

FIXED POINT THEOREMS FOR A GENERALIZED ALMOST CONTRACTIVE MAPPINGS IN ORDERED METRIC SPACES FOR INTEGRAL TYPE

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Abstract

In this paper, the existence theorems of fixed points and common fixed points for two weakly increasing mappings satisfying a new condition in ordered metric spaces are proved. Our results extend, generalize and unify most of the fundamental metrical fixed point thaorems in the literature in Integral type mappings.

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1. Introduction and Preliminaries

Fixed point theory plays basic role in application of various branches of mathematics from elementary calculus and linear algebra to topology and analysis. Fixed point theory is not restricted to mathematics and this theory has many application in other disciplines. This theory is closely related to game theory, military, economics, statistics and medicine.

Theorem 1.1 (Banach's contraction principle) Let (X, d) be a complete metric space, c∈(0,1) and f: X→X be a mapping such that for each x, y ∈X,
d(fx, fy) ≤ cd (x, y) Then f has a unique fixed point a ∈X, such that for each
x∈ X, $\lim_{n\to\infty} f^n(x) = a.$

Theorem1.2 (Branciari) Let (X, d) be a complete metric space ,c $\in (0,1)$ and
let f : X→ X be a mapping such that for each x, y ∈X,
$\int_0^{d(fx,fy)} \emptyset(t)dt \le c \int_0^{d(x,y)} \emptyset(t)dt$ where $\emptyset$:[0,+ ∞) →[0,+ ∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0,+ ∞) , non-negative ,and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \emptyset(t)dt$, then f has a unique fixed point a ∈X, such that for each x ∈ X,
$\lim_{n\to\infty} f^n(x) = a.$

After the paper of Branciari, a lot of research works have been carried out on generalizing contractive condition of integral type for different contractive mappings satisfying various known properties. A fine work has been done by Rhoades [2] extending the result of Branciari by replacing the condition [1.2] by the following

$$\int_0^{d(fx,fy)} \emptyset(t)dt \le \int_0^{\max\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fx)}{2}\right\}} \emptyset(t)dt.$$

Due to its simplicity and usefulness, it has become a very popular tool in solving existence problems in many branches of mathematical analysis and its has many applications in solving nonlinear equations. Then, several authors studied and extended it in many direction;

Despite these important features, Theorem 1.1 suffers from one drawback: the contractive condition (1.1) forces T to be continuous on X. It was then natural to ask if there exist weaker contractive conditions which do not imply the continuity of T. In 1968, this question was answered in confirmation by Kannan[20], who extended Theorem 1.1 to mappings that need not be continuous on X.

On the other hand, Sess[35] introduced the notation of weakly commuting mappings, which are a generalization of commuting mappings, while Jungck[18] generalized the notation of weak commutativity by introducing compatible mappings and then weakly compatible mappings[19].

In 2004, Berinde[4] defined tha notion of a weak contraction mapping which is more general than a contraction mapping. However in [5] Berinde renamed it as an almost contraction mapping, which is more appropriate. Berinde[4] proved some fixed point theorems for almost contractions in complete metric spaces. Afterward, many authors have studied this problem and obtained significant results. Moreover Berinde[4] proved that any strict contraction, the Kannan[20] and Zamfirescu[43] mappings as well as a large class of quasi-contractions are all almost contractions.

Let T and S be two self mappings in a metric space(X,d). The mapping T is said to be a S – contraction if there exists k $\in$ (0,1) such that d (Tx, Ty) $\leq$ k d(Sx, Sy) for all x, y $\in$ X.

In 2006, Al- Thagafi and Shahzad [1] proved the following theorem which is a generalization of many known results.

Theorem 1.3. Let E be a subset of a metric space (X,d) and S, T be two self maps of E such that T(E) $\subseteq$ S (E). Suppose that S and T are weakly compatible, T is an S- contraction and S(E) is complete. Then S and T have a unique common fixed point in E.

Recently Babu et al. [2] defined the class of mappings satisfying condition(B) as follows.

Definition 1.4. Let (X,d) be a metric space. A mapping T: X$\rightarrow$X is said to satisfy condition (B) if there exist a constant δ $\in$(0,1) and some L $\geq$ 0such that

$$d(Tx,Ty) \leq \delta\, d(x,y) + L \min\{d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\}$$

for all x,y $\in$ X.

