On Normal Fuzzy Soft Group

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Abstract

In this paper, we introduce the concept of normal fuzzy soft group. We also define the level subsets of a normal fuzzy soft subgroup and discussed some of its properties.

Keywords Fuzzy group, Fuzzy Soft Group, Fuzzy Normal, Normal Fuzzy Soft Group, Fuzzy normalizer.

1. Introduction

There are various types of uncertainties in the real world, but some classical mathematical tools may not be appropriate to model these uncertainties. Many complicated problems in economics, engineering, social sciences, medical sciences and many other fields involve uncertain data. These problems, which one comes face to face with in life, cannot be solved using classical mathematic methods. In classical mathematics, a mathematical model of an object is devised and the notion of the exact solution of this model is determined. Because of that, the mathematical model is too complex, the exact solution cannot be found. There are several well-known theories to describe uncertainty. For instance, fuzzy set theory [1], rough set theory [10] and other mathematical tools. But all of these theories have their inherit difficulties as pointed out by Molodtsov[5]. To overcome these difficulties, Molodtsov introduced the concept of a soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties affecting the existing methods. The theory of soft sets has rich potential for applications in several directions, a few of which were demonstrated by Molodtsov in his pioneer work [5]. Rosenfeld[11] have the new idea of fuzzy subgroups.

Akta.H, Çagman.N[2] an introduction to the new definition of soft groups in a soft set depending on inclusion relation and intersection of sets. Aslam and Qurashi [8] extended the concept of soft group, and discussed some of their properties. They also defined normal soft group, cyclic soft group, abelian soft group, product of soft group, coset of a soft subgroup of a soft group.

In this paper we define a new different algebraic structure of normal fuzzy soft subgroups and study some of their properties.

2. Preliminaries

In this section, we first recall the basic definitions related to Fuzzy Sets, Fuzzy Group, Soft Sets and Fuzzy Soft Group which would be used in the sequel.

Definition: 2.1 Fuzzy Sets[8]

Let G be any sets. A mapping $\alpha : G \to [0,1]$ is called fuzzy sets in G.

Definition: 2.2 Fuzzy Subsets[1]

Let X be any non empty sets. A fuzzy subset α of X is a function $\alpha : X \to [0,1]$.

Definition: 2.3 Soft Group [12]

Let X be a group and α be a soft set over X then α be a soft set over X . Then α is said to be a soft group over X iff $F(a) < X, \forall a \in A$.

Definition: 2.4 Normal Fuzzy Subgroup [6]

Let G be a group. A fuzzy subgroup α of G is said to be normal if for all $x, y \in G$,

$$\alpha(xyx^{-1}) = \alpha(y) \text{ or } \alpha(xy) \geq \alpha(yx).$$

3. Normal Fuzzy soft Subgroups

In this section, we define normal fuzzy soft groups and study some of their basic properties.

Definition: 3.1 Fuzzy Soft Subgroup

A fuzzy set α is called a fuzzy soft subgroup of a group G, if for $x, y \in G$,

(i) $\alpha(xy) \geq T\{\alpha(x), \alpha(y)\}$

(ii) $\alpha(x^{-1}) \geq \alpha(x)$.

Definition: 3.2 Normal Fuzzy Soft Subgroup

Let G be a group. A fuzzy soft subgroup α of G is said to be normal fuzzy soft subgroup, if for all $x, y \in G$ and $\alpha(xyx^{-1}) = \alpha(y)$ or $\alpha(xy) \geq \alpha(yx)$.

Theorem: 3.2.1

Let α, β and γ be three fuzzy soft subgroup of G .Then $\alpha \cap \beta \cap \gamma$ is a fuzzy soft subgroup of G .

Proof:

Let α, β and γ be three fuzzy soft subgroup of G .

(i) $(\alpha \cap \beta \cap \gamma)(xy^{-1}) = T\{(\alpha \cap \beta)(xy^{-1}), \gamma(xy^{-1})\}$

$$\geq T\{\, T\,[(\alpha \cap \beta)(x),\ (\alpha \cap \beta)(y^{-1})],\ T\,[\gamma(x),\ \gamma(y^{-1})]\,\}$$

$$\geq T\{\, T\,[(\alpha \cap \beta)(x),\ \gamma(x)],\ T\,[(\alpha \cap \beta)(y^{-1}),\ \gamma(y^{-1})]\,\}$$

$$= T\,\{\,(\alpha \cap \beta \cap \gamma)(x),\ (\alpha \cap \beta \cap \gamma)(y^{-1})\,\}.$$

Thus $(\alpha \cap \beta \cap \gamma)(xy^{-1}) \geq T\,\{\,(\alpha \cap \beta \cap \gamma)(x),\ (\alpha \cap \beta \cap \gamma)(y^{-1})\,\}$.

