Common Fixed Point Theorem for ψ-weakly commuting maps in L-Fuzzy Metric Spaces for integral type

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Abstract
In this paper, we proved a common fixed point theorem ψ-weakly commuting maps in L-Fuzzy Metric Spaces for integral type inequality.
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1. Introduction

In 1922, Let (X, d) be a complete metric space, c∈ (0, 1) and f: X→X be a mapping such that for each x, y ∈ X, d $(fx, fy) \leq$ c d(x, y) Then f has a unique fixed point a ∈ X, such that for each x ∈ X, $\lim_{n\to\infty} f^n x = a$ by S. Banach [23].As a generalization of fuzzy sets introduced by L.A.Zadeh [14], K. Atanassov [13] introduced the idea of intuitionistic fuzzy set. Fixed point and common fixed point properties for mappings defined on fuzzy metric spaces by [5], [6], [8], [15], [16], Intuitionistic fuzzy metric spaces by [7], [21]. A. George and P.Veeramani [2] modified the concept of fuzzy metric space introduced by I. Kramosil and J. Michalek [10] and defined a Hausdorff topology on this fuzzy metric space by [12]. Most of the properties which provide the existence of fixed points and common fixed points are of linear contractive type conditions. L-fuzzy metric spaces have been studied by many authors [11], [24]. H. Adibi et al.[9] introduced the concept of compatible mappings and proved common fixed point theorems for four mappings satisfying some conditions in L-fuzzy metric spaces. In the sequel, we shall adopt the usual terminology, notation and conventions of L-fuzzy metric spaces introduced by R. Saadati et al. [19] which are a generalization of fuzzy metric spaces and intuitionistic fuzzy metric spaces [20]. R. Saadati, S.Sedghi and H. Zhou [22] by a common fixed point theorem ψ-weakly commuting maps in L-Fuzzy Metric Spaces

2. Preliminaries

Definition 2.1 [1]: Let (X, d) be a complete metric space, c∈ (0, 1) and f: X→X be a mapping such that for each x, y ∈ X,

$$\int_0^{d(fx,fy)} \varphi(t)dt \leq c \int_0^{d(x,y)} \varphi(t)dt$$

where φ: [0,+∞) →[0,+∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0,+∞) , non negative, and such that for each ε > o, $\int_0^{\varepsilon} \varphi(t)dt,$ then f has a unique fixed point $a \in X$ such that for each $x \in X, \lim_{n\to\infty} f^n x = a.$

B.E.Rhoades [4], extending the result of Branciari by replacing the above condition by the following

$$\int_0^{d(fx,fy)} \varphi(t)dt \leq c \int_0^{max\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fx)}{2}\right\}} \varphi(t)dt.$$

Definition 2.2[11] Let L = (L, $\leq_L$) be a complete lattice, and U a nonempty set called a universe.

An L-fuzzy set A on U is defined as a mapping A : U → L. For each u in U, A(u) represents the degree (in L) to which u satisfies A.

Lemma 2.1[8]. Consider the set L^* and the operation $\leq_{L^*}$ defined by:

$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0,1]^2$ and $x_1 + x_2 \le 1\}$, $(x_1, x_2) \le_{L^*} (y_1, y_2) \leftrightarrow x_1 \le y_1$ and $x_2 \ge y_2$, for every
$(x_1, x_2), (y_1, y_2) \in L^*$. Then $(L^*, \le_{L^*})$ is a complete lattice.
Classically, a triangular norm T on ([0, 1],$\le$) is defined as an increasing, commutative, associative mapping T: $[0,1]^2 \to [0,1]$ satisfying T(1, x) = x, for all x $\in$ [0, 1]. These definitions can be straightforwardly extended to any lattice L = (L, $\le_L$) Define first 0_L = inf L and 1_L = sup L.

Definition 2.3[19]. A triangular norm (t-norm) on L is a mapping T: $L^2 \to L$ satisfying the following conditions:
(i) $(\forall x \in L)(T(x, 1_L) = x)$; (boundary condition)
(ii) $(\forall (x, y) \in L^2)(T(x, y) = T(y, x))$; (commutativity)
(iii) $(\forall (x, y, z) \in L^3)(T(x, T(y, z)) = T(T(x, y), z))$; (associativity)
(iv) $(\forall (x, x', y, y') \in L^4)(x \le_L x'$ _ and $y \le_L y' \Rightarrow T(x, y) \le_L T(x', y'))$;(monotonicity)
A t-norm $\mathcal{T}$ on L is said to be continuous if for any x, y $\in$ X and any sequences $\{x_n\}$ & $\{y_n\}$ which converge to x and y we have

$$\lim_{n\to\infty} \mathcal{T}(x_n, y_n) = \mathcal{T}(x, y)$$

Definition 2.4 [7].A t-norm $\mathcal{T}$ on L^* is called t-represent able if and only if there exist a t-norm T and a t-co norm S on [0, 1] such that, for all $(x_1, x_2), (y_1, y_2) \in L^*$. $\mathcal{T}(x, y) = (T(x_1, y_1), S(x_2, y_2))$.

