

MODELLING EXCHANGE RATE VOLATILITY OF THE GHANA CEDI TO THE US DOLLAR USING GARCH MODELS.

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ABSTRACT

The study examines exchange rate volatility with GARCH models using monthly exchange rate data from January 1990 to November 2013. Simple rate of returns is employed to model the exchange rate volatility of Ghana Cedi-United States Dollar. The models included both symmetric and asymmetric models that capture the most common stylized facts about returns such as volatility persistence and leverage effect. The result identified EGARCH (2, 2) as the overall best fitted model. This model has the least AIC of -6.28 and SIC of -6.16. Diagnostic test of the models residuals with the Ljung-Box test, the ARCH-LM test and the ACF plots revealed that the models are free from higher order autocorrelation and conditional heteroscedasticity separately. Our results also revealed persistence of volatility and the non-existence of leverage effects as shown by the asymmetric models.

Keywords: Leptokurtic, volatility persistence, leverage effect.

1. **Introduction**

Exchange rate is one of the macroeconomic variables that play an essential role in the management of most economies. Changes in exchange rates have pervasive effects, with consequences for prices, wages, interest rates, production levels, and employment opportunities, and thus with direct or indirect implications for the welfare of virtually all economic participants.

Exchange rate refers to the number of one currency required to purchase one unit of another currency while volatility is a measure for variation of price of a financial instrument over time. There are various possible factors that could account for exchange rate volatility. Froot and Rogoff (1991) noted that increases in government consumption tend to increase the relative price of nontradables which forms a large proportion of government spending. This was collaborated by De Gregorio et al. (1994). Stancik (2007) also outlined the sources of exchange rate volatility as domestic and foreign money supply, inflation, level of output and the exchange rate regime. The study seeks to modelled exchange rate volatility between the Ghana cedi and the US dollar.

2.Materials and Methods

2.1 Data and Source

The data for this study was monthly exchange rate data of the Ghana cedi to the US dollar from January, 1990 to November 2013. The data was obtained from the Bank of Ghana database. 276 data points was used in the estimation and the remaining 11 data point was also used in our out of sample forecast.

2.2 Unit Root Test

In order to make inferences on time series, the data must be weakly stationary. A weakly stationary time series is one who's first and second moments are invariant of time. We check for stationarity by using The Augmented Dicker Fuller Test and Philip Perrons test.

2.2.1 Augmented Dicker Fuller Test

We used the Augmented Dickey Fuller (ADF) Test to determine whether the times series has a unit root (non-stationary) or is weakly stationary. This test is based on the assumption that the series follows a random walk with model.

$$y_t = \gamma_1 y_{t-1} + e_t$$

The null hypothesis for this test is :

$H_0: \gamma = 0$, the existence of unit root and the alternative hypothesis is

$H_1: \gamma < 0$, the non-existence of unit root. The test statistic for the ADF test is given by

$$ADF = \frac{\gamma}{\mathrm{SE}(\widehat{\gamma})}$$

Where $\hat{\gamma}$ denote the Least Squares estimates of γ and $SE(\hat{\pi})$ is the standard error. The null hypothesis is rejected if the test statistic is greater than the critical value.

2.2.2 Philip-Perron (PP) Test

The PP test is similar to the ADF test with regards to the statement of its hypothesis. This test corrects the statistic for serial correlation and possible Heteroscedastic error terms. The test is based on the regression equation

$$\Delta Y_t = \alpha + \pi Y_{t-1} + \delta t + \varepsilon_t$$

Where Y_t is the time series, α is the intercept, π is the coefficient of interest, *t* is the time or trend variable and ε_t is the disturbance term. The Ordinary Least Squares standard errors are adjusted for serial correlation in the disturbance term ε_t .We fail to reject the null hypothesis of the existence of unit root if the test statistic is less than the critical value

2.3 Testing for Heteroscedasticity

One of the most significant issues before applying the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) methodology is to first examine the residuals for evidence of heteroscedasticity. We will employ the Ljung-Box Statistics test and the ARCH LM test

2.3.1 ARCH-LM Test

The ARCH-LM test proposed by Engle (1982) is used to test for the presence of conditional heteroscedasticity in the model residuals. In summary, the test procedure is performed by first obtaining the residuals from the ordinary least squares regression of the conditional mean equation which might be an autoregressive (AR) process, moving average (MA) process or a combination of AR and MA processes; (ARMA) process. After obtaining the residuals, the next step is regressing the squared residuals on a constant and q lags. The null hypothesis is:

