Some Common Random Fixed Point Results for Expansive Mappings in a Cone Metric Space

PIYUSH M. PATEL[(1)]
Research Scholar of Sainath University,Ranchi(Jarkhand). Asst. Professor in Saffrony Institute of Technology, Mehsana

RAMAKANT BHARDWAJ[(2)]
Associate Professor in Technocrats Institute of Technology and Science, Bhopal M.P).

S. S. RAJPUT
Professor, Govt. P.G. College, Gadarwara, (MP)

SUNIL GARGE
Senier Scientific Officer , MPCST, Bhopal

1. Abstract:

The purpose of this work is to extend and generalize some common random fixed point theorems for Expansive type mappings in complete cone metric spaces. We are attempting to generalize the several well- known recent results.

Key wards: common fixed point, cone metric space, random variable

2. Introduction and Preliminaries

Recently, non-convex analysis has found some applications in optimization theory, and so there have been some investigations about non-convex analysis, especially ordered normed spaces, normal cones and Topical functions (for example[1,2,3]). In these efforts an order is introduced by using vector space cones. Huang and Zhang used this approach in [1]; they defined cone metric spaces by substituting an ordered normed space for the real numbers. In this paper, we shall show that there are no normal cones with normal constant M < 1, and for each k > 1 there are cones with normal constant M > k. Also, by providing non-normal cones and omitting the assumption of normality in some results of [1] . Huang and Zhang [1] introduced the concept of cone metric space by replacing the set of real numbers by an ordered Banach space. They prove some fixed point Theorems for contractive mappings using normality of the cone. The results in [1] were generalized by Sh. Rezapour and Hamlbarani [4] omitted the assumption of normality on the cone, which is a milestone in cone metric space. In this manuscript, the known results [5] are extended to cone metric spaces where the existence of common fixed points for expansive type mappings on cone metric spaces is investigated.

Definition 2.1. Let E be a real Banach space and let P be a subset of E .then P is called Cone if and only if

i. P is closed and $P \neq \{0\}$;

ii. $\alpha,\beta \in R, \alpha,\beta \geq 0, f(t), g(t) \in P \Rightarrow \alpha f(t) + \beta g(t) \in P$;

iii. $f(t) \in P, -f(t) \in P \Rightarrow f(t) = 0$.

Definition 2.2. Let X be non empty set and $d : X \times X \times \Omega \to E$ a mapping such that

$(p1)$ $0 \le d(f(t), g(t)), \forall\, f(t), g(t) \in X, d(f(t), g(t)) = 0 \Leftrightarrow f(t) = g(t)$
$(p2)$ $p(f(t), g(t)) = p(g(t), f(t)); \forall\, f(t), g(t) \in X$
$(p3)$ $p(f(t), g(t)) \le p(f(t), h(t)) + p(h(t), g(t)); \forall\, f(t), g(t), h(t) \in X, \forall t \in \Omega$

then d is called a cone metric on X and (X,d) is called cone metric space on X

Example 2.3. Let $E = R^2, P = \{(f(t), g(t)) \in E, f(t), g(t) \ge 0\}$ and $X = Y$,defined by $d(f(t), g(t)) = (\alpha|f(t)-g(t)|, \beta|f(t)-g(t)|, \gamma|f(t)-g(t)|)$ where $\alpha, \beta, \gamma \ge 0$ is constant then (X,d) is cone metric space.

Definition 2.4. Let (X,d) be a cone metric space $f(t) \in X$ and $\{f_n(t)\}$ be a sequence in X. Then

i. $\{f_n(t)\}_{n\ge1}$ converges to $f(t)$ whenever to every $c \in E$ with $0 \ll c$ there is a natural number N such that $f(t) << c$ for all $n \ge N$.

ii. $\{f_n(t)\}_{n\ge1}$ is said to be a Cauchy sequence if for every $c \in E$ with $0 << c$ there is a natural number N such that $d(f_n(t), f_m(t)) \ge c$ for all $n,m \ge N$.

iii. (X,d) is called a complete cone metric space if every Cauchy sequence in X is convergent in X.

Definition 2.5. Let (X,d) *be a cone metric space,P be a cone in real Banach space E,if*

i $\alpha \in P$ and $\alpha << c$ for some $k \in [0,1]$, then $\alpha = 0$.

ii $u \le v, v \ll w$, then $u \ll w$.

