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On Pairwise #πgs - Closed maps in Bitopological Spaces

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Abstract

In this paper we introduce the class of closed sets namely (1, 2)* - generalized$^{(1,\,2)^*\text{-}\,\pi gb}$ semi -closed sets (briefly(1,2)*- #πgs closed sets) and discuss some of their properties in bitopological spaces.Further, we define and study a new class of generalized maps called (1,2)* generalized$^{(1,\,2)^*\text{-}\,\pi gb}$ semi-closed maps(briefly(1,2)*-#πgs- closed maps). Also, we give some characterizations and applications of it.

1.Introduction

Fukutake[4] introduced and studied the concept of (1,2)* genralized closed ((1,2)* g-closed)sets, M.Lellis Thivagar and O.Ravi[9],El- Tantawy and Abu-Donia [3]introduced the notions of (1,2)* generalized semi-closed (briefly (1,2)* gs-closed) set , (1,2)* generalized α -closed sets (briefly (1,2)* α g -closed) set , (1,2)* semi generalized -closed (briefly (1,2)* sg-closed) set, (1,2)* α- generalized -closed (briefly (1,2)* gα -closed) set in bitopological spaces respectively. Arockiarani and K. Mohana[1],[2], Ravi.O , Lellis Thivagar and M.Joseph Isreal[20]introduced the concepts of (1,2)*-π-generalized closed (briefly (1,2) * - πg – closed set), (1,2)*-π-generalized α-closed (briefly (1, 2)*-πgα-closed set) and obtain some of their properties. Ravi.O,Pious Missier and Salai Parkunan [19],Kamaraj,M.[8]introduced the notion of (1,2)* semi -generalized –star-closed (briefly (1,2)* sg*-closed) set, (1,2)* α-genralized semi- closed ((1,2)* αgs-closed)sets

Ravi.O and Lellis Thivagar [15], introduced the concepts of (1,2)* - rg closed set , P. E. Long and L. L. Herington[12],Y.Gnanambal[5], studied (1, 2)*- regular-closed sets, (1, 2)*- gpr-closedsets.Ravi.O,Pious Missier[18], Jeyanthi.V and Janaki.C.[7] , introduced the concepts of (1,2)*- rw closed, (1,2) *- rwg closed), (1,2)*- πwg-closed sets, (1, 2)*-rgα - closed set. Sreeja,and Janaki,C.[22] introduced the concepts of (1, 2)* generalized b-closed set (briefly (1, 2)* gb closed) , (1,2)*-π -generalized b- closed (briefly (1,2)* - πgb – closed set) in bitopological spaces.

The purpose of this paper is to introduce a new class of closed sets , namely (1, 2)*- generalized$^{(1,\,2)^*\text{-}\,\pi gb}$ - semi-closed sets in bitopological spaces ,we have elementary properties of this class,also we study its relations with the classes of $\tau_1\tau_2$ –closed, (1, 2)* α-closed set , (1, 2)* semi-closed set, (1, 2)* g-closed, (1, 2)* g*-closed, (1, 2)* pre-closed, (1, 2)* gp -closed ,(1, 2)* gsp-closed , (1, 2)* πgp-closed, (1, 2)* π –closed, (1, 2)* w -closed , (1, 2)*- rw –closed,(1, 2)* gb-closed, (1, 2)* gs-closed, (1, 2)* sg-closed, (1,2)* sg*-closed, (1, 2)* αg-closed, (1, 2)* gα-closed ,(1, 2)* πgb –closed, (1, 2)* πgα –closed,(1, 2)*- πwg –closed, (1, 2)*rg-closed, (1, 2)*rwg-closed ,(1,2)* αgs-closed). The notion of (1, 2)*- #πgs- closed set and (1,2)*- #πgs- open set and its different characterizations are given in this paper ,also we provide some propositions and examples .

We present and study a new class of generalized maps namely generalized$^{(1,\,2)^*\text{-}\,\pi gb}$ semi - closed maps and(1, 2)*-#πgs - irresolute as applications, also we provide several properties of this concepts and to investigate its relationships with certain types of closed maps**.**Several results concerning these types of maps are introduced

2. Preliminaries

Throughout the present paper (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) (or simply X, Y, Z) denote bitopological spaces.

Before entering into our work we recall the following definitions:

Definition 2.1: [12] A subset B of of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-open if
B= $U_1 \cup U_2$ wehere $U_1 \in \tau_1$ and $U_2 \in \tau_2$.
The complement of $\tau_1\tau_2$ -open set is $\tau_1\tau_2$ -closed.

Remark 2.2:[12] $\tau_1\tau_2$-open subset of X need not necessarily from a topology

Definition 2.3: [12] Let A be a subset of (X, τ_1, τ_2) ,then
(1) The $\tau_1\tau_2$-closure of A, denoted by $\tau_1\tau_2$- cl(A) is defined by:
$\tau_1\tau_2$-closure (A)= ∩{F/ A⊆F and F is $\tau_1\tau_2$-closed}

(2) The $\tau_1\tau_2$- interior of A, denoted by $\tau_1\tau_2$-int(A) is defined :
$\tau_1\tau_2$-interior of (A)= ∪{U/ U⊆A and U is $\tau_1\tau_2$-open}

(3) The (1, 2)* α- closure of A(resp. (1, 2)* semi- closure, (1, 2)* b- closure)and is denoted by (1, 2)*α cl (A) (resp. (1, 2)* s cl (A) , (1, 2)*bcl (A))is defined :
(1, 2)*α cl (A))= ∩{F/ A⊆F and F is (1, 2)*- α- closed} (resp. (1, 2)*scl (A) , (1, 2)*bcl (A)).

(4) The (1, 2)* α- interior of A(resp. (1, 2)* semi- interior, (1, 2)* b- interior), denoted by (1, 2)* (1, 2)*sint (A) (resp. αint(A), (1, 2)*bint (A)) is defined :
(1, 2)*sint(A) =∪{ U/ U⊆ A and U is (1, 2)*- s-open }(resp. (1, 2)* αint (A) , (1, 2)*bint (A))

Clearly (1, 2)* bcl (A) ⊆ (1, 2)*scl (A) ⊆ (1, 2)* ⊆ α cl (A) ⊆ (1, 2)* cl (A)

Definition 2.4. A subset A of a bitopological space (X, τ_1, τ_2) is called a

(1) (1, 2)*α- open set[10] if A⊆ $\tau_1\tau_2$ - int ($\tau_1\tau_2$ - cl($\tau_1\tau_2$ - int (A)))

(2) (1, 2)*- semi-open set[10] if A⊆ $\tau_1\tau_2$- cl($\tau_1\tau_2$ - int(A))

(3) (1, 2)*-preopen set [12] if A⊆ $\tau_1\tau_2$ -int ($\tau_1\tau_2$ -cl (A))

(4) (1, 2)*-b-open [10] if A ⊆ $\tau_1\tau_2$- cl($\tau_1\tau_2$- int(A)) ∪ $\tau_1\tau_2$- int ($\tau_1\tau_2$- cl(A)).

(5) (1, 2)*-regular open [15] if A = $\tau_1\tau_2$-int($\tau_1\tau_2$- cl(A)).
6).(1,2)*- regular α-open in X [18]if there is a (1,2)* - regular open set U such that U ⊆ A⊆ $\tau_1\tau_2$- αcl(U).

7).(1,2)* - regular semi open set[12]if there is a (1,2)* - regular open set U in X, such that U ⊆A ⊆ $\tau_1\tau_2$ –cl(U)

8) $\tau_1\tau_2$- π- open[1] if A is the finite union of (1, 2)*-regular open sets.The complement of $\tau_1\tau_2$- π- open is said to be $\tau_1\tau_2$-π- closed.

Definition 2.5. A subset A of a bitopological space (X, τ_1, τ_2) is called a

1).(1, 2)* generalized closed set(briefly (1, 2)*g-closed) [17] if $\tau_1\tau_2$ -cl(A) ⊆ U whenever A ⊆ U and U is $\tau_1\tau_2$ -open se in X.
2).(1, 2)*Strongly generalized closed set (briefly (1, 2)* g*-closed[5] if (1, 2)* cl(A) ⊆U whenever A ⊆ U and U is (1, 2)* g-open se in X.
3).(1, 2)* generalized α- closed set (briefly (1, 2)* gα-closed [12] if (1, 2)* αcl(A) ⊆ U whenever A ⊆ U and U is (1, 2)* α-open in X.
4).(1, 2)* α-generalized closed set (briefly (1, 2)* αg-closed [12]if (1, 2)* αcl(A) ⊆ U whenever A ⊆ Uand U is $\tau_1\tau_2-$ open in X.
5).(1, 2)* generalized semi- closed set (briefly (1, 2)* gs-closed [21] if (1, 2)* scl(A) ⊆ U whenever A ⊆ U and U $\tau_1\tau_2-$ open in X.
6).(1, 2)* semi-generalized closed set (briefly (1, 2)*- sg-closed [9] if(1, 2)*scl(A) ⊆ U whenever A ⊆ U and U is (1, 2)* semi-open in X.
7).(1, 2)* semi-generalized -star closed set (briefly (1, 2)*- sg*-closed [19] if $\tau_1\tau_2$ −cl(A) ⊆ U whenever A ⊆ U and U is (1, 2)* semi-open in X.
8). (1, 2)* weakly- generalized closed set (briefly(1, 2)* wg-closed) [18]if $\tau_1\tau_2$ -cl($\tau_1\tau_2$-int(A)) ⊆ U whenever A ⊆ U and $\tau_1\tau_2-$ U is open in X..
8).(1,2)* - rω- closed set [18] if $\tau_1\tau_2$ -cl(A) ⊆ U,whenever A ⊆ U and U is (1,2)* - regular semi open set in X.
9). (1,2)* regular -weakly generalized closed (briefly (1,2)*- rwg closed)[18] if $\tau_1\tau_2$ −cl($\tau_1\tau_2$-int (A)) ⊆U whenever A⊆ U and U⊆ (1,2)*- regular open in X
10).(1, 2)* regular generalized -closed set (briefly(1, 2)* rg- closed) [15] if $\tau_1\tau_2$ -cl(A) ⊆ U whenever A ⊆ U and U is (1, 2)* regular -open in X.