H_0: There is no heteroscedasticity in the model residuals

against

H_1: There is heteroscedasticity in the model residuals

The test statistic is

$$\mathrm{LM} = nR^2$$

where n is the number of observations and R^2 is the coefficient of determination of the auxiliary residual regression.

$$e_t^2 = \beta_0 + \beta_1 e_{t-1}^2 + \beta_2 e_{t-2}^2 + \ldots + \beta_q e_{t-q}^2 + v_t$$

where e_t is the residual. The null hypothesis is rejected when the p-value is less than the level of significance and is concluded that there is heteroscedasticity.

2.3.2 Ljung – Box Test

The null hypothesis (H_0) for this test is that the first lags of the autocorrelation function of the series is zero against the alternative hypothesis (H_1) that not all the first lags of the autocorrelation function of the series is zero.

The test statistic is given as;

$$Q = T(T+2)\sum_{k=1}^{m}(T-k)^{-1}r_k^2$$

where

r_k^2 represent the residual autocorrelation at lag k

T is the number of residua\\m is the number of time lags included in the test

Q is also asymptotically as chi-square with degrees of freedom under the null hypothesis. The decision rule is to reject the null hypothesis of non-autocorrelation of the residuals if the p- value of Q is less than the significance level.

2.4 Volatility Modelling Techniques.

The study employed both symmetric and asymmetric models. In the symmetric models, the conditional variance only depends on the magnitude, and not the sign, of the underlying asset, whereas in the asymmetric models the shocks of the same magnitude, positive or negative, have diverse effect on future volatility.

2.4.1 The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model

The GARCH model introduces and use the lagged conditional variance terms as autoregressive terms. The standard GARCH (p, q) process is specified as:

$$a_t = \sigma_t\epsilon_t \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}\alpha_i a_{t-1}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2$$

$$where$$

$$\alpha_o > 0, \alpha_i \geq 0, \beta_j \geq 0, \sum_{i=1}^{\max(p,q)}(\alpha_i + \beta_j) < 1.$$

2.4.2 The Exponential GARCH (EGARCH) Model

This model captures asymmetric responses of the time-varying variance to shocks and, at the same time, ensures that the variance is always positive. It was developed by Nelson (1991) with the following specification. An EGARCH (p, q) model can be written as

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{p}\alpha_i\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \int\frac{2}{\pi}\right| + \sum_{j=1}^{q}\beta_j\log(\sigma_{t-1}^2) + \sum_{k=1}^{r}\gamma_k\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right|$$

Where γ is the asymmetric response parameter or leverage parameter.

2.3 Models estimation and selection

The study made use of the maximum likelihood estimation approach to estimate the parameters of the models and the best models selected based on the Akaike Information Criterion (AIC) and the Schwarz Bayesian Information (SIC) Criterion. The best model is the one with least values of AIC and SIC.

2.4 Model Diagnostics

The selected models were tested to determine whether or not they properly represent the data set. The diagnostic check on the residuals of the fitted models was to examined whether they are white noise series or not. The Ljung Box test, the ARCH-LM test and the ACF plots of the standardised residuals and squared residuals were applied to the residuals of the best models to determine whether they are random and their variance homoscedastic.

2.5 Results and Discussions

The distribution is positively skewed and the excess kurtosis of 11.86 shows that the series is leptokurtic in nature. The skewness indicates non-normality and this is supported by the Jarque Berra statistic of 1271.46 with an associated p-value of zero as showed in Table 1. To provide better economic and statistical interpretation for the exchange rate data as indicated by Tsay (2005), the data was converted to returns by taking the log difference. We then checked for stationarity by using the Augmented Dicker Fuller test and the Philips- Perrons test as shown in Tables 2 and 3 respectively, both tests confirms that the data is stationary.

We fitted a mean equation with various ARMA (p, q) models and selected an ARMA (1, 1) as the best mean equation based on AIC and SIC values as shown in Table 4. A test for heteroscedasticity was performed on the residuals of the mean equation with the ARCH LM test and the Ljung Box test as shown in Tables 5 and 6 respectively.

The order determination of the models was done by examining the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots of the squared returns and the squared residuals series.

