

A NOTE ON PRECONTINUOUS AND ALMOST PRECONTINUOUS MAPPINGS

P.R.S. Choudhary
Department of Mathematics & Computer Science, Govt. Model Science College, Jabalpur, M.P.

Abstract

Recently, a new class of functions between topological spaces called precontinuous functions [7] has been introduced and studied (see also [2], [4], [6]). In the present paper, author has studied the construction of these functions and explores some general criteria regarding the constructive aspects of these functions.

KEY WORDS AND PHRASES : Regular opensets, Preopensets, Precontinuity, Almost precontinuity, s-topological space, p-topological space.

MSC : primary 54C20, 54A35, secondary o3E50, 54C45, 54D15, 54G05

1. PREREQUISITIES :

Let G be subset of an arbitrary topological space $(X, \mathcal{T})$. The closure and the interior of G will be denoted by *Cl(G)* and Int (G) respectively. Throughout this paper, we denote Int *(Cl(G))* by the Bourbaki notation $\alpha(G)$.

Let $(X, \mathcal{T})$ and $(Y, \mathcal{T}')$ be two arbitrary topological spaces and $f : X \to Y$. Denote E = $f^{-1}(G)$ (pre–image of G under the mapping f) whenever G is open in Y.

1.1 Definition : A subset G of X is said to be regular open if $G = \alpha(G)$ (cf. [7],[3] and [4]).

Every regular open set is an open set but the converse is not necessarily true.

1.2 Definition : a subset G of X is said to be preopen [1] if $G \subseteq \alpha(G)$ (cf [3] and [7]).

Every open set is preopen but the converse is not necessarily true. A closed set can not be preopen.

1.3 Definition : A function f is said to be precontinuous [7] (see also [3], [5]) if E is preopen in X.

Every continuous mapping is precontinuous but the converse is not necessarily true.

1.4 Definition : A function f is said to be almost precontinuous [7] if E is preopen in X whenever G is regular open in Y.

Every precontinuous mapping is almost precontinuous but the converse is not necessarily true.

1.5 Definition : Let X be any non-empty set and the topology $\mathcal{T}$ consists of ϕ, X, A and CA. Then (X, $\mathcal{T}$) is said to be a simple (or s)-topological space. Here C stands for the complement of the set A in X.

1.6 Definition : Let X be any non-empty set and τ consists of ϕ and the collection $\{V_a\}$ of subsets of X such that $a \in V_a$ for fixed $a \in$ X. Then (X, $\mathcal{T}$) is said to be the point (or p)-topological space.

2. SOME CHARACTERISTICS PROPERTIES OF p-TOPOLOGICAL SPACE :

Let P(G) and C_a(G) denote the power set and the cardinality of the set G respectively.

2.1 If ***F*** denotes the corresponding family of closed sets in the p-topological space, then it may be verified easily that

$$(\mathcal{T} \cup \boldsymbol{F})_{\sim \phi, X} = P(X) \qquad (2.1)$$

$$C_a(\mathcal{T} \cup \boldsymbol{F}) = C_a(P(X)) + 2 \qquad (2.2)$$

If X consists of n-elements, then it may be verified easily that

$$C_a(\mathcal{T}) = C_a(F) = 2^{n-1} + 1 \qquad (2.3)$$

2.2 It is a T_o space which is not T_1.

2.3 Every open set ($\neq \phi$, X) of this space is preopen and the only regular open sets in this space are ϕ and X.

2.4 It is a connected space.

3. MAIN RESULTS

Let (X, $\mathcal{T}$) be the topological space and ***F*** be the corresponding family of closed sets in (X, $\mathcal{T}$). This suppose that (Y, $\mathcal{T}'$) be another arbitrary topological space.

3.1 Theorem : A function f : X $\rightarrow$ Y is precontinuous if for each open set G in Y, E is not contained in F_a ($\neq$ X) for each $F_a \in$ F.

Proof. Case 1. If E $\in \mathcal{T}$ for all open sets G in Y, the f is continuous and hence it would be precontinuous (cf. definition 1.3).