11) (1,2)* regular - generalized α- closed set [18](briefly (1,2)* - rgα - closed set) if $\tau_1\tau_2$-αcl (A) ⊆ U whenever A⊆ U and U is (1,2)*- regular- α -open set in X.
12). (1, 2)* generalized b-closed set (briefly (1, 2)* gb closed) [22] if (1, 2)* bcl(A) ⊆ U whenever A ⊆ U and U is (1, 2)* -open in X.
13). (1, 2)* α- generalized semi-closed set (briefly (1, 2)* αgs closed)[8] if (1, 2)* αcl(A) ⊆ U whenever A ⊆ U and U is (1, 2)* semi-open in X
14).(1, 2)*π- generalized -closed (briefly (1, 2)* πg-closed) [20]if $\tau_1\tau_2$ -cl(A) ⊆U whenever A⊆ U and U is $\tau_1\tau_2-$ π-open in X.

15).(1, 2)*π -generalized α-closed (briefly (1, 2)* πgα-closed)[2] if (1, 2)* αcl(A)⊆U whenever A⊆ U and U is $\tau_1\tau_2-$ π-open in X.

16).(1, 2)* generalized pre-closed(briefly ((1, 2)*- gp-closed) [22] if (1, 2)* pcl(A) ⊆U whenever A ⊆ U and U is $\tau_1\tau_2-$ open in X

17).(1, 2)* generalized semi-pre -closed(briefly ((1, 2)*- gsp-closed) [22] if (1, 2)* spcl(A) ⊆U whenever A ⊆ U and U is $\tau_1\tau_2-$ open in X

18).(1, 2)* π- generalized p-closed(briefly ((1, 2)*-πgp-closed) [22] if (1, 2)* pcl(A) ⊆U whenever A ⊆ U and U is $\tau_1\tau_2-$ π-open in X

19).(1,2) *-πwg- closed set[7] in X if $\tau_1\tau_2$ − cl ($\tau_1\tau_2$ -int(A)) ⊆ U whenever A⊆ U and U is $\tau_1\tau_2-$ π-open in X.

20).(1, 2)* π generalized b-closed(briefly ((1, 2)*-πgb-closed) [22] if (1, 2)* bcl(A) ⊆U whenever A ⊆ U and U is $\tau_1\tau_2-$ π-open in X.

The complements of the above mentioned sets are called their respective open sets.

Definition 2.6: A map f :(X, τ_1, τ_2) → (Y, σ_1, σ_2) from bitopological space X into bitopological space Y is called :
1) (1, 2)*- continuous (briefly (1, 2)*- continuous)[13] if f^{-1}(V) is (1, 2)* - closed in X for every $\sigma_1\sigma_2$ – closed set V in Y.
2) (1, 2)*-b- irresolute (briefly (1, 2)*-b- irresolute)[20] if f^{-1} (V) is (1, 2)* b- closed in X for every (1, 2)* b-closed set V in Y.
3) (1, 2)*- closed map (briefly (1, 2)*- closed)[17] if f(F) is $\sigma_1\sigma_2$ – closed in Y for every $\tau_1\tau_2$ −closed set F in X.
4) (1, 2)* generalized closed map (briefly (1, 2)*-g- closed)[14] if every f (F) is (1, 2)*-g- closed in Y for every $\tau_1\tau_2$- closed set F in X.
5) (1, 2)* Strongly generalized closed map (briefly(1, 2)*-g*- closed)[6] if every f (V) is (1, 2)*-g*- closed in Y for every $\tau_1\tau_2$- closed set V in X.
6) (1, 2)* generalized semi- closed map (briefly(1, 2)*-gs- closed) [19] if every f (V) is (1, 2)*-gs- closed in Y for every $\tau_1\tau_2$- closed set V in X.
7) (1, 2)* semi- generalized closed (briefly(1, 2)*-sg- closed) [10] if every f (V) is (1, 2)*-sg- closed in Y for every $\tau_1\tau_2$- closed set V in X.
8) (1, 2)* α - generalized closed map (briefly (1, 2)*- α g- closed) [10] if every f (V) is (1, 2)*-α g- closed in Y for every $\tau_1\tau_2$- closed set Vin X.
9) (1, 2)* generalized α - closed map (briefly (1, 2)*-g α - closed) [10] if every f (V) is (1, 2)* -g α - closed in Y for every $\tau_1\tau_2$- closed set Vin X.
10) (1, 2)* weakly generalized closed map (briefly (1, 2)* wg-closed) [18] if every f (V) is (1, 2)*- wg - closed in Y for every $\tau_1\tau_2$- closed set Vin X.
11) (1, 2)* rω - closed map (briefly (1, 2)*-rω - closed) [18] if every f (V) is (1, 2)*-rω - closed in Y for every $\tau_1\tau_2$- closed set Vin X.
12) (1, 2)* regular generalized -closed map (briefly(1, 2)*-rg - closed) [12] if every f (V) is (1, 2)*-rg - closed in Y for every $\tau_1\tau_2$- closed set Vin X.
13) (1, 2)* regular - generalized α- closed map (briefly (1, 2)*-rgα - closed) [12] if every f (V) is (1, 2)*-rgα - closed in Y for every $\tau_1\tau_2$- closed set Vin X.
14) (1, 2)* generalized b-closed map (briefly (1, 2)*-gb - closed) [10] if every f (V) is (1, 2)*-gb- closed in Y for every $\tau_1\tau_2$- closed set Vin X.

15) (1, 2)* α- generalized semi-closed set map (briefly (1, 2)* αgs closed) [8] if every f (V) is (1, 2)*- αgs - closed in Y for every $\tau_1\tau_2$- closed set Vin X.
16) (1, 2)* π -generalized -closed map (briefly (1, 2)*-πg - closed) [1] if every f (V) is (1, 2)*-πg- closed in Y for every $\tau_1\tau_2$- closed set Vin X.
17) (1, 2)* π -generalized α-closed map (briefly (1, 2)*-πgα- closed) [1] [if every f (V) is (1, 2)*-πgα- closed in Y for every $\tau_1\tau_2$- closed set Vin X.
18) (1, 2)* π- generalized p-closed map (briefly (1, 2)*-πgp - closed) [20] if every f (V) is (1, 2)*-πgp- closed in Y for every $\tau_1\tau_2$- closed set Vin X.
19) (1, 2)* πwg - closed map (briefly (1, 2)*- πwg- closed) [7] if every f (V) is (1, 2)*- πwg- closed in Y for every $\tau_1\tau_2$- closed set V in X.
20) (1, 2)* $\pi-$generalized b- closed map (briefly (1, 2)*- πgb- closed) [22] if every f (V) is (1, 2)*- πgb - closed in Y for every $\tau_1\tau_2$- closed set V in X.
21) pre(1, 2)* gs- closed [21]if every f (V) is (1, 2)*- gs - closed in Y for every (1, 2)*- gs closed set V in X.
Definition 3.1: A subset A of a bitopological space (X, τ_1, τ_2) is called (1, 2)* #π- generalized semi -closed sets (briefly (1,2)*- #πgs closed sets) if $\tau_1\tau_2$-scl(A) ⊆U whenever A ⊆U and U ∈ (1,2)* #π where #π is (1,2)*-πgb-open set in X.

3.(1, 2)* - #πgs -closed sets

Definition 3.1: A subset A of a bitopological space (X, τ_1, τ_2) is called (1, 2)* - generalized**(1, 2)*-** πgb semi - closed sets (briefly (1,2)*- #πgs closed sets) if $\tau_1\tau_2$-scl(A) ⊆ U whenever A ⊆ U and U is (1,2)*- πgb-open set in X.
The class of all (1, 2)*- #πgs -closed subset of (X, τ_1, τ_2) is denoted by (1, 2)*- #πGSC (x).

Definition 3.2: A subset A of (X, τ_1, τ_2) is called (1, 2)*-#πgs - open if and only if its compliment is (1, 2)*-#πgs - closed in (X, τ_1, τ_2)

The class of all (1, 2)*- #πgs - open subset of X is denoted by (1, 2)*- #πGSO(x).

Remark 3.3: (1, 2)*-scl (X-A) = X- (1, 2)*-sint (A)

Theorem 3.4:
i) Every $\tau_1\tau_2$ -closed set is (1, 2)*-#πgs -closed set.
ii) Every $\tau_1\tau_2$-π-closed set is (1, 2)*-#πgs -closed set.
iii) Every (1, 2)* semi-closed set is (1, 2)*-#πgs -closed set.
iv) Every (1, 2)* α -closed set is (1, 2)*-#πgs -closed set.