They proved a fixed point theorem for such mappings in complete metric spaces. They also discussed quasi-contraction, almost contraction and the class of mappings that satisfy condition (B) in detail.

In recent year, Ciric [15] defined the following class of mappings satisfying an almost generalized contractive condition.

Definition 1.5. Let (X,d) be a metric space, and let S, T: X$\rightarrow$X . A mapping T is called an almost generalized contraction if there exist δ $\in$[0,1) and

L $\geq$ 0 such that

$$d(Tx,Sy) \leq \delta\, M(x,y) + L \min\{d(x,Tx), d(y,Sy), d(x,Sy), d(y,Tx)\}$$

for all x,y $\in$ X. where

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Sy), \frac{d(x,Sy)+d(y,Tx)}{2}\right\}$$

Definition 1.6. Let (X,d) be a metric space, and let S, T: X$\rightarrow$ X, are said to be compatible of type (A) if

$$\lim_{n\to\infty} d(TSx_n, SSx_n) = 0 \text{ and } \lim_{n\to\infty} d(STx_n, TTx_n) = 0,$$

Whenever { x_n } is a sequence in X such that $\lim\limits_{n\to\infty} Sx_n = \lim\limits_{n\to\infty} Tx_n = z$

For some z $\in$ X.

Proposition 1.7. Let (X,d) be a metric space, and let S, T: X→ X, are compatible of type (A) and $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n$ = z for some z ∈ X. Then we have

(1) $\lim_{n\to\infty} TSx_n$ = Sz if S is continuous at z.

(2) STz = TSz and Sz = Tz if S and T are continuous at z.

Definition 1.8. A pair (S,T) of self- mappings on X is said to be weakly compatible if S and T commute at their coincidence point. A point y ∈ X is called a point of coincidence of two self-mappings S and T on X if there exists a point x ∈ X such that y = Tx = Sx.

Definition 1.9. Let X bea nonempty set. Then (X,d, ≤) is called an ordered metric space iff:

(1) (X,d) is a metric space,

(2) (X, ≤) is partial ordered.

2. Main Results

Theorem 2.1. Let (X, ≤) be a partially ordered set and suppose that there exists a metric d on X such that the metric (X,d) is complete.

Let A, B, S, T: X→ X be four mappings with respect to ≤ satisfying the following

(i) A(X) ⊆ T(X) and B(X) ⊆ S(X),

(ii) The pairs {A, S} and {B, T} are compatible of type (A),

(iii) One of A, B, S and T is continuous,

(iv) There exists δ ∈[0,1) and L ≥ 0 such that

$$\int_0^{d(Ax,By)} \emptyset(t)dt \leq \delta \int_0^{M(x,y)} \emptyset(t)dt.$$

$$+ L \int_0^{\min\{d(Sx,Ax),d(Ty,By),d(Sx,By),d(Ty,Ax)\}} \emptyset(t)dt \qquad (2.1.1)$$

Where $M(x,y) = \max\left\{d(Sx,Ty), \frac{d(Sx,Ax)+d(Ty,By)}{2}, \frac{d(Sx,By)+d(Ty,Ax)}{2}\right\}$

for all comparable elements x,y ∈ X.also $\emptyset$:[0,+ ∞) →[0,+ ∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0,+ ∞) , non-negative ,and such that for each $\varepsilon > 0, \int_0^{\varepsilon} \emptyset(t)dt$ .Then A, B, S and T have a unique common fixed point in X.

Proof. Suppose x_0 ∈ X is arbitrary. Let us construct a sequence $\{y_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Tx_{2n+1} \text{ and}$$

$$y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}\text{, for all } n \geq 0.$$