Case 2. If E $\notin \mathcal{T}$ and given E $\not\subseteq F_a$ ($\neq$ X) for each $F_a \in$ ***F***, it may be verified easily that α(E)=X. Hence, E $\subseteq \alpha$(E) and E is preopen for every open set G in Y. Thus, we assure the precontinuity of f.

Remark. The condition of theorem 3.1 is not necessary.

3.1 Example. Let X {a,b,c,d}, $\mathcal{T}_1$ = { ϕ , X,{a}}, {a,b,c}, {b,c},

F_1 = {X, ϕ,{b,c,d}, {d}, {a,d}}, $\mathcal{T}_2$ = {ϕ, X,{a}} Define f : (X, $\mathcal{T}_1$) $\to$ (X, $\mathcal{T}_2$) such that f(x) = x. It may be observed that f^{-1}{b}={b}$\subseteq${b,c,d}$\in \boldsymbol{F}_1$ and f is precontinuous mapping which is not continuous.

3.2 Thoerem : Let (X, $\mathcal{T}$) be the s-topological space. Then a mapping f : X$\to$Y is precontinuous.

Proof. Case 1. If E=ϕ, X, A or CA for each open set G in Y, then f is continuous and hence precontinuous.

Case 2. If E$\subseteq$X, A or CA, then α(E)=X, A or CA and we have E$\subseteq\alpha$(E) and hence f is precontinuous.

3.3 Theorem : Let (X, $\mathcal{T}$) be the p-topological space and f : X$\to$Y. Then (i) If f is continuous then it is precontinuous. (ii) If f is not continuous, then it can never be precontinuous.

Proof. (i) Follow directly by the definition.
(ii) Let if possible f is not continuous but it is precontinuous. There exists at least one open set G in Y such that E is preopen (not open) in X. In view of equation (2.1), E$\in$**F** and since E is preopen, we get E$\subseteq$Int(E) which is not true. We therefore conclude that f is not precontinuous.

3.4 Theorem : Let (X, $\mathcal{T}$) be the trivial topological space and (Y,$\mathcal{T}'$) be any arbitrary topological space. Then f : X$\to$Y is always precontinuous which may or may not be continuous.

Proof. Case 1. If E=ϕ or X for each open set G in Y then f is continuous and consequently, it is precontinuous.

Case 2. If E$\subseteq$X, then obviously E $\subseteq\alpha$(E) = X. and hence f is pre continuous.

3.5 Theorem : Let (X, $\mathcal{T}$) be the Hausdorff space and f : X$\to$Y where (Y, $\mathcal{T}'$) be any arbitrary topological space. Then (i) If f is continuous, then it is precontinuous. (ii) If f is not continuous, then it can not be precontinuous if E is either finite or compact subset of X for at least one open set G in Y.

Proof. (i) holds by definition.
(ii) Let if possible f is precontinuous but not continuous. We therefore have that E is preopen (not open) for at least one open set G in Y.

Case 1. E is finite and X is Hausadorff (cf Theorem 6.8 of [5]), therefore E is closed. Since E is preopen, we conclude that E $\subseteq$ Int(E) which is not true. Thus f is not precontinuous is this case.

Case 2. E is compact and X is Hausdorff (cf. Theorem 5.3 of [5]), hence E is closed. As proceeded in case 1, we again conclude that f is not precontinuous

Remark. Theorem 3.5 holds even when X is not Hausdorff.

3.2 Example. Let X be an infinite set, let $\mathcal{T}_F$ be the collection of all subsets U if X such that X–U is either finite or is all of X [5]. Then, it may be checked easily that $\mathcal{T}_F$ is a topology on X. The corresponding collection F is closed sets consists of all finite sets and the set X. X is not Hausdorff. If we define f : (X, $\mathcal{T}$) $\to$ (X, $\mathcal{T}'$) and if E is finite in X, then it is closed and hence f is not precontinuous.

3.6 Theorem : Let (X, $\mathcal{T}'$) be a p-topological space. Then f : X$\to$Y in almost precontinous.

Proof. The only regular open sets in Y are ϕ and Y. Their pre-images under, viz. ϕ and X are open (preopen) in X and therefore f is almost precontinuous. (cf. [7]).

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