Proof:

i) Let A be any $\tau_1\tau_2$ −closed set and U be any (1,2)*- πgb open set containing A. Since A is (1,2)*closed set ,then $\tau_1\tau_2$-cl(A) =A ⊆U. Since $\tau_1\tau_2$-scl(A) ⊆ $\tau_1\tau_2$-cl(A) ⊆ U, implies that $\tau_1\tau_2$-scl(A)⊆ U. Hence A is (1, 2)*-#πgs -closed.

ii) Let B be $\tau_1\tau_2$-π-closed set **.**Since (Every $\tau_1\tau_2$-π-closed set is $\tau_1\tau_2$ −closed set) ,then A is $\tau_1\tau_2$ −closed set and by step (i) A is (1, 2)*- #πgs -closed set.

iii) Let A be a (1, 2)*-semi-closed in(X, τ_1, τ_2) ,such that A ⊆U ,where U is (1, 2)*-πgb - open set. Since A is (1, 2)* semi -closed set .This implies that (1, 2)*- scl(A) ⊆ $\tau_1\tau_2$-cl(A) ⊆ U, (1, 2)*-scl(A) ⊆U. Therefore A is (1, 2)*- #πgs -closed set.

iv) Let A be a (1, 2)*-α-closed set **.**Since (Every (1, 2)*-α-closed set is (1, 2)*-semi-closed) ,hence A is (1, 2)*-semi-closed and by step (iii) A is (1, 2)*- #πgs -closed set.

The converse of 3.4 need not be true as seen from the following examples.
Example 3.5:
1)Let X = {a, b, c} and τ_1= {X, φ,{a,b},{b}} and τ_2 = {X, φ ,{a }} . $\tau_1\tau_2$− open={X, φ,{a,b},{a},{b}}and $\tau_1\tau_2$− closed={X, φ,{c},{a,c},{b,c}}. Then the set {a} is (1, 2)*-#πgs -closed ,but is not $\tau_1\tau_2$ -closed set (resp. (1, 2)*-semi-closed , (1, 2)* α-closed) sets in (X, τ_1, τ_2) .

2) Let X = {a, b, c} and τ_1= { X, {b} } and τ_2 = { X, φ,{c }}.
$\tau_1\tau_2$− open={X, φ,{b},{c},{b,c }} and $\tau_1\tau_2$− closed ={X, φ,{a},{a,b}{a,c}},then set {b,c } is (1, 2)* #πgs -closed ,but is not (1, 2)* rwg –closed and the set {b } is (1, 2)*-#πgs -closed ,but is not $\tau_1\tau_2$-π-closed set.

Remark 3.6: The concepts of (1, 2)* g-closed (resp. (1, 2)* g*-closed)sets and (1, 2)* #πgs -closed sets are in general independent as seen from the following examples.

Example 3.7:
1)Let X = {a, b, c} and τ_i= { X, φ,{a,c}} and τ_j = { X, φ,{b,c}}. $\tau_1\tau_2$− open={X, φ, {a,c}{ b,c}} and $\tau_1\tau_2$− closed ={X, φ,{a},{b}}.Then the set {a,b } is (1, 2)* g-closed(resp. (1, 2)* g*-closed) but is not (1, 2)* #πgs -closed .

2) Let X = {a, b, c} and τ_i= { X, φ,{a},{b},{a,b}} and τ_j = { X, φ,{a}, {a,b}, {a,c}}. $\tau_1\tau_2$− open={X, φ, {a},{b},{a,b},{a,c}} and $\tau_1\tau_2$− closed ={X, φ,{b},{c},{a,c}{b,c} }.Then the set {a } is (1, 2)* #πgs -closed but is not (1, 2)* g-closed(resp. (1, 2)* g*-closed) .

Remark 3.8: (1, 2)* pre-closed set and (1, 2)* #πgs -closed set are independent as seen from the following two examples.

Example 3.9:
1) Let X, τ_1 and τ_2 be as in Example (3.5)(1). Then the set { c } is (1, 2)*- #πgs - closed but is not (τ_i, τ_j)− pre-closed .

2) Let X = {a, b, c} and τ_i= { X, φ,{a,c}} and τ_j = { X, φ,{b}}.
$\tau_1\tau_2$− open={X, φ,{b },{a,c}} and $\tau_1\tau_2$− closed ={X, φ,{b},{a,c}}.Then the set {b,c } is (1, 2)*- pre - closed but is not (1, 2)*- #πgs -closed set.

Theorem 3.10: Every (1, 2)* #πgs -closed set is (1, 2)*- gb -closed set.
Proof: Let A be any (1, 2) * #πgs -closed set and U is $\tau_1\tau_2$- open set such that A⊆U. Since every (1, 2)*-semi-closed set is (1, 2)* b –closed and A is (1, 2)* #πgs -closed set then (1, 2)* bcl(A) ⊆ (1, 2)* scl(A) ⊆ U,so (1, 2)* bcl(A) ⊆ U. Hence A is (1, 2)*- gb -closed set.

The converse of 3.10 need not be true as seen from the following example.

Example 3.11: Let X, τ_1 and τ_2 be as in Example (3.13),the set {c } is (1, 2)* gb-closed but is not (1, 2)* #πgs -closed .

Theorem 3.12: Every (1, 2)* #πgs -closed set is (1, 2)* πgb -closed set.
Proof: Let A be any (1, 2)* #πgs -closed set in (X, τ_1, τ_2) such that A⊆U ,where U is $\tau_1\tau_2$ - π-open set. Since A is (1, 2)* #πgs - closed set ,(1, 2)* scl(A) ⊆ U and ,hence (1, 2)* bcl(A) ⊆(1, 2)* scl(A) ⊆ U , bcl(A) ⊆ U .Then A is πgb -closed set.

The following example show that the converse of the above theorem is not true :

Example 3.13: Let X = {a, b, c} and τ_1= { X, φ,{a }} and τ_2 = { X, φ,{a},{a,b },{a,c}}.
$\tau_1\tau_2$− open={X, φ,{a},{a,b },{a,c}} and $\tau_1\tau_2$− closed ={X, φ,{b},{c},{b,c }}.Then the set {a,c } is (1, 2)* πgb -closed set but is not (1, 2)* #πgs -closed set.

Remark 3.14: (1, 2)* rg-closed sets and (1, 2)* #πgs -closed sets are in general independent as seen from the following two example.

Example 3.15
1) Let X, τ_1 and τ_2 be as in Example (3.13),the set {a,b } is (1, 2)* rg-closed but is not (1, 2)* #πgs -closed .
2)Let X = {a, b, c} and τ_1= { X, φ,{a }} and τ_2 = { X, φ,{b}}.
$\tau_1\tau_2$− open={X, φ,{a},{ b },{a,b}} and $\tau_1\tau_2$− closed ={X, φ,{c},{a,c},{b,c }}.Then the set { b } is (1, 2)* #πgs -closed set but not(1, 2)* rg -closed set.

Theorem 3.16: Every (1, 2)* #πgs -closed set is
1) (1, 2)* sg -closed set (resp. (1, 2)* αg-closed set).
2) (1, 2)* gs-closed set(resp. (1, 2)*gα -closed set).

Proof(1): Let A be any $(1,2)*$ #πgs -closed set in (X, τ_1, τ_2) such that A⊆U ,where U is(1, 2)* semi- open set . Since A is (1, 2)* #πgs -closed set, we have (1, 2)* scl(A) ⊆ U.Then A is (1, 2)* sg-closed set.
Proof(2): Suppose that A is $(1,2)*$ #πgs -closed set.By part (1) A is (1, 2)* sg-closed set. Since every (1, 2)* sg-open (resp. (1, 2)*gα- open set) is (1, 2)* gs-open(resp. (1, 2)* αg- open set) . Thus A is (1, 2)* gs-closed set (resp. (1, 2)* αg - open set).

The following example show that the converse of the above theorem s not true :

Example 3.17: Let X = {a, b, c} and τ_1= { X, φ,{b}} and τ_2 = { X, φ,{b,c }}.
$\tau_1\tau_2-$ open={X, φ,{b},{b, c }} and $\tau_1\tau_2-$ closed ={X, φ,{a},{a,c}}.Then the set {b,c}is (1, 2)* #πgs -closed set, but is not (1, 2)* sg -closed (resp. (1, 2)* gs -closed set, (1, 2)* αg-closed set (1, 2)*gα -closed set) .

Remark 3.18: (1, 2)* πgα -closed (resp. (1, 2)* πg -closed)sets and (1, 2)* #πgs -closed sets are in general independent as seen from the following two example.

Example 3.19:
1) Let X = {a, b, c} and τ_1= { X, φ } and τ_2 = { X, φ,{ a,c }}.
$\tau_1\tau_2-$ open={X, φ,{a,c}} and $\tau_1\tau_2-$ closed ={X, φ,{b}}.Then the set {a ,b} is (1, 2)* πgα -closed set (resp. (1, 2)* πg -closed), but is not(1, 2)* #πgs -closed set .
2) Let X, τ_1 and τ_2 be as in Example (3.15),the set {b,c } is (1, 2)* #πgs -closed, but is not (1, 2)* πgα -closed(resp. (1, 2)* πg -closed) sets.