Now

$$M(x_{2n}, x_{2n+1}) = \max\left\{\begin{array}{c} d(Sx_{2n}, Tx_{2n+1}), \frac{d(Sx_{2n},Ax_{2n})+d(Tx_{2n+1},Bx_{2n+1})}{2}, \\ \frac{d(Sx_{2n},Bx_{2n+1})+d(Tx_{2n+1},Ax_{2n})}{2} \end{array}\right\}$$

$$= \max\left\{\begin{array}{c} d(y_{2n-1}, y_{2n}), \frac{d(y_{2n-1},y_{2n})+d(y_{2n},y_{2n+1})}{2}, \\ \frac{d(y_{2n-1},y_{2n+1})+d(y_{2n},y_{2n})}{2} \end{array}\right\}$$

$$= \max\left\{d(y_{2n-1}, y_{2n}), \frac{d(y_{2n},y_{2n+1})}{2}, \frac{d(y_{2n-1},y_{2n+1})}{2}\right\}$$

$$\leq \max\left\{d(y_{2n-1}, y_{2n}), \frac{d(y_{2n},y_{2n+1})}{2}, \frac{d(y_{2n-1},y_{2n})+d(y_{2n},y_{2n+1})}{2}\right\}$$

$$= \max\left\{d(y_{2n-1}, y_{2n}), \frac{d(y_{2n},y_{2n+1})}{2}\right\}$$

Therefore $M(x_{2n}, x_{2n+1}) \leq \max\left\{d(y_{2n-1}, y_{2n}), \frac{d(y_{2n},y_{2n+1})}{2}\right\}$.

Since x_n and x_{n+1} are comparable then by taking y_{2n} for x and y_{2n+1} for y in (2.1.1), it follows that

$$\int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt = \int_0^{d(Ax_{2n},Bx_{2n+1})} \emptyset(t)dt.$$

$$\leq \delta \int_0^{M(x_{2n},x_{2n+1})} \emptyset(t)dt.$$

$$+ L\int_0^{\min\left\{\begin{array}{c} d(Sx_{2n},Ax_{2n}),d(Tx_{2n+1},Bx_{2n+1}),d(Sx_{2n},Bx_{2n+1}), \\ d(Tx_{2n+1},Ax_{2n}) \end{array}\right\}} \emptyset(t)dt$$

$$\leq \delta \int_0^{\max\left\{d(y_{2n-1},y_{2n}), \frac{d(y_{2n},y_{2n+1})}{2}\right\}} \emptyset(t)dt.$$

$$+ L\int_0^{\min\left\{\begin{array}{c} d(y_{2n-1},y_{2n}),d(y_{2n},y_{2n+1}),d(y_{2n-1},y_{2n+1}), \\ d(y_{2n},y_{2n}) \end{array}\right\}} \emptyset(t)dt$$

$$\leq \delta \int_0^{\max\left\{d(y_{2n-1},y_{2n}), \frac{d(y_{2n},y_{2n+1})}{2}\right\}} \emptyset(t)dt.$$

$$+ L\int_0^{\min\{d(y_{2n-1},y_{2n}),d(y_{2n},y_{2n+1}),d(y_{2n-1},y_{2n+1}),0\}} \emptyset(t)dt$$

Thus

$$\int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt \leq \delta \int_0^{\max\left\{d(y_{2n-1},y_{2n}), \frac{d(y_{2n},y_{2n+1})}{2}\right\}} \emptyset(t)dt.$$

If $\max\left\{d(y_{2n-1},y_{2n}),\frac{d(y_{2n},y_{2n+1})}{2}\right\} = d(y_{2n-1},y_{2n})$, Then

$$\int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt \leq \delta \int_0^{d(y_{2n-1},y_{2n})} \emptyset(t)dt$$

In case $\max\left\{d(y_{2n-1},y_{2n}),\frac{d(y_{2n},y_{2n+1})}{2}\right\} = \frac{d(y_{2n},y_{2n+1})}{2}$ for some n, we have

$$\int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt \leq \frac{\delta}{2} \int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt.$$

Which is contradiction

Therefore we have

$$\int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt \leq \delta \int_0^{d(y_{2n-1},y_{2n})} \emptyset(t)dt$$

Similarly, it can be proved that

$$\int_0^{d(y_{2n+1},y_{2n+2})} \emptyset(t)dt \leq \delta \int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt$$

So

$$\int_0^{d(y_n,y_{n+1})} \emptyset(t)dt \leq \delta \int_0^{d(y_{n-1},y_n)} \emptyset(t)dt \tag{2.1.2}$$

$$\leq \delta^2 \int_0^{d(y_{n-2},y_{n-1})} \emptyset(t)dt$$

$$\leq \ldots\ldots\ldots\ldots\ldots$$

$$\leq \delta^n \int_0^{d(y_0,y_1)} \emptyset(t)dt, \text{ for all } n \geq 1.$$

It is obvious that the following inequality holds for $m > n$.

