_1 > 0, \lambda_2 > 0, \dots, \lambda_m > 0$ and $\lambda_1 . \lambda_2 \dots . \lambda_m = \lambda^m$

Let $c_i = \frac{\lambda_i}{\lambda}$ for $i = 1, 2, \dots, m$. Then since $c_1 . c_2 \dots c_m = 1$

using lemma 1.5 $f(c_1 E(x_1) . c_2 E(x_2) \dots c_m E(x_m)) \leq \prod_{i=1}^m c_i f(E(x_i))$ and using Definition 1.3 we get

$$f(\lambda_1 E(x_1) . \lambda_2 E(x_2) \dots \lambda_m E(x_m)) \leq \prod_{i=1}^m \lambda_i f(E(x_i))$$

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