Remark 3.20: The concepts of (1, 2)* gp -closed (resp. (1, 2)* gsp-closed , (1, 2)* πgp-closed) sets and (1, 2)* #πgs -closed set are in general independent as seen from the following example.

Example 3.21:
1) Let (X, τ_1 , τ_2) be as in Example (3.5)(1),Then the set {a,b } is (1, 2)* #πgs -closed set but ,is not (1, 2)* gp -closed (resp. (1, 2)* gsp-closed , (1, 2)* πgp-closed) sets
2) Let (X, τ_1 , τ_2)be as in Example (3.15),the set {b,c } is(1, 2)* gp -closed (resp. (1, 2)* gsp-closed , (1, 2)* πgp-closed) sets, but is not (1, 2)* #πgs -closed

Remark 3.22: The concepts of (1, 2)* sg*-closed (resp. (1, 2)* rw -closed)sets and (1, 2)* #πgs -closed set are in general independent as seen from the following example.

Example 3.23:
1) Let X = {a, b, c} and τ_1= { X, {a} } and τ_2 = { X, φ,{b,c }}.
$\tau_1\tau_2-$ open={X, φ,{a},{b,c }} and $\tau_1\tau_2-$ closed ={X, φ,{a},{b,c }}.Then the set {a ,c} is (1, 2)* - sg* -closed set (resp. (1, 2)* rw -closed) , but is not(1, 2)* #πgs -closed set .
2) Let (X, τ_1 , τ_2) be as in Example (3.17),the set {c } is (1, 2)* #πgs -closed ,but is not (1,2)* sg* -closed(resp. (1, 2)* rw -closed) sets.

Remark 3.24: The concepts of (1, 2)* πgw -closed (resp.(1, 2)* rwg –closed) and (1, 2)* #πgs -closed set are in general independent as seen from the following examples.

Example 3.25:
1)Let X = {a, b, c} and τ_1= { X, φ,{b }} and τ_2 = { X, φ,{a},{a,c}}.
$\tau_1\tau_2-$ open={X, φ,{a},{ b },{a,b},{a,c}} and $\tau_1\tau_2-$ closed ={X, φ,{b},{c},{a,c},{b,c}}.Then the set { b,c } is (1, 2)* #πgs -closed set but not(1, 2)* πgw -closed set and the set {a,b } is(1, 2)* πgw -closed set,but is not (1, 2)* #πgs -closed set.

2) Let (X, τ_1 , τ_2) be as in Example (3.5)(2),the set {b } is (1, 2)* #πgs -closed ,but is not (1, 2)* rwg –closed and the set {b,c } is(1, 2)* rgw -closed set ,but is not (1, 2)* #πgs -closed set.

Remark 3.26: The concepts of (1, 2)* rgα –closed(resp.(1, 2)* αgs –closed,(1, 2)* wg–closed) and (1, 2)* #πgs -closed set are ingeneral independent as seen from the following example.

Example 3.27:
1) 1)Let X = {a, b, c} and τ_1= { X, φ,{a },{a,b}} and τ_2 = { X, φ,{b},{b,c}}.

$\tau_1\tau_2-$ open={X, φ,{a},{b},{a,b},{b,c}} and $\tau_1\tau_2-$ closed ={X, φ,{a},{c},{a,c},{b,c }}.Then the set {a,c } is (1, 2)* rgα –closed, but is not (1, 2)* #πgs -closed set.

2) Let (X, τ_1 , τ_2) be as in Example (3.5)(1),the set {a } is is (1, 2)* #πgs -closed set ,but is not (1, 2)* -rgα -closed **.**

3) Let (X, τ_1 , τ_2) be as in Example (3.23),the set {c } is (1, 2)* αgs -closed ,but is not (1, 2)* #πgs -closed set.

4) Let (X, τ_1, τ_2)be as in Example (3.5)(1),the set {b }is (1, 2)* #πgs -closed set,but is not (1, 2)* -αgs -closed **.**

5) Let (X, τ_1 , τ_2)be as in Example (3.9)(2),the set {c } is is (1, 2)* wg -closed set ,but is not(1, 2)* -#πgs -closed **.**

6) Let (X, τ_1 , τ_2)be as in Example (3.25),the set {b } is is (1, 2)* #πgs -closed set but not(1, 2)* -wg-closed**.**

Theorem 3.28: If A and B are (1, 2)*- #πgs - closed in (X, τ_1, τ_2),then A ∩ B is also (1, 2)*-#πgs - closed in X.
Proof: Suppose that A ∩ B ⊆ U where U is (1, 2)*- #π - open set in X ⇒ A ⊆ U and B ⊆ U.Since A and B are (1, 2)*- #πgs - closed in X ⇒ scl(A) ⊆ U and scl(B) ⊆ U then scl(A) ∩ scl(B) ⊆ U .But scl(A ∩ B) ⊆ scl(A) ∩ scl(B) **.** Therefore A ∩ B is (1, 2)*-#πgs - closed in X.

Remark 3.29:
i)The the union of two (1, 2)*- #πgs closed set may not be an(1, 2)*- #πgs closed set.
ii) The the intersection of two (1, 2)*- #πgs open set may not be an(1, 2)*- #πgs open set.

Example 3.30: Let (X, τ_1 , τ_2)be as in Example (3.27)
(1,2)*-#πgs closed ={X, φ,{a},{b},{c},{a,c}},{b,c}}and the subsets{a} ,{b}are (1, 2)*- #πgs closed sets but {a}∪{b}={a,b} is not (1, 2)*- #πgs closed set.Also, (1, 2)*- #πgs open = {X, φ,{a},{c},{a,b},{a,c},{b,c}}, the subsets{a,c} ,{b,c}are (1, 2)*- #πgs open set but {a,c}∩{ b,c}={b} is not (1, 2)*- #πgs open set.

Theorem 3.31: If A is (1, 2)*- πgb - open and (1, 2)*- #πgs - closed in (X, τ_1, τ_2),then A is (1, 2)* -sg-closed .
Proof: Suppose that A is (1, 2)*- #πgs - closed, (1, 2)*- πgb -open and A⊆A ⇒ (1, 2)*-scl(A) ⊆ A.Since A ⊆ (1, 2)*-scl(A) ⇒ A= (1, 2)*-scl(A).Therefore A is (1, 2)*-sg-closed.

Theorem 3.32: If a set A is (1, 2)*- #πgs - closed in bitopological space then (1, 2)* -scl(A) –A does not contain any non empty (1, 2)*- πgb -closed set.
Proof: Let F be a non empty (1, 2)*- πgb -closed set such that F⊆ (1, 2)* scl(A)-A ⇒ A⊆ X-F where X-F is (1, 2)*- πgb–open. Since A is (1, 2)*- #πgs –closed set in X and X-F is (1, 2)*- πgb –open, then (1, 2)* -scl(A) ⊆ X-F ⇒ F ⊆ X- (1, 2)* -scl(A) .We get F⊆ (1, 2)*- scl (A)∩(X-(1, 2)*-scl (A)) =φ which is a contradiction. Therefore (1, 2)*-scl(A) does not contain any non empty (1, 2)*- πgb -closed set.

Corollary 3.33: Let A be (1, 2)* - #πgs -closed in (X, τ_1, τ_2). Then A is (1, 2)* sg-closed iff (1, 2)* - scl(A)-A is πgb - closed.
Proof: ⇒ Let A be (1, 2)* #πgs -closed in X and (1, 2)* - sg-closed⇒ (1, 2)* - scl(A) = A

⇒(1, 2)* -scl(A)-A= φ which is πgb - closed.

Conversely: Let (1, 2)* - scl(A)-A be an πgb– closed set in X and A be (1, 2)* #πgs -closed in X By Theorem 3.32 (1, 2)* -scl(A) –A does not contain any non empty (1, 2)*- πgb -closed set⇒ (1, 2)*-scl (A) = φ ⇒ (1, 2)*-scl (A) = A.Then A is (1, 2)* sg-closed.

Theorem 3.34: If A is any (1, 2)* #πgs -closed in (X, τ_1, τ_2) and B is any set such that A ⊆B⊆ (1, 2)*-scl (A), then B is (1, 2)* #πgs -closed set in (X, τ_1, τ_2) .
Proof: Let U be any (1, 2)* πgb-open in X such that B ⊆ U .Since A ⊆ B implies that A ⊆U.Since A is (1, 2)* #πgs –closed ⇒(1, 2)*-scl(A) ⊆ U, also B ⊆ (1, 2)*-scl (A) ⇒ (1, 2)*-scl (B) ⊆ (1, 2)*-scl((1, 2)*-scl (A))= (1, 2)*-scl (B) ⊆ U ⇒ (1, 2)*-scl (B) ⊆U becomes B is also (1, 2)* #πgs -closed set.