Definition 2.5[19]. A negation on L is any decreasing mapping N: L $\to$ L satisfying N (0_L) = 1_L and N (1_L) = 0_L. If N (N(x)) = x, for all x $\in$ L, then N is called an involutive negation.

Definition 2.6[19]. The 3-tuple (X, M, $\mathcal{T}$) is said to be an L-fuzzy metric space if X is an arbitrary (non-empty) set, $\mathcal{T}$ is a continuous t–norm on L and M is an L-fuzzy set on $X^2 \times (0, \infty)$satisfying the following conditions for every x, y, z in X and t, s in $(0, \infty)$:
(a) M(x, y, t) $>_L 0_L$;
(b) M(x, y, t) = 1_L for all t > 0 if and only if x = y;
(c) M(x, y, t) = M(y, x, t);
(d) $\mathcal{T}$(M(x, y, t), M(y, z, s)) $\le_L$ M(x, z, t + s);
(e) M(x, y, ·) : $(0, \infty) \to$ L is continuous.
Let (X, M, $\mathcal{T}$) be an L-fuzzy metric space. For t $\in (0, \infty)$, we define the open ball B(x, r, t) with center x $\in$ X and radius r $\in L \setminus \{0_L, 1_L\}$, as B(x, r, t) = {y ∈ X :M(x, y, t) $>_L$ N(r)}. A subset A $\subseteq$ X is called open if for each x $\in$ A, there exist t > 0 and r $\in L \setminus \{0_L, 1_L\}$ such that B(x, r, t) $\subseteq$ A. Let $\mathcal{T}_M$ denote the family of all open subsets of X. Then $\mathcal{T}_M$ is called the topology induced by the L-fuzzy metric M.

Example 2.1 [21]. Let (X, d) be a metric space. Denote $\mathcal{T}(a, b) = (a_1 b_1, \min(a_1 + b_2, 1))$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in L^* and let M and N be fuzzy sets on $X^2 \times (0,\infty)$ be defined as follows:
$M_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)) = \left(\frac{t}{t+md(x,y)}, \frac{d(x,y)}{t+d(x,y)}\right)$, in which $m > 1$.Then (X, $M_{M,N}$,$\mathcal{T}$) is an intuitionistic fuzzy metric space.

Example 2.2 [19]. Let X = N. Define $\mathcal{T}(a, b) = (\max(0, a_1 + b_1 - 1), a_2 + b_2 - a_2 b_2)$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in L^*, and let M (*x*, *y*, *t*) on $X^2 \times (0,\infty)$ be defined as follows:

$$M(x,y,t) = \begin{cases} \left(\frac{x}{y}, \frac{y-x}{y}\right) & \text{if } x \le y \\ \left(\frac{x}{y}, \frac{x-y}{x}\right) & \text{if } y \le x \end{cases}$$

for all x, y $\in$ X and t $>$ 0. Then (X, M, $\mathcal{T}$) is an L-fuzzy metric space.
Let (X, M, $\mathcal{T}$) be an L-fuzzy metric space. For t$\in (0, \infty)$, we define the open ball B(x, r, t) with center x $\in$ X and radius r $\in L \setminus \{0_L, 1_L\}$, as B(x, r, t) = {y ∈ X :M(x, y, t) $>_L$ N(r)}. A subset A $\subseteq$ X is called open if for

each x ∈ A, there exist t > 0 and r ∈ L \ {0_L,1_L} such that B(x, r, t) ⊆ A. Let $\mathcal{T}_M$ denote the family of all open subsets of X. Then $\mathcal{T}_M$ is called the topology induced by the L-fuzzy metric M.

Lemma 2.2 [9]. Let (X, M, $\mathcal{T}$) be an L-fuzzy metric space. Then M(x, y, t) is non decreasing with respect to t, for all x, y in X.