(ii) $(\alpha \cap \beta \cap \gamma)(x) = \{(\alpha \cap \beta)(x),\ \gamma(x)\}$

$$= \{[\alpha(x), \beta(x)], \gamma(x)\}$$

$$= \{[\alpha(x^{-1}), \beta(x^{-1})], \gamma(x^{-1})\}$$

$$= \{(\alpha \cap \beta)(x^{-1}), \gamma(x^{-1})\}$$

$$= \{(\alpha \cap \beta \cap \gamma)(x^{-1})\}.$$

Hence $\alpha \cap \beta \cap \gamma$ is a fuzzy soft subgroup of G.

Theorem: 3.2.2

The intersection of any three normal fuzzy soft subgroups of G is also a normal fuzzy soft subgroup of G.

Proof:

Let α, β and γ be three normal fuzzy soft subgroups of G.

By above theorem 3.2.1, $\alpha \cap \beta \cap \gamma$ is a fuzzy soft subgroup of G.

Now for all x, y in G, we have

$$(\alpha \cap \beta \cap \gamma)(yxy^{-1}) = T\{(\alpha \cap \beta)(yxy^{-1}), \gamma(yxy^{-1})\}$$

$$= T\{[\alpha(yxy^{-1}), \beta(yxy^{-1})], \gamma(yxy^{-1})\}$$

$$= T\{[\alpha(x), \beta(x)], \gamma(x)\}$$

$$= T\{(\alpha \cap \beta)(x), \gamma(x)\}$$

$$= (\alpha \cap \beta \cap \gamma)(x).$$

Hence $\alpha \cap \beta \cap \gamma$ is a normal fuzzy soft subgroup of G.

Remark: 3.2.3

If Let $(\alpha \cap \beta)_i, i \in \Delta$ are normal fuzzy soft subgroup of G, then $\cap_{i\in\Delta} (\alpha \cap \beta)_i$ is a normal fuzzy soft subgroup of G.

Theorem: 3.2.4

Let $\alpha \cap \beta$ is a normal fuzzy soft subgroup of G, then for any $y \in G$, we have

$$(\alpha \cap \beta)(y^{-1}xy) = (\alpha \cap \beta)(yxy^{-1}).$$

Proof:

Let $\alpha \cap \beta$ is a normal fuzzy soft subgroup of G, then for any $y \in G$

Now

$$\begin{aligned}(\alpha \cap \beta)(y^{-1}xy) &= (\alpha \cap \beta)(xy^{-1}y)\\ &= (\alpha \cap \beta)(x)\\ &= (\alpha \cap \beta)(yy^{-1}x)\\ &= (\alpha \cap \beta)(yxy^{-1}).\end{aligned}$$

Hence the theorem.

Theorem: 3.2.5

If $\alpha \cap \beta$ is a normal fuzzy soft subgroup of G, then g($\alpha \cap \beta$)g $^{-1}$ is also a normal fuzzy soft subgroup of G, for all $g \in G$.

Proof:

If $\alpha \cap \beta$ is a normal fuzzy soft subgroup of G, then $g(\alpha \cap \beta)g^{-1}$ is a subgroup of G.

Now

$$\begin{aligned}g(\alpha \cap \beta)g^{-1} &= (\alpha \cap \beta)(g^{-1}(yxy^{-1})g)\\ &= (\alpha \cap \beta)(yxy^{-1})\\ &= (\alpha \cap \beta)(x)\\ &= (\alpha \cap \beta)(gxg^{-1})\\ &= g(\alpha \cap \beta)g^{-1}(x).\end{aligned}$$

Hence the theorem.

Definition: 3.3 Level Subset

Let $\alpha \cap \beta$ be a fuzzy soft subgroup of a group G. For any $t \in [0,1]$, we define the level subset of $\alpha \cap \beta$ is the set,

$$(\alpha \cap \beta)^t = \{x \in G/(\alpha \cap \beta)(x) \geq t\}.$$

Theorem: 3.3.1

Let G be a group and $\alpha \cap \beta$ be a fuzzy subset of G. Then $\alpha \cap \beta$ is a normal fuzzy soft subgroup of G iff the level subsets $(\alpha \cap \beta)^t$, $t \in [0,1]$ are subgroup of G.

Proof:

Let $\alpha \cap \beta$ is a normal fuzzy soft subgroup of G and the level subset

$$(\alpha \cap \beta)^t = \{x \in G/(\alpha \cap \beta)(x) \geq t \in [0,1]\}.$$

Let $x, y \in (\alpha \cap \beta)^t$ .Then $(\alpha \cap \beta)(x) \geq t \,\&\, (\alpha \cap \beta)(y) \geq t$.

Now

$$\begin{aligned}(\alpha \cap \beta)(xy^{-1}) &\geq T\{(\alpha \cap \beta)(x),(\alpha \cap \beta)(y^{-1})\}\\ &= T\{(\alpha \cap \beta)(x),(\alpha \cap \beta)(y)\}\\ &\geq T\{t,t\}.\end{aligned}$$

Therefore, $(\alpha \cap \beta)(xy^{-1}) \geq t$

This implies $xy^{-1} \in (\alpha \cap \beta)^t$.