Theorem 3.35: If A is (1, 2)*-#πgs -closed and (1, 2)* πgb -open set in (X, τ_1, τ_2) and B is any set such that B ⊆ A ⊆X. Then B is (1, 2)*- #πgs - closed relative to A iff B is (1, 2)*-#πgs -closed in X.
Proof: ⇒ Let B be (1, 2)*- #πgs -closed in A and B ⊆ A⊆ X where A is (1, 2)*- #πgs - closed and (1, 2)* πgb -open set in X . Let B ⊆U where U is (1, 2)* πgb-open in X .Since B ⊆ A ⇒ B = B∩A ⊆ U∩A ⇒ (1, 2)*-scl(B) = (1, 2)*- scl_A(B) ⊆ U∩A ⊆U . Therefore B is (1, 2)*-#πgs -closed in X .
Conversely :Suppose that B is (1, 2)*-#πgs -closed in X. To prove that B is (1, 2)*-#πgs -closed relative to A. Let B⊆G where G is (1, 2)* πgb -open set in A⇒ G=U∩A where U is (1, 2)* πgb-open set in X ⇒ B⊆ G=U ∩ A ⊆ U .Since B be (1, 2)*-πgb-closed in X ⇒ (1, 2)*-scl(B) ⊆ U ,

(1, 2)*-scl_A(B) ⊆ A ∩ (1, 2)*-scl(B) ⊆U∩A=G and scl_A(B) ⊆ G.Hence B is (1, 2)*-#πgs -closed relative to A.

Theorem 3.36: A subset A of a bitopological space (X, τ_1, τ_2)is (1, 2)*- #πgs - open iff F⊆ (1, 2)*-sint (A) whenever F is (1, 2)*-πgb -closed subset of X and F⊆ A .

Proof: ⇒: Suppose that A is is (1, 2)*- #πgs - open in X whenever F is (1, 2)*-πgb -closed and F⊆ A ⇒ X-A ⊆ X-F where X-F is (1, 2)*-πgb -open. Since X-A is (1, 2)*- #πgs closed and X-F is (1, 2)*-πgb –open ⇒ (1, 2)*-scl (X-A) ⊆ X-F .By remark (3.3) (1, 2)*-scl (A-X) = X- (1, 2)*-sint (A) ⊆ X-F. Thus F ⊆ (1, 2)*- sint (A).

Conversely: Suppose that F ⊆ (1, 2)*-sint (A) and F ⊆A whenever F is (1, 2)*- πgb-closed.Let X-A ⊆ U ,where U is (1, 2)*-πgb -open⇒ X-U ⊆ A where X-U is (1, 2)*-πgb -closed. This implise X-U⊆ (1, 2)*-sint (A) ⇒ X- (1, 2)*-sint (A) ⊆ U ⇒ (1, 2)*-scl (X-A) ⊆ U ⇒ X-A is (1, 2)*-#πgs -closed . Then A is (1, 2)*-#πgs -open set in X.

Theorem 3.37: If A is (1, 2)*-#πgs -open and (1, 2)*- sint (A) ⊆ B ⊆ A then B is (1, 2)*-#πgs -open.

Proof: Since (1, 2)*-sint (A) ⊆ B ⊆A⇒ X-A ⊆ X-B ⊆ X- (1, 2)*-sint A , by (3.3) X-A ⊆ X-B ⊆ (1, 2)*- scl (X-A) and X-A is (1, 2)*-#πgs closed, by Theorem (3.34) (X-A) ⊆ (X-B) ⊆ (1, 2)*- scl (X-A) ⇒ (X-B) is (1, 2)*-#πgs - closed. Thus B is (1, 2)*-#πgs -open.

Theorem 3.38: A subset A of a bitopological space (X, τ_1, τ_2) is (1, 2)*- #πgs - closed if and only if (1, 2)*-scl(A)− A is (1, 2)*-#πgs - open set .

Proof: ⇒ Let A is (1, 2)*- #πgs - closed and F is any (1, 2)*- πgb closed such that F ⊆ (1, 2)*-scl (A) –A.By Theorem (3.32) F is empty . Then F ⊆ (1, 2)*- sint[(1, 2)*- scl(A)− A].Thus by Theorem (3.36) (1, 2)*- sint (A) –A is (1, 2)*-#πgs - open set .

Conversely: Suppose that(1, 2)*- scl(A)− A is (1, 2)*-#πgs - open set in X and A ⊆U where U is (1, 2)*-πgb-open set ⇒ (1, 2)*-scl(A) ∩ (X-U) ⊆ (1, 2)*-scl(A) ∩ (X-A) =(1, 2)*-scl(A) –A ,then (1, 2)*-scl(A) ∩ (X-U) is (1, 2)*- πgb - closed subset of (1, 2)*- scl(A)− A .Therefore by Theorem (3. 36) (1, 2)*-scl(A) ∩ (X-U) ⊆ (1, 2)*- sint[(1, 2)*- scl(A)− A]= φ, it follows that (1, 2)*- scl(A) ⊆ U.Then A is (1, 2)*- #πgs - closed.

Theorem 3.39: For any x in a bitopological space (X, τ_1, τ_2) then {x} is either (1, 2)*- πgb- closed set or (1, 2)*-#πgs - closed set .

Proof: Suppose that {x} is not(1, 2)*- πgb- closed set in X ⇒X-{x} is not (1, 2)*- #π- open set in X,implies that X is only (1, 2)*- πgb- open set of X containing X-{x} ,then (1, 2)*-scl(X-{x}) ⊆ X ⇒ X-{x}is (1, 2)*-#πgs - closed set . Hence {x}is (1, 2)*-#πgs - open set .

4. (1, 2)*- #πgs- continuous and (1, 2)*- #πgs- irresolute maps

Definition 4.1: A map f :(X, τ_1, τ_2) → (Y, σ_1, σ_2) from bitopological space X into bitopological space Y is called :
1) generalized$^{(1, 2)*- \pi gb}$ semi - continuous (briefly (1, 2)*- #πgs continuous) if f^{-1}(V) is (1, 2)* - #πgs closed in X for every $\sigma_1\sigma_2$ –closed set V in Y.
2) generalized$^{(1, 2)*- \pi gb}$ semi - closed (briefly (1, 2)*- #πgs - closed) if every f (V) is (1, 2)*- #πgs - closed in Y for every $\tau_1\tau_2$- closed set V in X.
3) generalized$^{(1, 2)*- \pi gb}$ semi -open (briefly (1, 2)*-#πgs - open) if every f (U) is (1, 2)* #πgs - open in Y for every $\tau_1\tau_2$- open set U in X.

Definition 4.2: A map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called (1, 2)*-#πgs - irresolute if f^{-1}(V) is (1, 2)*-#πgs - open in (X, τ_1, τ_2) for every (1, 2)*- #πgs -open set V in (Y, σ_1, σ_2) .

Remark 4.3: A map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (1, 2)*-#πgs - irresolute iff the inverse image of every (1, 2)*-#πgs -closed in (Y, σ_1, σ_2) is (1, 2)*-#πgs -closed in (X, τ_1, τ_2).

Theorem 4.4. Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) be map from bitopological space X into bitopological space Y**.**Then the following statements are equivalent.
i) f is (1, 2)*- #πgs continuous.
ii) The inverse image of each $\sigma_1\sigma_2$ –open set of Y is (1, 2)* - #πgs open in X

Proof:i) ⇒ii) Let A is any $\sigma_1\sigma_2$ –open subset of Y ⇒ A^c is $\sigma_1\sigma_2$ –closed,by hypothesis $f^{-1}(A^c)$ is (1, 2)* - #πgs closed in X ,but $f^{-1}(A^c) = (f^{-1}(A))^c$ so that $f^{-1}(A)$ is (1, 2)*-#πgs - open in X.
ii) ⇒i) Let B be any $\sigma_1\sigma_2$ – closed subset of Y ⇒ B^c is $\sigma_1\sigma_2$ – open subset of Y⇒$f^{-1}(B^c)$ is (1, 2)*-#πgs - open in X ,but $f^{-1}(B^c) = (f^{-1}(B))^c$ ⇒ $f^{-1}(B)$ is (1, 2)* - #πgs closed in X.Thus f is (1, 2)*- #πgs continuous.

Theorem 4.5. Every (1, 2)*-continuous map is (1, 2)*- #πgs - continuous map .
Proof: Let f be (1, 2)*- continuous map and F be a $\tau_1\tau_2$-closed set in Y.By Theorem 3.4.F is (1, 2)*-#πgs -closed in Y .Since f is (1, 2)*-continuous map ⇒ $f^{-1}(V)$ is (1, 2)*-#πgs -closed in X ⇒
f is (1, 2)*-#πgs - continuous.

The converse of above theorem may not be true in general as seen in the following example.

Example 4.6: Consider X=Y={a,b,c}, τ_1= {X, φ,{a},{a,b}} and τ_2 = {X, φ, {b},{b,c}} .So the sets in{ X φ,{a},{b},{a,b},{b,c} } are $\tau_1\tau_2$ –open sets in X,{X,φ,{a},{c},{a,c},{b,c}} are $\tau_1\tau_2$ –closed.Let σ_1 ={ φ ,Y,{a}} and σ_2={ φ,Y,{a,c}}. So the sets in {Y,φ, {a},{a,c}} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {b},{b,c}} are σ_1 σ_2 –closed. Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a)= a ,f(b)= b,f(c) = c. Then f is (1, 2)*- #πgs - continuous map ,but it is not (1, 2)*-continuous map since the inverse image of (1, 2)*- closed sets {b }={b} is not (1, 2)*-closed set in X.

Theorem 4.7: Every (1, 2)*-#πgs- irresolute map is (1, 2)*-#πgs- continuous.
Proof: Suppose that f is (1, 2)*-#πgs-irresolute and F be a $\tau_1\tau_2$-closed set in Y.By Theorem 3.4.F is (1, 2)*-#πgs -closed in Y .Since f is (1, 2)*-#πgs - irresolute ⇒ $f^{-1}(V)$ is (1, 2)*-#πgs -closed in X . Thus f is (1, 2)*-#πgs - continuous.