Definition 2.6. *Let* (X,d) *be a cone metric space and P* ***be*** *a cone metric space in real Banach space E and* $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha \ge 0$.

If $f_n(t) \to f(t), g_n(t) \to g(t), h_n(t) \to h(t), p_n(t) \to p$ *in* X and $\alpha_1 d(f_n(t), f(t)) + \alpha_2 d(g_n(t), g(t)) + \alpha_3 d(h_n(t), h(t)) + \alpha_4 d(p_n(t), p)$. then $\alpha = 0$

3. Main Result:

Theorem 3.1. Let (X,d) be a complete cone metric space with respect to a cone P containing in a real Banach space E. Let T, S be any two surjective self mappings of X satisfy

$$d(T(f(t),t), S((g(t),t))) \ge \alpha d(f(t), T(f(t),t)) + \beta d(g(t), S(g(t),t)) +$$
$$\gamma d(f(t), g(t)) + k[d(f(t), S(g(t),t)) + d(g(t), T(f(t),t))] \qquad ---3.1.1$$

for each $f(t), g(t) \in X$, $f(t) \neq g(t)$ where α ,β, γ, $k \geq 0$, α + β + γ >1+2k, β + γ > k and γ >2k. Then T and S have a unique common random fixed point.

Prof. Let $f_0(t)$ be arbitrary point in X.since T, S are subjective mapping, there exit a point $f_1(t) \in T^{-1}(f_0(t), t)$ and $f_2(t) \in S^{-1}(f_1(t), t)$ that is $T(f_1(t), t) = f_0(t)$ and $S(f_2(t), t) = f_1(t)$. In this way, we define the sequence $\{f_n(t)\}$ with $f_{2n+1}(t) \in T^{-1}(f_{2n}(t), t)$ and $f_{2n+2}(t) \in S^{-1}(f_{2n+1}(t), t)$.that means

$T(f_{2n+1}(t), t) = f_{2n}(t)$ and

$$S(f_{2n+2}(t), t) = f_{2n+1}(t) \quad for\, n = 0,1,2,3..... \qquad --- 3.1.2$$

Note that if $f_{2n}(t) = f_{2n+1}(t)$ for some $n \geq 0$ then $f_{2n}(t)$ is fixed point of T, S

Now putting $f(t) = f_{2n+1}(t)$ and $g(t) = f_{2n+2}(t)$ in 3.1.1 we have

$$\begin{aligned} d\left(T(f_{2n+1}(t), t), S(f_{2n+2}(t), t)\right) = {} & \alpha d\left(f_{2n+1}(t), T(f_{2n+1}(t), t)\right) \\ & + \beta d\left(f_{2n+2}(t), T(f_{2n+2}(t), t)\right) \\ & + \gamma d\left(f_{2n+1}(t), f_{2n+2}(t)\right) \\ & + k\begin{bmatrix} d\left(f_{2n+1}(t), S(f_{2n+2}(t), t)\right) \\ +d\left(f_{2n+2}(t), S(f_{2n+1}(t), t)\right) \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \Rightarrow d\left(f_{2n}(t), f_{2n+1}(t)\right) \geq {} & \alpha d\left(f_{2n+1}(t), f_{2n}(t)\right) \\ & + \beta d\left(f_{2n+2}(t), f_{2n+1}(t)\right) \\ & + \gamma d\left(f_{2n+1}(t), f_{2n+2}(t)\right) \\ & + k\left[d\left(f_{2n+1}(t), f_{2n+1}(t)\right) + d\left(f_{2n+2}(t), f_{2n}(t)\right)\right] \end{aligned}$$

$$\begin{aligned} \Rightarrow d\left(f_{2n}(t), f_{2n+1}(t)\right) \geq {} & \alpha d\left(f_{2n+1}(t), f_{2n}(t)\right) \\ & + \beta d\left(f_{2n+2}(t), f_{2n+1}(t)\right) \\ & + \gamma d\left(f_{2n+1}(t), f_{2n+2}(t)\right) \\ & + k\left[d\left(f_{2n+2}(t), f_{2n+1}(t)\right) + d\left(f_{2n+1}(t), f_{2n}(t)\right)\right] \end{aligned}$$

$$\Rightarrow d\left(f_{2n}(t), f_{2n+1}(t)\right) \geq [\alpha + k] d\left(f_{2n+1}(t), f_{2n}(t)\right) + [\beta + \gamma + k] d\left(f_{2n+2}(t), f_{2n+1}(t)\right)$$

$$\Rightarrow d\left(f_{2n+2}(t), f_{2n+1}(t)\right) \leq \frac{[1 - \alpha - k]}{[\beta + \gamma + k]} d\left(f_{2n}(t), f_{2n+1}(t)\right) \qquad --------3.1.3$$