$$d(y_n, y_{n+m}) \leq \sum_{i=1}^{m} d(y_{n+i-1}, y_{n+i})$$

$$\leq \sum_{i=1}^{m} \delta^{n+i-1} d(y_0, y_1)$$

$$\leq \frac{\delta^n}{1-\delta} d(y_0, y_1)$$

$$\int_0^{d(y_n,y_{n+m})} \emptyset(t)dt \leq \frac{\delta^n}{1-\delta} \int_0^{d(y_0,y_1)} \emptyset(t)dt$$

Hence

$$\int_0^{d(y_n,y_{n+m})} \emptyset(t)dt = 0 \text{ as } n \to \infty. \tag{2.1.3}$$

Now we prove that $\{y_n\}$ is a Cauchysequence. Suppose it is not.Then there exists an $\varepsilon > 0$ and sub sequence $\{x_{m(p)}\}$and $\{x_{n(p)}\}$such that

M(p) $<$ n(p) $<$ m(p+1) with

$$d(y_{n(p)}, y_{m(p)}) \geq \varepsilon,$$
$$d(y_{n(p)-1}, y_{m(p)} < \varepsilon \qquad (2.1.4)$$

Now

$$d(y_{m(p)-1}, y_{n(p)-1}) \leq d(y_{m(p)-1}, y_{m(p)}) + d(y_{m(p)}, y_{n(p)-1})$$
$$< d(y_{m(p)-1}, y_{m(p)}) + \varepsilon \qquad (2.1.5)$$

From (2.1.3), (2.1.5), we get

$$\lim_{p\to\infty} \int_0^{d(y_{m(p)-1}, y_{n(p)-1})} \varphi(t)dt \leq \int_0^{\varepsilon} \varphi(t)dt \qquad (2.1.6)$$

Using (2.1.2), (2.1.4), and (2.1.6) we get ,

$$\int_0^{\varepsilon} \varphi(t)dt \leq \int_0^{d(y_{n(p)}, y_{m(p)})} \varphi(t)dt$$
$$\leq k\int_0^{d(y_{n(p)-1}, y_{m(p)-1})} \varphi(t)dt$$
$$\leq k\int_0^{\varepsilon} \varphi(t)dt$$

Which is contradiction,

Hence we conclude that $\{y_n\}$ is a Cauchy sequence. Since X is complete. The sequence $\{y_n\}$ converges to a point z in X and subsequences $\{Ax_{2n}\}$, $\{Sx_{2n}\}$,$\{Bx_{2n-1}\}$ and $\{Tx_{2n+1}\}$ also converges to z.

Now suppose that T is continuous. Since B and T are compatible of type (A), then by Proposition 1.7, we have

$$B\,Tx_{2n+1},\ T\,Tx_{2n+1} \to Tz \text{ as } n \to \infty.$$

Putting x = x_{2n} and y = Tx_{2n+1} in (2.1.1), we have

$$\int_0^{d(Ax_{2n}, BTx_{2n+1})} \emptyset(t)dt$$

$$\leq \delta \int_0^{\max\left\{ d(Sx_{2n}, TTx_{2n+1}), \frac{d(Sx_{2n}, Ax_{2n}) + d(TTx_{2n+1}, BTx_{2n+1})}{2}, \frac{d(Sx_{2n}, BTx_{2n+1}) + d(TTx_{2n+1}, Ax_{2n})}{2} \right\}} \emptyset(t)dt.$$

$$+ L \int_0^{\min\left\{ d(Sx_{2n}, Ax_{2n}), d(TTx_{2n+1}, BTx_{2n+1}), d(Sx_{2n}, BTx_{2n+1}), d(TTx_{2n+1}, Ax_{2n}) \right\}} \emptyset(t)dt$$