The converse of above theorem may not be true in general as seen in the following example.

Example 4.8: Consider X=Y={a,b,c}, τ_1= {X, φ,{b}} and τ_2 = {X, φ, {a},{a,c}} .So the sets in{ X φ,{a},{b},{a,b},{a,c} } are $\tau_1\tau_2$ –open sets in X,{X,φ,{b},{c},{a,c},{b,c}} are $\tau_1\tau_2$ –closed.Let σ_1 ={Y,φ,{a},{a,b}} and σ_2={ φ,Y,{b}}. So the sets in {Y,φ, {a},{b},{a,b}} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {c},{a,c}} {b,c}} are σ_1 σ_2 –closed. Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a)= a ,f(b)= b,f(c) = c. Then f is (1, 2)*- #πgs - continuous map,but it is not (1, 2)*- #πgs -irresolute since the inverse image of (1, 2)*-#πgs - closed sets {a } in Y is not (1, 2)*-#πgs -closed set in X.

Remark 4.9: Composition of two (1, 2)*-#πgs -continuous maps need not be (1, 2)*-#πgs -continuous.
Example 4.10: Let X = Y = Z = {a,b,c}, τ_1 = {X, φ,{a}} and τ_2 = {X, φ, {a,c}} .So the sets in{X,φ,{a},{a,c}}are$\tau_1\tau_2$ –open sets in X,{X,φ,{b},{b,c}} are $\tau_1\tau_2$ –closed.Let σ_1 ={Y,φ,{a},{a,b}} and σ_2={ φ,Y,{b}}. So the sets in {Y,φ, {a},{b},{a,b}} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {c},{a,c},{b,c}} are σ_1 σ_2 –closed. Let η_1={ Z,φ,{b,c}} and η_2={Z, φ}.So $\eta_1\eta_2$ –open={Z, φ,{b,c}} and $\eta_1\eta_2$ –closed={Z, φ,{a}}.Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a)= a ,f(b) = b,f(c) = c. Define g: (Y, σ_1, σ_2)→(Z, η_1,η_2) by g(a)= a, g(b) = c, g(c) = b. Then f and g are
(1, 2)*-#πgs -continuous but g o f^{-1}({a})=f^{-1} (g $^{-1}$ ({a})) = f^{-1} ({a})= {a} which is not (1, 2)*- #πgs - closed in (X, τ_1, τ_2).Then g o f is not (1, 2)*-#πgs -continuous.

Definition 4.11. A subset B of bitopological space (X, τ_1, τ_2) is said to be (1,2)*- #πgs - neighborhood(briefly ((1,2)*- #πgs -nbh) of a point x in X if there exists a (1,2)*- #πgs -open set U of X such that x ∈ U⊆ B.The family of all (1,2)*- #πgs -nbh of x is denoted by $N_{(1,2)*-\#\pi gs}$
Remark 4.12:. Every $\tau_1\tau_2$–nbh is (1,2)*- #πgs -nbh but the converse is not true in general as seen in the following example.

Example 4.13: Let X = {a, b, c} and τ_1= { X, φ,{b}} and τ_2 = { X, φ,{ c }}. So the sets in
{X, φ,{b},{c },{b,c}} are $\tau_1\tau_2$ –open sets in X , (1, 2)*$_{\#\pi gs}$ O(x).={X,φ,{b},{c},{a,b}{a,c}{b,c}} Then the set {a,c } is (1,2)*- #πgs -nbh of a but not $\tau_1\tau_2$–nbh of a.

Theorem 4.14.The set U is (1,2)*- #πgs - open in bitopological space (X, τ_1, τ_2) if and only if is (1,2)*- #πgs -nbh of each of its points.
Proof: ⇒ For each x ∈ U**,** there is a #πgs - open set **U** such that x ∈ U⊆ U ,it is clearly U is (1,2)*- #πgs -nbh of each of its points.
Conversely: Suppose that U is (1,2)*- #πgs -nbh of each of its points.

i.If U = φ ⇒ U is (1,2)*- #πgs - open.
ii. If U ≠ φ ⇒ For each x ∈ U**,** there is a #πgs - open set B_x such that x ∈ B_x⊆ U,then ∪ B_x ⊆ U.
and if x ∈ U⇒ x ∈ B_x for some B_x ∈ (1, 2)*- $_{\#\pi gs}$O(x) and B_x ⊆ U ⇒U ⊆ ∪ B_x ⇒ U= ∪B_x.Thus U is (1,2)*-#πgs - open .

Theorem 4.15 . Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) then
1.f is (1, 2)*-#πgs -continuous.
2. For every x ∈ X and every open set B of Y containing f(x) ,there is a #πgs - open set A of X containing x such that f(A) ⊆ B
3. For every x ∈ X**,** the inverse of every nbh of f(x) is (1,2)*- #πgs -nbh of x.
4. For every x ∈ X**,** and every $\mathcal{H}$ nbh of f(x) **,** there is a #πgs - nbh B of x such that f(B) ⊆ $\mathcal{H}$

Proof:
(1) ⇒ (2). Let x ∈ X and B be an open set inY such that f(x) ∈ B ,by hypothesis , f^{-1}(B) is (1,2)*-#πgs - open in X and x ∈ f^{-1}(B) .If f^{-1}(B) =A ⇒ A is (1, 2)*-#πgs - open in X containing x such that f(A) ⊆ B.

(2) ⇒ (1). Suppose that x ∈ X and B is open set in Y such that x ∈ f^{-1}(B) ⇒ f(x) ∈ B, by (2) there is a #πgs - open set A in X containing x such that f(A) ⊆ B⇒ x ∈ A ⊆ f^{-1}(B) .Hence f^{-1}(B) is (1, 2)*-#πgs - open in X ,so f is (1, 2)*-#πgs -continuous.

(1) ⇒ (3). Let $\mathcal{H}$ be any nbh of f(x) ⇒ there exists an U open set of Y such that f(x) ∈ U ⊆ $\mathcal{H}$ ⇒ x ∈ f^{-1}(U) ⊆ f^{-1}($\mathcal{H}$).Since f is (1, 2)*-#πgs -continuous and U open set .This implies that f^{-1}(U) is (1, 2)*-#πgs - open in X.Thus f^{-1}($\mathcal{H}$) is (1,2)*- #πgs –nbh of x .

(3) ⇒ (1). Let U be an open of Y , $\mathcal{H}$ be any nbh of f(x) .Let f^{-1}($\mathcal{W}$) is (1,2)*- #πgs -nbh of x.
i. If f^{-1}(U) = φ ⇒ it is (1, 2)*-#πgs - open in X.
ii. If f^{-1}(U) ≠ φ ,Let x ∈ f^{-1}(U) ⇒ f(x) ∈ U ⇒ U is a nbh of f(x). By hypothesis f^{-1}(U) is (1, 2)*-#πgs -nbh of x.Thus f^{-1}(U) is(1,2)*- #πgs -nbh of each of its points and hence it is(1, 2)*-#πgs - open in X.

(3) ⇒ (4). Let x ∈ X**,** $\mathcal{H}$ be any nbh of f(x) ⇒ $\mathcal{U}$ = f^{-1}($\mathcal{H}$) is (1,2)*- #πgs -nbh of x and f^{-1}($\mathcal{U}$) =f(f^{-1}($\mathcal{H}$)) ⊆ $\mathcal{H}$.

(4) ⇒ (2). Let x ∈ X and V be an open set containing f(x) then Vis a nbh of f(x) ⇒ there exists a #πgs - nbh B of x such that x ∈ B and f(B) ⊆V ⇒ there exists a #πgs -open set U of X such that x ∈ U⊆ B .Then f(U) ⊆ f(B) ⊆V.

Theorem 4.16: Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g: (Y, σ_1, σ_2)→(Z, η_1,η_2) be two mappings and let gof : (X, τ_1, τ_2) → (Z, η_1,η_2) be (1, 2)* - #πgs -closed map.Then
1) If f is (1, 2)*-continuous and surjection then g is (1, 2)* - #πgs -closed.
2) If g is (1, 2)* - #πgs -irresolute and injective then f is (1, 2)* -#πgs -closed.
Proof:
1) Let A be $\sigma_1\sigma_2$-closed in Y.Since f is (1, 2)*-continuous ⇒f^{-1}(A) is $\tau_1\tau_2$-closed in X. Since gof is (1,2)* - #πgs -closed ⇒ gof (f^{-1}(A)) = g (f (f^{-1}(A))) =g (A) is (1, 2)* - #πgs closed in Z.Therefore g is (1, 2)* - #πgs - closed map.
2) Let A be $\tau_1\tau_2$-closed in X.Since gof is (1, 2)*-#πgs –closed ⇒ (gof)(A) is (1, 2)*-#πgs -closed in Z.Since g is (1, 2)*-#πgs -irresolute ⇒ g^{-1}((gof)(A)) = f(A) is(1, 2)*-#πgs -closed in Y.Thus f is (1, 2)* - #πgs -closed map

Theorem 4.17. Every (1, 2)*- closed map is (1, 2)*- #πgs -closed map.
Proof: Suppose that V is $\tau_1\tau_2$-closed in X .Since f is (1, 2)*- closed map ⇒ f(V) is $\sigma_1\sigma_2$-closed set in Y.By Theorem 3.4., f(V) is (1, 2)*-#πgs -closed set in Y . Thus f is (1, 2)*- #πgs -closed map

The following example show that the converse of the above proposition is not true :

Example 4.18:. Consider X=Y={a,b,c}, τ_1={X, φ,{b},{b,c}} and τ_2 ={X, φ, {a},{b},{a,b}} .So the sets in{ X φ,{a},{b},{a,b},{b,c}} are $\tau_1\tau_2$ –open sets in X,{X,φ,{a},{c},{a,c},{b,c}} are $\tau_1\tau_2$ –closed.Let σ_1 ={Y,φ,{a}} and σ_2 ={ φ,Y,{b}}. So the sets in {Y,φ,{a}, {b},{a,b}} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {c},{a,c},{b,c}} are σ_1 σ_2 –closed. Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by by f(a)= a ,f(b)= b,f(c) = c.Then f is (1, 2)*- #πgs - closed map ,but it is not (1, 2)*- closed map , since {a} is $\tau_1\tau_2 -$ closed in (X, τ_1, τ_2) , but f({a})={a} is not (1, 2)* σ_1 σ_2 –closed set in (Y, σ_1, σ_2) .