Where $h = \dfrac{[1 - \alpha - k]}{[\beta + \gamma + k]} < 1 \quad as \beta + \gamma + k \geq 1 + 2k$

In general $d\left(f_{2n}(t), f_{2n+1}(t)\right) \le hd\left(f_{2n-1}(t), f_{2n}(t)\right)$

$\Rightarrow d\left(f_{2n}(t), f_{2n+1}(t)\right) \le h^{2n} d\left(f_{2n-1}(t), f_{2n}(t)\right)$

So for every positive integer p we have

$$
\begin{aligned}
\Rightarrow d\left(f_{2n}(t), f_{2n+p}(t)\right) &\le d\left(f_{2n}(t), f_{2n+1}(t)\right) + d\left(f_{2n+1}(t), f_{2n+2}(t)\right) \\
&+ \ldots\ldots\ldots + d\left(f_{2n+p-1}(t), f_{2n+p}(t)\right) \\
&\le \left(h^{2n} + h^{2n+1} + \ldots\ldots + h^{2n+p-1}\right) d\left(f_0(t), f_1(t)\right) \\
&= h^{2n}\left(1 + h + h^2 + \ldots\ldots + h^{p-1}\right) d\left(f_0(t), f_1(t)\right) \\
&< \frac{h^{2n}}{1-h} d\left(f_0(t), f_1(t)\right) \qquad -------3.1.4
\end{aligned}
$$

Therefore $\{f_{2n}(t)\}$ is Cauchy sequence ,which is complete space in X there exist $f^*(t) \in X$ such that $\{f_{2n}(t)\} \to f^*(t)$ .since T is surjective map. There exists point $g(t) \in X$ such that $g(t) = T^{-1}\left(f^*(t), t\right)$ that means $T\left(g(t), t\right) = f^*(t)$ $\quad ---3.1.5$

Now consider

$$
\begin{aligned}
d\left(f_{2n}(t), f^*(t)\right) &= d\left(T\left(f_{2n}(t), t\right), g(t)\right) \\
&\ge \alpha d\left(f_{2n+1}(t), T\left(f_{2n+1}(t), t\right)\right) + \beta d\left(g(t), T\left(g(t), t\right)\right) \\
&+ \gamma d\left(f_{2n+1}(t), g(t)\right) + k\begin{bmatrix} d\left(f_{2n+1}(t), T\left(g(t), t\right)\right) + \\ d\left(g(t), T\left(f_{2n+1}(t), t\right)\right) \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow d\left(f^*(t), f^*(t)\right) &\ge \alpha d\left(f^*(t), f^*(t)\right) + \beta d\left(g(t), f^*(t)\right) + \gamma d\left(f^*(t), g(t)\right) + \\
&+ k\left[d\left(f_{2n+1}(t), T\left(g(t), t\right)\right) + d\left(g(t), T\left(f_{2n+1}(t), t\right)\right)\right]
\end{aligned}
$$

$\Rightarrow 0 \ge [\alpha + \gamma + k] d\left(f^*(t), g(t)\right)$

$\Rightarrow d\left(f^*(t), g(t)\right) = 0 \; as \; \alpha + \gamma + k \ge 0$

$\Rightarrow f^*(t) = g(t).$ $\qquad ---3.1.6$

Here $f^*(t)$ is random fixed point of T as $T\left(g(t), t\right) = f^*(t) = g(t)$