We fitted several ARCH models and selected the best ARCH model based on their AIC and SIC values. This was done for the rest of the models and ARCH (3), GARCH (2, 3), EGARCH (2, 2) and TGARCH (2, 3) models were selected as best models in their respective categories. These results are shown in Tables 7 to 10.1.

The significance of α_i in ARCH(3)and α_i *and* β_j in GARCH (2, 3) indicates that lagged conditional variance and squared disturbance has an impact on the conditional variance, in other words this means that news about volatility from the previous periods has an explanatory power on current volatility.

There is evidence of weakly stationarity in volatility of the monthly exchange rate in the GARCH (2,3) model as the sum of the ARCH parameters and the GARCH parameter are less than one,(i.e.0.380209+0.346804-.473063+.235592+.496889 = 0.986431). This implies that there is volatility persistence in the monthly exchange rate. The persistence in the volatility in the monthly exchange rate means that the impact of new shocks or information on the monthly exchange rate will last for a longer period.

The EGARCH (2, 2) is covariance stationary since the sum of the GARCH parameters (β_j) are less than one. It also provides evidence to the effect that the volatility in the current month's rate of exchange rate is perfectly explained by the volatility in the previous month's exchange rate.

Moreover, there was the existence of asymmetric effects on the volatility of the monthly exchange returns. Consequently positive shocks (news) and negative shocks (news) would have different impacts on the volatility of the monthly exchange rates. However, there was no evidence of leverage effects in the two asymmetric models as the leverage parameter(γ) is positive in the EGARCH (2, 2) and negative in the TGARCH (2, 3). The absence of leverage effects indicates that the impact of a positive shock on the volatility of the monthly exchange rate exceeds that of a negative shock of equal magnitude. From the results, a positive shock would have an impact of 0.6119 on exchange rate in the EGARCH(2,2) model and 0.8428 in the TGARCH(2,3) model while a negative shock of the same magnitude would have an impact of -0.7495 in the EGARCH(2,2) model and 0.2126 in the TGARCH(2,3) model respectively.

This is consistent with the findings of Giot (1999), Olewe (2009) and Bala and Asemota (2013). The EGARCH (2, 2) model was selected as the overall best model when it was compared with the other models based on their AIC and SICS values as shown in Table 11. The conditional mean and conditional variance equations of the EGARCH (2, 2) are given below.

$y_t = 0.004208 + 0.858562y_{t-1} + a_t - 0.2752071\varepsilon_{t-1}, a_t = \sigma_t\varepsilon_t$ and

$$\ln(\sigma_t^2) = -0.912221 - 0.363771\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \int \frac{2}{\pi}\right| + 0.294954\left|\frac{\varepsilon_{t-2}}{\sigma_{t-2}} - \int \frac{2}{\pi}\right| + 0.680713\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + 0.801330\log(\sigma_{t-1}^2) + 0.112875\log(\sigma_{t-2}^2)$$

Dropping the insignificant parameter, the EGARCH (2, 2) model reduces to.

$$\ln(\sigma_t^2) = -0.912221 - 0.363771\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \int \frac{2}{\pi}\right| + 0.294954\left|\frac{\varepsilon_{t-2}}{\sigma_{t-2}} - \int \frac{2}{\pi}\right| + 0.680713\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + 0.801330\log(\sigma_{t-1}^2)$$

The selected model was diagnosed using the Univariate ARCH LM test, the Ljung Box test and the ACF plots of the residuals and were found to be adequate.

The evaluation of the forecasted results using the Chi-Square goodness of fit test shows that there is no significance difference between the expected and the observed values as shown in Table 15

4.6 Conclusions

In this study, the exchange rate volatility between the Ghana cedis and US dollars from January, 1990 to November, 2013 was studied. The results revealed that volatility is persistent. The study also found that there is an adverse asymmetric reaction with good news increasing the volatility more than bad news.

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APPENDIX

Table1: Descriptive Statistics for exchange rate return series

Mean	0.014756
Median	0.006496
Maximum	0.147983
Minimum	-0.016216
Std. Dev.	0.022585
Skewness	2.655882
Kurtosis	11.85897
Jarque-Bera	1271.463
Probability	0.000000
Sum	4.220180
Sum Sq. Dev.	0.145377

Figure1: Histogram of descriptive statistics.