Theorem 4.19. Every (1, 2)*- #πgs -closed map is (1, 2)*- gb –closed(resp. (1, 2)*- πgb –closed ,(1, 2)*- sg -closed map((1, 2)*- gs –closed, (1, 2)* -gα -closed, (1, 2)*- α g-closed,) map.
Proof: Follows from theorems(3.10),(3.12), (3.16).

The converse of above theorem may not be true in general as seen in the following example.
Example 4.20:Consider X=Y={a,b,c}, τ_1= {X, φ,{a},{b,c}} and τ_2 = {X, φ, {c},{a,c}} .So the sets in{ X φ,{a},{c},{a,c},{b,c} } are $\tau_1\tau_2$ –open sets in X,{X,φ,{a},{b},{a,b},{b,c}} are $\tau_1\tau_2$ –closed.Let σ_1 ={Y,φ,{a}} and σ_2 ={ φ,Y,{b,c}}. So the sets in {Y,φ, {a},{b,c}} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {a},{b,c}} are σ_1 σ_2 –closed. Define f: f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a)= a ,f(b)= b,f(c) = c. Then f is (1, 2)*- sg -closed map(resp. (1, 2)*- gs –closed, (1, 2)* -gα -closed, (1, 2)*- α g-closed, (1, 2)*- gb –closed, (1, 2)*- πgb –closed) map,but it is not(1, 2)*- #πgs -closed map since {b}is $\tau_1\tau_2$ − closed in (X, τ_1, τ_2) but f({b})={b} is not (1, 2)*- #πgs –closed in (Y, τ_1, τ_2) .

Remark 4.21:

- **i.** (1, 2)*- #πgs -closed map and (1, 2)*- g -closed (resp. (1, 2)*- g^* -closed,(1, 2)* αgs closed, (1, 2)*-rgα – closed) map**s** are in general independent.
- **ii.** (1, 2)*- #πgs -closed map and(1, 2)*- pre-closed (resp. (1, 2)* gp -closed ,(1, 2)* gsp-closed , (1, 2)* πgp-closed) maps are in general independent
- **iii.** (1, 2)*- #πgs -closed map and (1, 2)*- πwg -closed map are in general independent
- **iv.** (1, 2)*- #πgs -closed map and (1, 2)* sg*-closed (resp. (1, 2)*- rw –closed, (1, 2)* πg -closed ,(1, 2)*- πgα -closed and (1, 2)*- rg -closed)map are in general independent.
- **v.** (1, 2)*- #πgs -closed and (1, 2)*- wg -closed map are in general independent.
- **vi.** (1, 2)*- #πgs -closed and (1, 2)*- rg -closed map are in general independent.

Example 4.22: It is clear that in **4.20**:,f is (1, 2)*- g -closed map (resp. (1, 2)*- g^* -closed ,(1, 2)*- pre-closed , (1, 2)* gp -closed ,(1, 2)* gsp-closed , (1, 2)* πgp-closed,(1, 2)*- πwg –closed, (1, 2)*- rw –closed, (1, 2)*- wg –closed, (1, 2)* πg –closed, (1, 2)*- πgα –closed,(1, 2)* αgs closed, (1, 2)*-rgα - closed) maps , but it is not (1, 2)*- #πgs -closed map .

Example 4.23: Let (X, τ_1 , τ_2) , (Y, σ_1, σ_2) and f be as in Example (4.8),then f is (1, 2)*- #πgs -closed map ,but it is not (1, 2)*- g -closed (resp. (1, 2)*- g^* -closed,(1, 2)* αgs closed, (1, 2)*-rgα - closed) map,since {b} is $\tau_1\tau_2$ − closed in X , but f({b})={b} is not (1, 2)*- g –closed set (resp. (1, 2)*- g^* -closed,(1, 2)* αgs closed, (1, 2)*-rgα - closed) maps set in Y .

Example 4.24:. Consider X=Y={a,b,c}, τ_1= {X, φ,{a},{a,b}} and τ_2 = {X, φ, {b},{b,c}} .So the sets in{ X φ,{a},{b},{a,b},{b,c} } are $\tau_1\tau_2$ –open sets in X,{X,φ,{a},{c},{a,c},{b,c}} are $\tau_1\tau_2$ –closed.Let σ_1 ={Y,φ,{a}} and σ_2 ={ φ,Y,{b}}. So the sets in {Y,φ, {a},{b},{a,b}} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {c},{a,c},{b,c}} are σ_1 σ_2 –closed. Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a)= a ,f(b)= b,f(c) = c.Then f is (1, 2)*- #πgs -closed map , but it is not (1, 2)*- pre- closed (resp. (1, 2)* gp -closed ,(1, 2)* gsp-closed , (1, 2)* πgp-closed) map, since {a} is $\tau_1\tau_2$ − closed in X , but f({a})={a} is not (1, 2)*- pre- closed map(resp. (1, 2)* gp -closed ,(1, 2)* gsp-closed , (1, 2)* πgp-closed)sets in Y .

Example 4.25:. Consider X=Y={a,b,c}, τ_1= {X, φ,{a},{b},{a,b}} and τ_2 = {X, φ, {a},{a,c}} .So the sets in{ X φ,{a},{b},{a,b},{a,c}} are $\tau_1\tau_2$ –open sets in X,{X,φ,{b},{c},{a,c},{b,c}} are $\tau_1\tau_2$ –closed.Let σ_1 ={Y,φ,{b}} and σ_2 ={ φ,Y,{a},{a,c}}. So the sets in {Y,φ,{a},{b},{a,b}, {a,c} } are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {b},{c},{a,c},{b,c}} are σ_1 σ_2 –closed. Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a)= a ,f(b)= b,f(c) = c.f is (1, 2)*- #πgs -closed map,but it is not (1, 2)*- πwg -closed map ,since {b,c} is $\tau_1\tau_2$ − closed in X , but f ({b,c})={b,c} is not (1, 2)* πwg -closed set in Y .

Example 4.26:. Consider X=Y={a,b,c}, τ_1= {X, φ,{b},{b,c}} and τ_2 = {X, φ, {c},{a,c}} .So the sets in{ X φ,{b},{c},{a,c},{b,c} } are $\tau_1\tau_2$ –open sets in X,{X,φ,{a},{b},{a,c},{b,c}} are $\tau_1\tau_2$ –closed.Let σ_1 ={Y,φ,{b}} and σ_2 ={ φ,Y,{c},{b,c}}. So the sets in {Y,φ, {b},{c},{b,c}} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {a},{a,b},{a,c}} are σ_1 σ_2 –closed. Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a)= a ,f(b)= b,f(c) = c.Then f is (1, 2)*- #πgs -closed map, but it is not(1, 2)* sg* -closed (resp. (1, 2)*- rw –closed, (1, 2)* πg -closed ,1, 2)*- πgα -closed) map ,since { b} is $\tau_1\tau_2$ − closed in X , but f({b})={ b} is not (1, 2)* sg*-closed (resp. (1, 2)*- rw –closed, (1, 2)* πg -closed ,(1, 2)*- πgα -closed) sets in Y .

Example 4.27: Let (X, τ_1, τ_2)be as in Example (4.26) and σ_1 ={Y,φ,{b}} and σ_2 ={ φ,Y,{c}}. So the sets in {Y,φ, {b},{c},{b,c}} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {a},{a,b},{a,c}} are σ_1 σ_2 –closed. Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a)= a ,f(b)= b,f(c) = c .Then f is (1, 2)*- #πgs -closed map ,but it is not(1, 2)* wg -closed map ,since { b} is $\tau_1\tau_2 -$ closed in X,but f({b})={ b} is not (1, 2)* wg -closed set in Y .