Uniqueness: Let $h(t)$ be any random fixed point of T that means $T\left(h(t), t\right) = h(t)$

then

$$\Rightarrow d\left(f^{*}(t),h(t)\right)=d\left(T\left(f^{*}(t),t\right),T\left(h(t),t\right)\right)$$
$$\geq \alpha d\left(f^{*}(t),T\left(h(t),t\right)\right)+\beta d\left(h(t),T\left(h(t),t\right)\right)+\gamma d\left(f^{*}(t),h(t)\right)$$
$$+k\left[d\left(f^{*}(t),h(t)\right)+d\left(h(t),f^{*}(t)\right)\right]$$
$$=k\left[d\left(f^{*}(t),h(t)\right)+d\left(h(t),f^{*}(t)\right)\right]+\gamma d\left(f^{*}(t),h(t)\right)$$
$$=[\gamma+2k]d\left(f^{*}(t),h(t)\right)$$
$$\Rightarrow d\left(f^{*}(t),h(t)\right)\geq\frac{1}{[\gamma+2k]}d\left(f^{*}(t),h(t)\right)$$

$\Rightarrow d\left(f^{*}(t),h(t)\right)=0\ as\ \gamma>2k$ as proposition 2.5(i)

Corollary 3.2 Let (x, d) be a complete cone metric space with respect to a cone P containing in a real Banach space E. Let T and S be any two surjective self mappings of X satisfying

$$d\left(T\left(f(t),t\right),S\left(g(t),t\right)\right)=\alpha d\left(f(t),T\left(f(t),t\right)\right)+\beta d\left(g(t),S\left(g(t),t\right)\right)+\gamma d\left(f(t),g(t)\right)$$

for each $f(t),g(t)\in X$, $f(t)\neq g(t)$ where α ,β, γ, $k\geq 0$, α + β + γ >1 Then T and S have a unique common random fixed point

Proof: The proof of the corollary immediately follows by putting $k = 0$ in the previous theorem.

Corollary 3.3. Let (X,d) be a complete cone metric space with respect to a cone P containing in a real Banach space$E$. Let T,S be any two surjective self mappings of X satisfy

$$d\left(T\left(f(t),t\right),S\left(g(t),t\right)\right)\geq k\left[d\left(f(t),S\left(g(t),t\right)\right)+d\left(g(t),T\left(f(t),t\right)\right)\right]$$ for each

$f(t),g(t)\in X$, $f(t)\neq g(t)$ where α ,β, γ, $k\geq 0$ then T and S have a unique common random fixed point

Proof: The proof of the corollary immediately follows by putting $\alpha=0,\ \beta=0\ and\ \gamma=0$ in the previous theorem.

References:

[1] Huang Long-Guang, Zhang Xian, Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl. 332 (2007) 1468–1476.

[2] H. Mohebi, Topical functions and their properties in a class of ordered Banach spaces, in: Continuous Optimization, Current Trends and Modern Applications, Part II, Springer, 2005, pp. 343–361.

[3] H. Mohebi, H. Sadeghi, A.M. Rubinov, Best approximation in a class of normed spaces with star-shaped cone, Numer. Funct. Anal. Optim. 27 (3–4) (2006) 411–436.

[4] Sh. Rezapour, R. Hamal barani, some notes on the paper "Cone metric spaces and Fixed Point Theorems of Contractive mappings. J.math.Anal.Appl.345 (2), (2008) 719-724.

[5] Sarla Couhan and Neeraj Msalviya,A Fixed Point Theorem for Expancsive Type Mappiming in Cone Metric Spaces.International Mathematical Forum, 6, 18, (2011) 891-897.

The IISTE is a pioneer in the Open-Access hosting service and academic event management. The aim of the firm is Accelerating Global Knowledge Sharing.

More information about the firm can be found on the homepage:
http://www.iiste.org

CALL FOR JOURNAL PAPERS

There are more than 30 peer-reviewed academic journals hosted under the hosting platform.

Prospective authors of journals can find the submission instruction on the following page: http://www.iiste.org/journals/ All the journals articles are available online to the readers all over the world without financial, legal, or technical barriers other than those inseparable from gaining access to the internet itself. Paper version of the journals is also available upon request of readers and authors.

MORE RESOURCES

Book publication information: http://www.iiste.org/book/

Academic conference: http://www.iiste.org/conference/upcoming-conferences-call-for-paper/

IISTE Knowledge Sharing Partners

EBSCO, Index Copernicus, Ulrich's Periodicals Directory, JournalTOCS, PKP Open Archives Harvester, Bielefeld Academic Search Engine, Elektronische Zeitschriftenbibliothek EZB, Open J-Gate, OCLC WorldCat, Universe Digtial Library , NewJour, Google Scholar