Definition 2.7[19]. A sequence $\{x_n\}_{n\epsilon N}$in an L-fuzzy metric space (X, M, $\mathcal{T}$) is called a Cauchy sequence, if for each ε ∈ L \ {0_L} and t > 0, there exists n_0 ∈ N such that for all m ≥ n ≥ n_0 (n ≥ m≥ n_0), M (x_m, x_n, t) $>_L$ N(ε). The sequence $\{x_n\}_{n\epsilon N}$ is said to be convergent to x ∈ X in the L-fuzzy metric space (X, M, $\mathcal{T}$) if $M(x_n, x, t) = M(x, x_n, t) = 1_L$ whenever $n \to \infty$ for every t > 0. A L-fuzzy metric space is said to be complete if and only if every Cauchy sequence is convergent.

Definition 2.8 [22] Let (X, M,$\mathcal{T}$) be an L-fuzzy metric space. M is said to be continuous on X^2 ×(0,∞) if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t)$$

whenever a sequence {(x_n, y_n, t_n)} in X^2 ×(0 ,∞) converges to a point (x, y, t) ∈ X^2 ×(0 ,∞) i.e.,

$$\lim_{n\to\infty} M(x_n, x, t) = lsim_{n\to\infty} M(y_n, y, t) = 1_L \ \& \ \lim_{n\to\infty} M(x, y, t_n) = M(x, y, t).$$

Lemma 2.3 [22] Let (X, M, $\mathcal{T}$) be an L-fuzzy metric space. Then M is continuous function on X^2 ×(0,∞).

Definition 2.9[22] Let $A \ and \ B$ be maps from an L-fuzzy metric space (X, M, $\mathcal{T}$) into itself. The maps $f \ and \ g$ are said to be weakly commuting if $\mathrm{M}\,(ABx, BAx, t) \ \geq_L \ \mathrm{M}\,(Ax, Bx, t)$ for each x in X & $t > 0.$

Definition 2.10[22]. Let $A \ and \ B$ be maps from an L-fuzzy metric space (X, M, $\mathcal{T}$) into itself. The maps $A \ and \ B$ are said to be ψ-weakly commuting if there exists a positive real function ψ: $(0, \infty) \to (0, \infty)$ such that $\mathrm{M}\,(ABx, BAx, t) \ \geq_L \ \mathrm{M}\,(Ax, Bx, \psi(t))$ for each x in X and $t > 0.$

Example 2.3[22]. Let X = R. Let $\mathcal{T}$(*a*, *b*) = ($a_1 b_1$, min ($a_1 + b_2$, 1)) for all *a* = (a_1, a_2) and *b* = (b_1, b_2) in L^* and let M and N be fuzzy sets on X^2 ×(0,∞) be defined as follows:

$$M_{M,N}(x, y, t) = \left((e^{\frac{|x-y|}{t}})^{-1}, \frac{\left(e^{\frac{|x-y|}{t}}\right)-1}{(e^{\frac{|x-y|}{t}})} \right)$$, for all $t > 0$. Then (X, $M_{M,N}$,$\mathcal{T}$) is an intuitionistic fuzzy metric space. Define $A(x) = 2x - 1, B(x) = x^2, then$

$$\mathrm{M}_{M,N}(x,y,t) = \left((e^{\frac{|x-y|}{t}})^{-1}, \frac{\left(e^{\frac{|x-y|}{t}}\right)-1}{(e^{\frac{|x-y|}{t}})} \right)$$

$$= \left((e^{\frac{2|x-y|^2}{t}})^{-1}, \frac{\left(e^{\frac{2|x-y|^2}{t}}\right)-1}{(e^{\frac{2|x-y|^2}{t}})} \right)$$

$$= \left((e^{\frac{2|x-y|^2}{t/2}})^{-1}, \frac{\left(e^{\frac{2|x-y|^2}{t/2}}\right)-1}{(e^{\frac{2|x-y|^2}{t/2}})} \right) = M_{M,N}(Ax, Bx, t/2)$$

$$<_{L^*}\left((e^{\frac{|x-y|^2}{t}})^{-1},\frac{\left(e^{\frac{|x-y|^2}{t}}\right)-1}{(e^{\frac{|x-y|^2}{t}})}\right)=M_{M,N}(Ax,Bx,t)$$

. Therefore, for $\psi(t)={}^{t}/_{2}$, $A\ and\ B\ are\ \psi$ weakly commuting. But $A\ and\ B\ are$ not weakly commuting since the exponential function is strictly increasing.