Example 4.28: Consider X=Y={a,b,c}, τ_1= {X, φ,{a},{a,b}} and τ_2 = {X, φ, {b},{b,c}} .So the sets in{ X φ,{a},{b},{a,b},{a,c},{b,c} } are $\tau_1\tau_2$ –open sets inX,{X,φ,{a}, {b},{c} ,{a,c} ,{b,c} } are $\tau_1\tau_2$ –closed.Let σ_1 ={Y,φ,{a},{b},{a,b}} and σ_2={φ,Y,{a,c}}. So the sets in {Y,φ, {a},{b}, {a,b}, {a,c} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ,{b},{c},{a,c},{b,c }} are σ_1 σ_2 –closed. Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a)= a , f(b)= b, f(c) = c. Then f is (1, 2)*- #πgs -closed map, but it is not(1, 2)* rg -closed map ,since { a} is $\tau_1\tau_2 -$ closed in X , but f({a})={ a} is not (1, 2)* rg - closed set in Y.

Theorem 4.29: For any bijection f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) , the following statements are equivalent.
(i) f^{-1}: (Y, σ_1, σ_2) →(X, τ_1, τ_2) is (1,2)*- #πgs -continuous.
(ii) f is a (1,2)*-#πgs -open map.
(iii) f is a (1,2)*-#πgs -closed map.
Proof: (i)⇒(ii).Let U be a $\tau_1\tau_2$-open set in X.Then X- U is $\tau_1\tau_2$-closed in X.Since f^{-1} is (1,2)*- #πgs continuous,by definition 4.1 $(f^{-1})^{-1}$(X-U)=f(X-U) is (1,2)*-πgb-closed in Y. Since f is bijection ,then f(X-U) =Y-f(U) , Y-f(U) is (1,2)*-#πgs -closed in Y ,so f(U) is (1,2)*- #πgs -open in Y.Hence f is a (1,2)*- #πgs -open map.
(ii) ⇒ (iii). Let V be a $\tau_1\tau_2$-closed set in X.Then X-V is $\tau_1\tau_2$-open in X.Since f is (1,2)*- #πgs -open, then f(X-V) is (1,2)*- #πgs -open in Y. Since f is bijection implies that f(X-V) =Y-f(V) . This show that f(V) is (1,2)*-#πgs -closed in Y.Thus f is a (1,2)*-#πgs -closed.
(iii)⇒ (i).Let F be $\tau_1\tau_2$-closed set in X.Since f:X→Y is (1,2)*- #πgs -closed, then f(V) is (1,2)*- #πgs -closed in Y.Since f(V) =$(f^{-1})^{-1}$(V) That is $(f^{-1})^{-1}$(V) is (1,2)*- #πgs -closed in Y. Therefore f^{-1} is (1,2)*-#πgs -continuous.

Theorem 4.30. If a map f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (1, 2) *- #πgs -closed then, (1, 2) *- #πgs cl(f(A)) ⊆ f($\tau_1\tau_2$ -cl(A)) for each subset A of X.
Proof: Let A ⊆ $\tau_1\tau_2$ -cl(A) ⇒ f(A) ⊆ f($\tau_1\tau_2$-cl(A)).Since f is (1, 2)* - #πgs closed, $\tau_1\tau_2$-cl(A) is $\tau_1\tau_2$ – closed in X ⇒ f ($\tau_1\tau_2$ -cl(A)) is (1, 2)* - #πgs closed in Y. By definition 3.1, (1, 2)*-#πgs - cl(f(A)) ⊆ f($\tau_1\tau_2$ cl(A)).

Converse need not be true as seen in the following example.
Example 4.31:. Consider X=Y={a,b,c}, τ_1= {X, φ, {a},{b}{ a,b}} and τ_2 = {X, φ,{b,c}} .So the sets in{ X φ,{a},{b},{a,b},{b,c} } are $\tau_1\tau_2$ –open sets in X,{X,φ,{a},{c},{a,c},{b,c}} are $\tau_1\tau_2$ –closed.Let σ_1 ={Y,φ,{a }} and σ_2={ φ,Y,{b,c}}. So the sets in {Y,φ, {a},{b,c }} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {a },{b,c}} are σ_1 σ_2 –closed. Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by by f(a)= a ,f(b)= b,f(c) = c.Then f is (1, 2)*-#πgs - cl(f(A)) ⊆ f($\tau_1\tau_2$ -cl(A)) ,but it is not (1, 2)*- #πgs -closed -

Theorem 4.32: Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) be (1,2)* - πgb -irresolute and pre (1,2)* -gs -closed map in X .If A is (1,2)*-#πgs -closed set in X ,then f (A) is (1, 2)*-#πgs -closed in Y.
Proof : Let f(A) ⊆ U where U is any (1, 2)* -πgb -open set in Y⇒ A⊆ f^{-1}(U) .Since f is (1, 2)*- πgb -irresolute ⇒ f^{-1}(U) is (1, 2)* -πgb -open set in X . Since A is (1,2)*-#πgs -closed set in X ⇒
(1, 2)* -scl(A) ⊆ f^{-1}(U) ⇒ f((1, 2)* -scl(A)) ⊆ U.Since f is pre (1,2)* -gs-closed and (1,2)* - scl(A) is (1,2)* -gs- closed set in X ⇒f((1, 2)* -scl(A)) is (1, 2)*- gs-closed in Y⇒ (1, 2)* -scl(f(A)) ⊆ (1, 2)* -scl(f((1, 2)* -scl(A))) = f((1, 2)* -scl(A)).We get (1, 2)*- scl (f(A)) ⊆ U.Hence f(A) is (1, 2)*- #πgs –closed.

Theorem 4.33: Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) be (1,2)* - gs -irresolute and pre-(1, 2)*- πgb -closed map in X. If B is (1,2)*-#πgs -closed set in Y ,then f^{-1}(B) is (1, 2)*-#πgs -closed in X.
Proof : Let B be any (1, 2)* -#πgs -closed set in Y and f^{-1}(B) ⊆ U where U is any #π -open set inY .Put V=Y-f(X-U) ⇒ V is #π -open set inY such that B ⊆V and f^{-1}(V) ⊆ U. Since B is (1,2)*-#πgs -closed set in Y ⇒(1,2)* - scl(B) ⊆V ,then ⇒ f^{-1}((1,2)* - scl(B)) ⊆ f^{-1}(V) ⊆ U. Since f is (1,2)* - gs –irresolute ⇒ f^{-1}((1,2)* - scl(B)) is (1,2)* - gs- closed set in X⇒ ((1,2)* - scl(f^{-1} (B)) ⊆ (1,2)* scl(f^{-1} (1,2)* -scl(B))) = (1,2)* - scl(B) ⊆U.Thus f^{-1}(B) is (1, 2)*-#πgs -closed in X.

Remark 4.34: Composition of two (1, 2)*-#πgs - closed maps need not be (1, 2)*-#πgs -closed map **.**
Consider the following example**:**
Example 4.35: Let X = Y = Z = {a,b,c}, τ_1= {X, φ,{a},{a,b},} and τ_2 = {X, φ,{b}, {b,c}} .So the set in{X,φ,{a},{b},{a,b},{a,c},{b,c}}are $\tau_1\tau_2 -$open sets in X,{X,φ, {a},{b},{ c},{a,c} ,{b,c}} are $\tau_1\tau_2 -$closed .

Letσ_1={Y,φ,{a},{b}{a,b}} and σ_2 ={ φ,Y,{a,c}}. So the sets in {Y,φ,{a} ,{b}, {a,b} ,{a,c} are $\sigma_1\sigma_2$ –open and the sets in {Y,φ, {b},{c},{a,c},{b,c}} are σ_1 σ_2 –closed. Let η_1={ Z,φ,{b}} and η_2={Z, φ,{a},{a,c}}.So $\eta_1\eta_2$ –open={Z, φ,{ a},{b} ,{a,b}, {a,c}, {b,c}} and $\eta_1\eta_2$ –closed={Z, φ,{b},{c},{a,c},{b,c}}.
Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g: (Y, σ_1, σ_2)→(Z, η_1,η_2) by the identity maps. Then f and g are (1, 2)*-#πgs –maps but not (1, 2)*-#πgs -map ,since {a} is $\tau_1\tau_2$ –closed in (X, τ_1, τ_2),but gof ({a})=g(f({a})= g({a}) ={a}which is not (1, 2)*- #πgs - closed in (Z, η_1,η_2).Therefore gof is not (1, 2)*-#πgs -map.

Theorem 4.36: Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g: (Y, σ_1, σ_2)→(Z, η_1,η_2) be two maps .Then

i) If f is (1,2)*- closed and g is (1,2)*-#πgs -closed ,then gof is(1,2)*-#πgs -closed **.**
ii) If f is (1,2)*- #πgs -closed and g is (1,2)*- πgb - irresolute and pre-(1, 2)*- gs-closed,then gof is(1,2)*-#πgs -closed **.**

Proof : i) Suppose that V is $\tau_1\tau_2$-closed in X .Since f is (1, 2)*- closed map ⇒ f(V) is $\sigma_1\sigma_2$-closed set in Y. Since g is (1, 2)*-#πgs -closed ⇒g(f(V)) =(gof)(V) is (1, 2)*- #πgs -closed map in Z . Therefore gof is an (1,2)*-#πgs -closed map **.**

ii) Let F be $\tau_1\tau_2$-closed in X .Since f is (1, 2)*- #πgs -closed map ⇒ f(F) is (1, 2)*- #πgs –closed in Y. Since g is(1,2)*-πgb- irresolute and pre-(1, 2)*- gs –closed ,then by Theorem 4.32 g(f(F)) =(gof)(F) is (1, 2)*- #πgs - closed map in Z . Hence gof is an (1,2)*-#πgs -closed map .

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