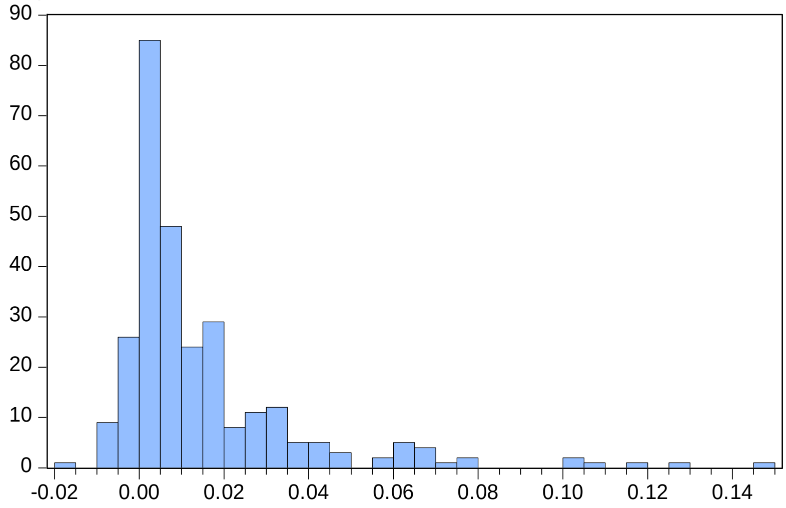

Table2: Augmented Dicker-Fuller test

Test	Constant	P-value	Constant + Trend	P-value
ADF	-4.087254	0.0012	-4.271757	0.0040
	Critical values			
1%	- 3.453400		-3.990935	
5%	-2.871582		-3.425841	
10%	-2.572193		-3.136090	

Table 3:Philips –Perrons Test

Test	Constant	P-value	Constant + Trend	P-value
PP	-9.243601	0.0000	-9.494340	0.0000
		Critical values		
1%	-3.453153		-3.990585	
5%	-2.871474		-3.425671	
10%	-2.572193		-3.135994	

Table 4: Selecting an appropriate mean equation

ARMA(p,q)	AIC	SIC
ARMA(1,1)	-5.1529*	-5.1134 *
ARMA(2,1)	-5.1463	-5.1067
ARMA(1,2)	-5.1420	-5.1025
ARMA(2,2)	-4.9528	-4.9131
ARMA(2,3)	-4.8690	-4.8303
ARMA(3,2)	-4.8313	-4.7915
ARMA(3,3)	-4.8405	-4.8007

Table5: Heteroskedasticity Test: ARCH LM test

F-statistic	5.188111	Probability	0.0000
Obs*R-squared	52.40494	Probability	0.0000

Table6: Test for Heteroscedasticity (ARCH effects) using Ljung Box test

Lags	Test statistic	P-value
6	86.94	0.000
12	104.58	0.000
18	105.90	0.000
24	110.51	0.000
36	119.14	0.000