3. Main Results

Theorem 3.1. Let (X, M, $\mathcal{T}$) be a left L-fuzzy metric space and let A and B be ψ weakly commuting self-mappings of X satisfying the following conditions:

(3.1.1) $A(X)\subseteq B(X)$;

(3.1.2) either A or B is continuous;

(3.1.3)

$$\int_0^{M\,(Ax,Ay,t)}\xi(t)dt\geq_L\int_0^{C\{M(Bx,By,t),M(Bx,Ax,t),M(Ax,By,t)\}}\xi(t)dt$$

where $C:L\rightarrow L$ is a continuous function such that $C(a)>_L a$ for each $a\in L\setminus\{0_L,1_L\}$, for every x, y in X .Then A and B have a unique common fixed point in X.

Proof. Let $x_0\in X$ be an arbitrary point in X. By (3.1.1), there exists $x_1\in X$ such that $Ax_0=Bx_1$. In general choose x_{n+1} such that $Ax_n=Bx_{n+1}$. Then for t > 0,

$$\int_0^{M(Ax_n,Ax_{n+1},t)}\xi(t)dt\geq_L\int_0^{C\{M(Bx_n,Bx_{n+1},t),M(Bx_n,Ax_n,t),M(Ax_n,Bx_{n+1},t)\}}\xi(t)dt$$

$$\geq_L\int_0^{C\{M(Ax_{n-1},Ax_n,t),M(Ax_{n-1},Ax_n,t),M(Ax_n,Ax_n,t)\}}\xi(t)dt$$

$$=\int_0^{M(Ax_{n-1},Ax_n,t)}\xi(t)dt$$

Thus,$\{M(Ax_n,Ax_{n+1},t);n\geq 0\}$is an increasing sequence in L and therefore, tends to a limit $a\leq_L 1_L$. we claim that $a=1_L$. For if $a<_L 1_L$, when $n\rightarrow\infty$ in the above inequality we get $a\geq_L C(a)>_L a$ a contradiction. Hence $a=1_L$, i.e.

$\lim_{n\rightarrow\infty}M(Ax_n,Ax_{n+1},t)=1_L$.

If we define (2.9) $c_n(t)=M(Ax_n,Ax_{n+1},t)$ then $\lim_{n\rightarrow\infty}c_n(t)=1_L$. Now, we prove that $\{Ax_n\}$ is a Cauchy sequence in $A(X)$. suppose that $\{Ax_n\}$ is not a Cauchy sequence in $A(X)$. For convenience, Let $y_n=Ax_n$ for n =1,2,3……Then there is an $\epsilon\in L\setminus\{0_L,1_L\}$ such that for each integer k, there exists integers $m(k)and\ n(k)$ with $m(k)>n(k)\geq k$ such that

(2.10) $d_k(t)=M(y_{n(k)},y_{m(k)},t)\leq N(\epsilon)$ for k = 1, 2, 3…..

We may assume that

Example.2.3 $M\left(y_{n(k)},y_{m(k)-1},t\right)>N(\epsilon)$,

by choosing $m(k)$ to be the smallest number exceeding $n(k)$ for which (2.10) holds. Using (2.9), we have

(3.1)

$$N(\epsilon)\geq d_k(t)\geq\mathcal{T}\left(M\left(y_{n(k)},y_{m(k)-1},{}^{t}/_{2}\right),M\left(y_{m(k)-1},y_{m(k)},{}^{t}/_{2}\right)\right)\geq\mathcal{T}\left(c_k\left(\frac{t}{2}\right),N(\epsilon)\right)$$

Hence, $d_k(t) \to N(\epsilon)$ for every $t > 0$ as $k \to \infty$.
We know that

$$d_k(t) = M\left(y_{n(k)}, y_{m(k)}, t\right)$$
$$\geq \mathcal{T}^2\{M\left(y_{n(k)}, y_{m(k)+1}, {}^t/_3\right), M\left(y_{n(k)+1}, y_{m(k)+1}, {}^t/_3\right), M\left(y_{n(k)+1}, y_{m(k)+1}, {}^t/_3\right)\}$$
$$\geq \mathcal{T}^2\left\{\left(c_k({}^t/_3), C\left(M\left(y_{n(k)}, y_{m(k)+1}, {}^t/_3\right)\right), c_k({}^t/_3)\right)\right\}$$
$$= \mathcal{T}^2\left\{\left(c_k({}^t/_3), C\left(d_k({}^t/_3)\right), c_k({}^t/_3)\right)\right\}$$