Taking the limit n $\to \infty$, we get

$$\int_0^{d(z, Tz)} \emptyset(t)dt$$

$$\leq \delta \int_0^{\max\left\{ d(z,Tz), \frac{d(z,z)+d(Tz,Tz)}{2}, \frac{d(z,Tz)+d(Tz,z)}{2} \right\}} \emptyset(t)dt.$$

$$+ L \int_0^{\min\{d(z,z),d(Tz,Tz),d(z,Tz)d(Tz,z)\}} \emptyset(t)dt$$

$$= \delta \int_0^{d(z,\ Tz)} \emptyset(t)dt$$

Which implies that Tz = z. Again by replacing x by x_{2n} and y by z in (2.1.1), we have

$$\int_0^{d(Ax_{2n},Bz)} \emptyset(t)dt$$

$$\leq \delta \int_0^{\max\left\{d(Sx_{2n},Tz),\frac{d(Sx_{2n},Ax_{2n})+d(Tz,Bz)}{2},\frac{d(Sx_{2n},Bz)+d(Tz,Ax_{2n})}{2}\right\}} \emptyset(t)dt.$$

$$+ L \int_0^{\min\left\{d(Sx_{2n},Ax_{2n}),d(Tz,Bz),d(Sx_{2n},Bz),d(Tz,Ax_{2n})\right\}} \emptyset(t)dt$$

,

Taking the limit $n \to \infty$, we get

$$\int_0^{d(z,\ Bz)} \emptyset(t)dt$$

$$\leq \delta \int_0^{\max\left\{d(z,z),\frac{d(z,z)+d(z,Bz)}{2},\frac{d(z,Bz)+d(z,z)}{2}\right\}} \emptyset(t)dt.$$

$$+ L \int_0^{\min\{d(z,z),d(z\ B,z),d(z,Bz)d(z,z)\}} \emptyset(t)dt$$

$$= \frac{\delta}{2} \int_0^{d(z,\ Bz)} \emptyset(t)dt,$$

Which implies that Bz = z. Since B(X) ⊆ S(X), there exists a point w in X such that Bz = Sw = z. Again by (2.1.1), we have

$$\int_0^{d(Aw,Bz)} \emptyset(t)dt$$

$$\leq \delta \int_0^{\max\left\{d(Sw,Tz),\frac{d(Sw,Aw)+d(Tz,Bz)}{2},\frac{d(Sw,Bz)+d(Tz,Aw)}{2}\right\}} \emptyset(t)dt.$$

$$+ L \int_0^{\min\left\{d(Sw,Aw),d(Tz,Bz),d(Sw,Bz),d(Tz,Aw)\right\}} \emptyset(t)dt$$

Taking the limit $n \to \infty$, we get

$$\int_0^{d(Aw,z)} \emptyset(t)dt$$

$$\leq \delta \int_0^{\max\left\{d(z,z),\frac{d(z,Aw)+d(z,z)}{2},\frac{d(z,z)+d(z,Aw)}{2}\right\}} \emptyset(t)dt.$$

$$+ L \int_0^{\min\{d(z,Aw),d(z,z),d(z,z)d(z,Aw)\}} \emptyset(t)dt$$

$$= \frac{\delta}{2} \int_0^{d(z,Aw)} \emptyset(t)dt,$$

Which implies that Aw = z. Since A and S are compatible of type (A), and Aw = Sw = z, then by Proposition 1.7, we have

Az = ASw = SAw =Sz.

By using (2.1.1) again, we have Az = z.

Therefore Az = Bz = Sz = Tz = z, that is z is a common fixed point of A, B, S and T. For uniqueness, let z' be another common fixed point such that

z $\neq z'$. Then

$$\int_0^{d(z,z')} \emptyset(t)dt = \int_0^{d(Az,Bz')} \emptyset(t)dt$$

$$\leq \delta \int_0^{\max\left\{d(Sz,Tz'),\frac{d(Sz,Az)+d(Tz',Bz')}{2},\frac{d(Sz,Bz')+d(Tz',Az)}{2}\right\}} \emptyset(t)dt.$$

$$+ L \int_0^{\min\{d(Sz,Az),d(Tz',By),d(Sz,Bz'),d(Tz',Az)\}} \emptyset(t)dt$$

$$\int_0^{d(z,z')} \emptyset(t)dt \leq \delta \int_0^{d(z,z')} \emptyset(t)dt$$

Which means that z = z'. Thus z is a unique common fixed point of A, B, S and T.