Thus $(\alpha \cap \beta)^t$ is a subgroup of G.

Conversely, Let us assume that $(\alpha \cap \beta)^t$ be a subgroup of G.

Let $x, y \in (\alpha \cap \beta)^t$ .Then $(\alpha \cap \beta)(x) \geq t \,\&\, (\alpha \cap \beta)(y) \geq t$.

Also, $(\alpha \cap \beta)(xy^{-1}) \geq t$, since $xy^{-1} \in (\alpha \cap \beta)^t$.

$$= T\{t,t\}$$

$$= T\{(\alpha \cap \beta)(x),(\alpha \cap \beta)(y)\}.$$

Therefore, $(\alpha \cap \beta)(xy^{-1}) \geq T\{(\alpha \cap \beta)(x),(\alpha \cap \beta)(y)\}$.

Hence $\alpha \cap \beta$ is a normal fuzzy soft subgroup of G.

Definition: 3.5 Fuzzy soft Normalizer

Let G be a group and $\alpha \cap \beta$ be a normal fuzzy soft subgroup of G.

Let $N(\alpha\cap\beta)=\{a\in G/(\alpha\cap\beta)(axa^{-1})=(\alpha\cap\beta)(x),\text{ for all } x\in G\}$.Then N($\alpha\cap\beta$) is called the fuzzy soft Normalizer of $\alpha\cap\beta$.

Theorem: 3.5.1

Let G be a group and $\alpha\cap\beta$ be a fuzzy subset of G. Then $\alpha\cap\beta$ is a normal fuzzy soft subgroup of G iff the level subsets $(\alpha\cap\beta)^t, t\in[0,1]$, are normal subgroup of G.

Proof:

Let $\alpha\cap\beta$ is a normal fuzzy soft subgroup of G and the level subsets $(\alpha\cap\beta)^t, t\in[0,1]$, is a subgroup of G.

Let $x\in G$ and $a\in(\alpha\cap\beta)^t$, then $(\alpha\cap\beta)(a)\geq t.$

Now,

$$(\alpha\cap\beta)(xax^{-1})=(\alpha\cap\beta)(a)\geq t$$

Since $\alpha\cap\beta$ is a normal fuzzy soft subgroup of G.

That is, $(\alpha\cap\beta)(xax^{-1})\geq t.$

Therefore, $xax^{-1}\in(\alpha\cap\beta)^t$.

Hence $(\alpha\cap\beta)^t$ is a normal subgroup of G.

Theorem: 3.5.2

If $\alpha\cap\beta$ is a normal fuzzy soft subgroup of G, iff $(\alpha\cap\beta)^t$ is an anti normal fuzzy soft subgroup of G.

Proof:

Assume $\alpha\cap\beta$ is a normal fuzzy soft subgroup of G. Then for all $x,y\in G.$

$$(\alpha\cap\beta)(xy)\geq T\{(\alpha\cap\beta)(x),(\alpha\cap\beta)(y)\}$$

$$\Leftrightarrow 1-(\alpha\cap\beta)^C(xy)\geq T\{[1-(\alpha\cap\beta)^C(x)],[1-(\alpha\cap\beta)^C(y)]\}$$

$$\Leftrightarrow (\alpha\cap\beta)^C(xy)\leq 1-T\{[1-(\alpha\cap\beta)^C(x)],[1-(\alpha\cap\beta)^C(y)]\}$$

$$\Leftrightarrow (\alpha\cap\beta)^C(xy)\leq \max\{(\alpha\cap\beta)^C(x),(\alpha\cap\beta)^C(y)\}.$$

By definition 3.1, $(\alpha\cap\beta)(x)=(\alpha\cap\beta)(x^{-1})$ for all x in G.

$$1-(\alpha\cap\beta)^C(x)=1-(\alpha\cap\beta)^C(x^{-1})$$

Therefore, $(\alpha\cap\beta)^C(x)=(\alpha\cap\beta)^C(x^{-1}).$

Hence $(\alpha \cap \beta)^C$ is an anti normal fuzzy soft subgroup of G.

Now suppose that,

If $\alpha \cap \beta$ is a normal fuzzy soft subgroup of G

$$\Leftrightarrow (\alpha \cap \beta)(xyx^{-1}) = (\alpha \cap \beta)(y)$$

$$\Leftrightarrow 1-[(\alpha \cap \beta)(xyx^{-1})] = 1-[(\alpha \cap \beta)(y)]$$

$$\Leftrightarrow (\alpha \cap \beta)^C(xyx^{-1}) = (\alpha \cap \beta)^C(y)$$

$\Leftrightarrow$ $(\alpha \cap \beta)^C$ is an anti normal fuzzy soft subgroup of G.

Hence $\alpha \cap \beta$ is a normal fuzzy soft subgroup of G, iff $(\alpha \cap \beta)^t$ is an anti normal fuzzy soft subgroup of G.

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