Figure2: ACF and PACF square returns

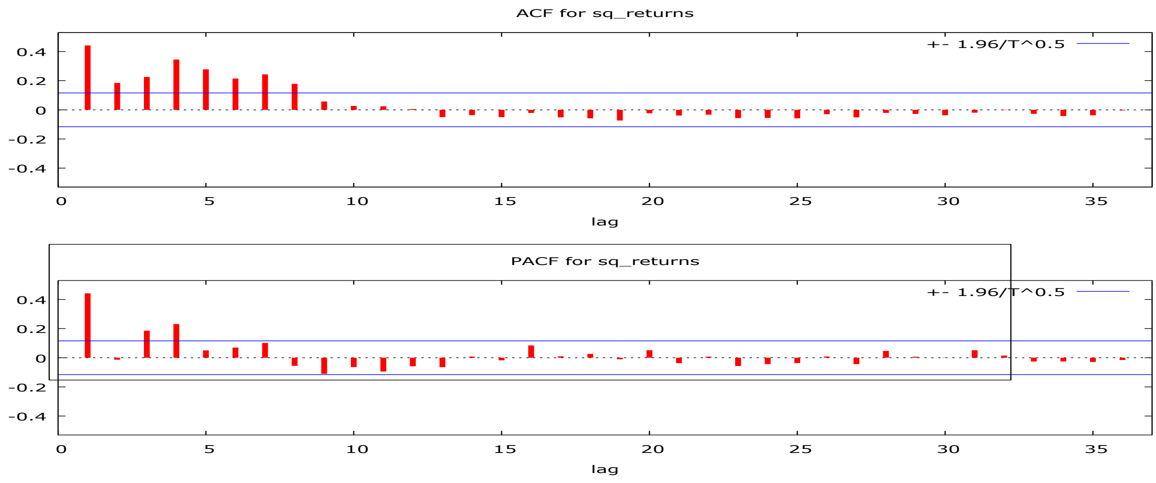

Figure 3: ACF and PACF of square residuals

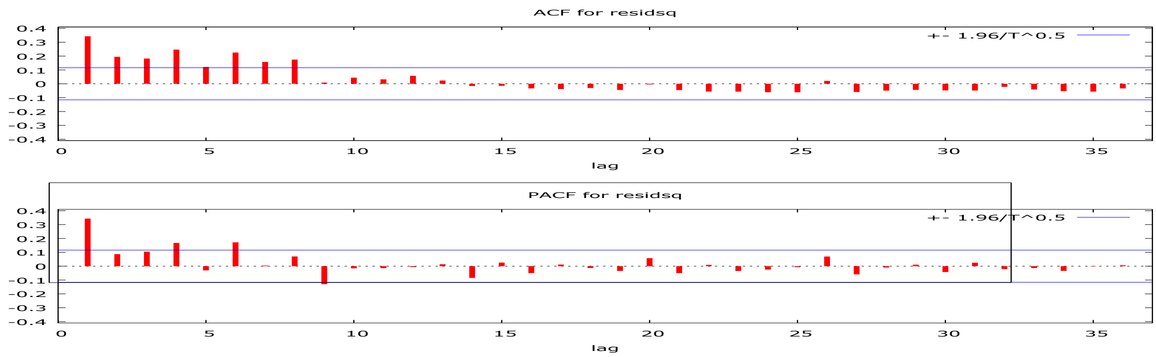

Table 7: Selecting the best ARCH model

Model	AIC	SIC
1	-5.7813	-5.7152
2	-5.8412	-5.7621
3	-5.9823*	-5.8900*

Table 7.1: Estimate parameters of ARCH (3)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.005098	0.002525	2.019196	0.0435
AR(1)	0.860566	0.042109	20.43665	0.0000
MA(1)	-0.323967	0.050841	-6.372141	0.0000
Variance Equation				
α_0	2.32E-05	3.16E-06	7.343816	0.0000
α_1	0.421938	0.103341	4.663574	0.0000
α_2	0.138041	0.057012	2.421236	0.0155
α_3	0.414510	0.095881	4.323171	0.0000

Table 8: Selecting the best GARCH model.

Model	**AIC**	**SIC**
GARCH(1,1)	-6.1092	-6.0301
GARCH(1,2)	-6.1044	-6.0121
GARCH(2,1)	- 6.1067	-6.0143
GARCH(2,2)	-6.0994	-5.9939
GARCH(2,3)	-6.2098 *	-6.0911*
GARCH(3,2)	-6.0924	-5.9737
GARCH(3,3)	-6.1198	-5.9880

Table 8.1: Estimate parameters of GARCH (2, 3)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.003093	0.001507	2.051682	0.0402
AR(1)	0.813910	0.040999	19.85195	0.0000
MA(1)	-0.311451	0.077618	-4.012621	0.0001
Variance Equation				
α_0	6.16E-06	1.77E-06	3.470828	0.0005
α_1	0.380209	0.069340	5.483240	0.0000
α_2	0.346804	0.057705	6.009926	0.0000
β_1	-0.473063	0.029876	-15.83425	0.0000
β_2	0.235592	0.032011	7.359693	0.0000
β_3	0.496889	0.036680	13.54657	0.0000

Table 9: Selecting the best EGARCH model

Model	AIC	SIC
EGARCH(1,1)	-6.2378	-6.1355
EGARCH(1,2)	-6.0969	-5.9914
EGARCH(2,1)	-6.0850	-5.9793
EGARCH(2,2)	-6.2816*	-6.1630*
EGARCH(2,3)	-6.0295	-5.8977
EAGRCH(3,2)	-6.0990	-5.9671
EGARCH(3,3)	- 6.2651	-6.1501