Thus, as $k \to \infty$ in the above inequality we have $N(\epsilon) \geq C\left(N(\epsilon)\right) > N(\epsilon)$ which is a contradiction. Thus, $\{Ax_n\}$ is a Cauchy and by the completeness of X, $\{Ax_n\}_n$ converges to z in X. Also $\{Bx_n\}_n$ converges to z in X. Let us suppose that the mapping A is continuous. Then $\lim_{n\to\infty} AAx_n = Az$ and $\lim_{n\to\infty} ABx_n = Az$. Further we have since A and B be ψ weakly commuting

$$\mathrm{M}\ (ABx, BAx, t) \ \geq_L \ \mathrm{M}\ (Ax, Bx, \psi(t))$$

On letting $n \to \infty$ in the above inequality we get $\lim_{n\to\infty} BAx_n = Az$, by lemma (2.3). We now prove that $z = Az$. Suppose $z \neq Az$ then $M(z, Az, t) <_L 1_L$. By (3.1.3)

$$\int_0^{M(Ax_n, AAx_n, t)} \xi(t)dt \geq_L \int_0^{C\{M(Bx_n, BAx_n, t), M(Bx_n, Ax_n, t), M(Ax_n, BAx_n, t)\}} \xi(t)dt$$

Letting $n \to \infty$ in the above inequality we get

$$\int_0^{M(z,Az,t)} \xi(t)dt \geq_L \int_0^{C\{M(z,Az,t), M(z,Az,t), M(z,Az,t)\}} \xi(t)dt >_L \int_0^{M(z,Az,t)} \xi(t)dt$$

a contradiction. Therefore, $z = Az$. Since $A(X) \subseteq B(X)$ we can find z_1 in X such that $z = Az = Bz_1$.
Now,

$$\int_0^{M\ (AAx_n, Az_1, t)} \xi(t)dt \geq_L \int_0^{C\{M(BAx_n, Bz_1, t), M(BAx_n, AAx_n, t), M(AAx_n, Bz_1, t)\}} \xi(t)dt$$

Letting $n \to \infty$ in the above inequality we get

$$\int_0^{M(Az, Az_1, t)} \xi(t)dt \geq_L \int_0^{C\{M(Az, Bz_1, t), M(Az, Az, t), M(Az, Bz_1, t)\}} \xi(t)dt \geq_L \int_0^{C(M(Az, Bz_1, t))} \xi(t)dt$$

Since $C(1_L) = 1_L$, this implies that $Az = Az_1, i.e.\ z = Az = Az_1 = Bz_1$. also for any $t > 0$, $\mathrm{M}(Az, Bz, t) = \mathrm{M}(ABz_1, BAz_1, t) \geq_L \mathrm{M}\left(Az_1, Bz_1, \psi(t)\right) = 1_L$ which again implies that $Az = Bz$. thus z is a common fixed point of A and B. Now, to prove uniqueness suppose $z \neq \dot{z}$ is another common fixed point of A and B. Then there exists $t > 0$ such that $M(z, \dot{z}, t) <_L 1_L$ and

$$\int_0^{M\ (z, \dot{z}, t)} \xi(t)dt = \int_0^{M\ (Az, A\dot{z}, t)} \xi(t)dt$$
$$\geq_L \int_0^{C\{M(Bz, B\dot{z}, t), M(Bz, Az, t), M(Az, B\dot{z}, t)\}} \xi(t)dt$$
$$= \int_0^{C\{M(z, \dot{z}, t)\}} \xi(t)dt >_L \int_0^{M(z, \dot{z}, t)} \xi(t)dt$$

Which is contradiction. Therefore, $z = \dot{z}$. i.e, z is a unique common fixed point A and B.

Example 2.4[22].Consider example 2.1 in which $X = [0,1]$.

Define $A(x) = 1$ and $B(x) = \begin{cases} 1, & if\ x\ is\ rational \\ 0, & if\ x\ is\ irrational \end{cases}$ on X. It is evident that $A(X) \subseteq B(X)$, A is continuous

and B is discontinuous. Define $C: L^* \to L^*$ by $C(a) = (\sqrt{a_1}, a_2{}^2)$, then $C(a) = (\sqrt{a_1}, a_2{}^2) >_{L^*} (a_1, a_2) = a$ for $0 < a_i < 1, i = 1,2$ and $M(Ax, Ay, t) \geq_{L^*} C(M(Bx, By, t))$ for all x, y in X, A and B be ψ weakly commuting. Thus all the conditions of last theorem are satisfied and 1 is a common fixed point of A and B.

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