REFERENCES

(1) Amit Singh, Darshana J. Prajapati, R.C.Dimri " Some Fixed Point Results of Almost Generalized Contractive mappings in Ordered Metric Spaces, International Journal of Pure and Applied Mathematics, Volume 86(5),(2013), 779-789.

(2) A.C.M. Ran, M.C.B. Reurings, A fixed point theorem in partially ordered sets and some applications to matrix equations, Proc.Amar.Math.Soc.,132(2004), 1435-1443.

(3) G.Jungck, Compatible mappings and common fixed points, Int.J.Math.Math.Sci.,9(4),(1986),771-779.

(4) G. Jungck, Common fixed points for noncontinuous nonself maps on non-metric spaces, Far East J.Math.Sci.,4(1996), 199-215.

(5) G. Jungck, P.P. Murty, Y.J.Cho, Compatible mappings of type (A), and common fixed points, Math.Japan.,38(2)(1993),381-390.

(6) G.V.R.Babu,M.L. Sandhya, M.V.R. Kameswari, A note on a fixed point theorem of Berinde on weak contractions, Carpathian J.Math.,24(1),(2008),08-12.

(7) I. Beg, M. Abbas, Coincidence point and invariant approximation for maps- pings satisfying generalized weak contractive condition, Fixed point Theory Appl.,(2006)pp.7.

(8) Lj.B. Ciric, A generalization of Banach's contraction principle, Proc.Amer.Math.Soc.,45(1974),267-273.

(9) Lj.B. Ciric, M.Abbas, R.Saadati, N.Hussain, Common fixed points of almost generalized contractive mappings in ordered metric spaces,Applied Mathematics and Computation, 217(12),(2011), 5784-5789.

(10) Lj.B. Ciric, N.Hussain , N.Cakic, Common fixed points for Ciric type

f- weak contraction with applications, Publ.Math.Debrecen,76(1),

(2010),31-49.

(11) Lj.B. Ciric , V. Rakocevic, S.Radenovic, M.Rajovic,R. Lazovic, Common fixed point theorems for non-self mappings in metric spaces of hyperbolic type,J.Comput.Appl.Math.,233(2010),2966-2974.

(12) M.A. Al-Thagafi, N. Shahzad, Noncommuting selfmaps and invariant approximations, Nonlinear Anal.,64(2006),2777-2786.

(13) M.Pacurar, Remark regarding two classes of almost contractions with unique fixed point, Creat.Math.Inform.,19(2),(2010),178-183.

(14) M. Pacurar,Iterative Methods for fixed point Approximation, Risoprint, Cluj- Napoca, (2009).

(15) R. Kannan, Some results on fixed points, Bull.Calcutta Math. Soc.,10(1968),71-76.

(16) S. Sessa, On a weak commutativity condition of mappings in fixed point consideration, Publ.Inst.Math.,32(1982),149-153.

(17) T. Zamfirescu, Fixed point theorems in metric spaces, Arch.Mat.,Basel,23(1972),292-298.

(18) V. Berinde, Approximating fixed points of weak contractions using the Picard iteration, Nonlinear Anal. Forum,9(1),(2004),43-53.

(19) V. Berinde, General constructive fixed point thaorems for Ciric-type almost contractions in metric spaces, Carpathian J.Math.,24(2),(2008),10-19.

(20) V. Berinde, Common fixed points of noncommuting almost contractions in cone metric spaces, Math.Commun.,15(1),(2010),229-241.

(21) V. Berinde, Approximating common fixed points of noncommuting almost contractions in metric spaces, Fixed Point Theory,11(2), (2010),179-188.

(22) V. Berinde, Some remarks on a fixed point theorems for Ciric-type almost contractions, Carpathian J.Math.,25(2),(2009),157-162.

(23) V. Berinde, Approximating common fixed points of noncommuting dis-continuous weakly contractive mappings in metric spaces, Carpathian J.Math.,25(1),(2009),13-22.

(24) Wutiphol Sintunavarat, Jong kyu kim and Poom Kumam " Fixed Point Theorems for A generalized almost $(\emptyset, \varphi)$- Contraction with respect to S in Ordered metric spaces.