Table 9.1: Parameters estimate of EGARCH (2, 2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.004208	0.001426	2.951852	0.0032
AR(1)	0.858562	0.034816	24.66001	0.0000
MA(1)	-0.275207	0.053857	-5.109988	0.0000
		Variance Equation		
α_0	-0.912221	0.219257	-4.160505	0.0000
α_1	-0.363771	0.116324	-3.127214	0.0018
α_2	0.294954	0.098256	3.001888	0.0027
γ	0.680713	0.103558	6.573260	0.0000
β_1	0.801330	0.168448	4.757132	0.0000
β_2	0.112875	0.157730	0.715624	0.4742

Table10: Selecting the best TGARCH model

Model	AIC	SIC
TGARCH(1,1)	-6.1169	-6.0246
TGARCH(1,2)	-6.1366	-6.0311
TGARCH(2,1)	- 6.0498	-5.9443
TGARCH(2,2)	-6.0154	-5.8967
TGARCH(2,3)	-6.2034*	-6.0715*
TGARCH(3,2)	-6.0386	-5.9067
TGARCH(3,3)	-5.9615	-5.8164

Table10.1: Parameters estimate of TGARCH (2, 3) model.

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.002014	0.001242	1.621231	0.1050
AR(1)	0.888971	0.031820	27.93761	0.0000
MA(1)	-0.534720	0.063594	-8.408295	0.0000
		Variance Equation		
α_0	1.37E-06	5.67E-07	2.407478	0.0161
α_1	0.606283	0.110655	5.479034	0.0000
γ	-0.630217	0.159273	-3.956839	0.0001
α_2	0.236483	0.050789	4.656219	0.0000
β_1	-0.244248	0.041125	-5.939128	0.0000
β_2	0.212095	0.037381	5.673826	0.0000
β_3	0.454835	0.042198	10.77847	0.0000

Table 11: Selecting the most appropriate model

Model	AIC	SIC
ARCH(3)	-5.9823	-5.8900
GARCH(2,3)	-6.2098	-6.0911
EGARCH(2,2)	**-6.2816***	**-6.1630***
TGARCH(2,3)	-6.2034	-6.0715

MODEL DIAGNOSTICS

Table12: Heteroskedasticity Test: ARCH LM for EGARCH(2,2)

F-statistic	0.247895	Probability	0.9954
Obs*R-squared	3.093091	Probability	0.9949

Table 13: Ljung Box test for EGARCH (2,2)

LAGs	test Statistic	P- value
6	1.3926	0.966
12	3.2794	0.993
18	4.7920	0.999
24	6.2208	1.000
36	10.047	1.000

Figure 4: ACF of standardised residuals and squared residuals of EGARCH (2,2)

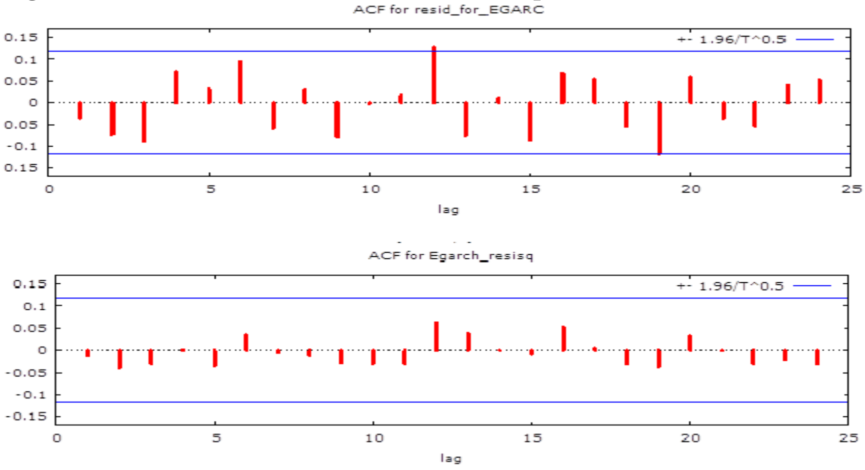

Table 14: Forecast values for EGARCH (2, 2) with corresponding observed values.

Year/Month	Observed	Forecasted
2013M01	0.002125	0.001332
2013/M02	0.001273	0.002202
2013M03	0.007710	0.001944
2013M04	0.006084	0.005628
2013M05	0.014637	0.005693
2013M06	0.003138	0.010700
2013M07	0.001283	0.005371
2013M08	0.003329	0.002822
2013M09	0.002502	0.003314
2013M10	0.034240	0.002967
2013M11	0.025833	0.021386

Table 15: Chi-square goodness of fit test

Critical values(table values) 5%	Test Statistic
18.307	0.3712