Fixed Point Theorem in Intuitionistic Fuzzy Metric Space by Using Occasionally Weakly Compatible Maps in Rational Form

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Abstract

In this paper we have generalized the result of Kamal Wadhwa and Hariom Dubey by using occasionally weakly compatible maps using intuitionistic fuzzy metric space. The concept of compatible maps introduced by Kramosil and Michalek and weakly compatible maps in fuzzy metric space is generalized by A. Al. Thagafi and Nasser Shahzad by introducing the concept of occasionally weakly compatible mappings.

Keywords: Fixed point, intuitionistic fuzzy metric space, compatible mappings.

Introduction

Fuzzy Set was introduced and defined by Zadeh. Kramosil and Michalek introduced fuzzy metric space, George and Veeramani modified the notion of fuzzy metric space with the help of continuous t- norm. Vasuki proved fixed point theorem for R- weakly commuting mapping. Pant introduced the new concept of common fixed point theorems.

Preliminaries

Definition: A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if * is satisfying the following conditions:

(a) * is commutative and associative;

(b) * is continuous;

(c) a * b = a for all a ∈ [0, 1];

(d) a* b ≤ c * d whenever a ≤ c and b ≤ d and a, b, c, d ∈ [0, 1].

Definition: A 3-tuple (X, M, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions ; for all x, y, z ∈ X, s, t > 0.

(FM-1)M (x, y, t) > 0;

(FM-2) M(x, y, t) =1 if and only if x = y;

(FM-3) M(x, y, t) = M(y, x, t);

(FM-4) M(x, y, t)* M(y, z, s) ≤ M(x, z, t + s);

(FM-5) M(x, y,.): (0, ∞)→[0, 1] is continuous.

Then M is called a fuzzy metric on X. The function M(x, y, t) denote the degree of nearness between x and y with respect to t.

Example: Let (X, d) be a metric space. Denote a *b =a b for a, b ∈ [0, 1] and let M_d be a fuzzy set on $X^2 \times (0, \infty)$ defined as follows:

$$M_d\,(x, y, t) = \frac{t}{t+d(x,\ y)}$$

Then (X, M_d, *) is a fuzzy metric space, we call this fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric space.

Definition: Let (X, M,*) be a fuzzy metric space, then

(a) A sequence $\{x_n\}$ in X is said to be convergent to x in X if for each ε > 0 and each t > 0, there exists $n_0 \in$ N such that M (x_n, x, t) > 1 − ε for all n ≥ n_0.

(b)A sequence $\{x_n\}$ in X is said to be Cauchy if for each ε > 0 and each t > 0, there exist $n_0 \in$ N such that M (x_n, x_m, t) > 1 − ε for all n, m ≥ n_0 .

(c)A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition: Two self-mappings A and S of a fuzzy metric space (X, M, *) are called compatible if $\lim_{n\to\infty}$ M (ASx_n, SAx_n, t) = 1, whenever { x_n} is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \text{ for some x in X.}$$

Definition: Two self-maps A and B of a fuzzy metric space (X, M, *) are called weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e. if Ax = Bx for some x ∈ X then ABx = BAx.

If self-maps A and B of a fuzzy metric space (X, M, *) are compatible then they are weakly compatible. Let (X, M,*) be a fuzzy metric space with the following condition:

(FM-6)$\lim_{t\to\infty}$ M (x, y, t) =1 for all x, y ∈ X.

Lemma: Let (X, M, *) be a fuzzy metric space. If there exists k ∈[0, 1] such that

$$M(x, y, kt) \geq M(x, y, t) \text{ then } x = y.$$

Lemma: Let $\{x_n\}$ be a sequence in a fuzzy metric space (X, M, *) with the condition (FM-6). If there exists k ∈[0, 1] such that

$$M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t) \text{ for all } t > 0 \text{ and } n \in N$$

Then { y_n } is a Cauchy sequence in X.

Lemma: Let X be a set A and B owc self maps of X. If A and B have a unique point of coincidence w = Ax = Bx, then w is the unique common fixed point of A and B.

Main Results

Theorem: Let (X, M, N, *, ◊) be a complete intuitionistic fuzzy metric space and let A, B, S, T be self mapping of X. Let the pairs (A, S) and (B, T) be owc and k > 1 then

$$M(Ax, By, kt) \leq \min \left\{ \begin{matrix} M(Sx,Ty,t), M(Sx,Ax,t), M(Ty,By,t), \\ M(Sx,By,t), M(Ty,Ax,t) \\ \frac{aM(Ax,Ty,t)+ b\, M(By,Sx,t)+ c\, M(Sx,Ty,t)}{a+b+c}, \\ \frac{M(By,Ty,t)+ M(Ax,Sx,t)}{2} \end{matrix} \right\} \quad (1)$$

$$N(Ax, By, kt) \geq \max \left\{ \begin{matrix} N(Sx,Ty,t), N(Sx,Ax,t), N(Ty,By,t), \\ N(Sx,By,t), N(Ty,Ax,t) \\ \frac{aN(Ax,Ty,t)+ b\, N(By,Sx,t)+ c\, N(Sx,Ty,t)}{a+b+c}, \\ \frac{N(By,Ty,t)+ N(Ax,Sx,t)}{2} \end{matrix} \right\} \quad (2)$$

For all x, y Є X and t > 0 such that Aw = Sw = w and a unique point z Є X such that Bz = Tz = z. Moreover z = w, so that there is a unique common fixed point of A, B, S, T.

Proof: Let the pairs (A, S) and (B, T) are OWC so there are points x, y Є X such that Ax = Sx and By = Ty, we claim that Ax = By. If not then by inequality (1) and (2)

$$M(Ax, By, kt) \le \min\left\{\begin{matrix} M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), \\ M(Sx, By, t), M(Ty, Ax, t) \\ \frac{aM(Ax,Ty,t) + b\,M(By,Sx,t) + c\,M(Sx,Ty,t)}{a+b+c}, \\ \frac{M(By,Ty,t) + M(Ax,Sx,t)}{2} \end{matrix}\right\}$$

$$N(Ax, By, kt) \ge \max\left\{\begin{matrix} N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), \\ N(Sx, By, t), N(Ty, Ax, t) \\ \frac{aN(Ax,Ty,t) + b\,N(By,Sx,t) + c\,N(Sx,Ty,t)}{a+b+c}, \\ \frac{N(By,Ty,t) + N(Ax,Sx,t)}{2} \end{matrix}\right\}$$

$$M(Ax, By, kt) \le \min\left\{\begin{matrix} M(Ax, By, t), M(Ax, Ax, t), M(By, By, t), \\ M(Ax, By, t), M(By, Ax, t) \\ \frac{aM(Ax,By,t) + b\,M(By,Ax,t) + c\,M(Ax,By,t)}{a+b+c}, \\ \frac{M(By,By,t) + M(Ax,Ax,t)}{2} \end{matrix}\right\}$$

$$N(Ax, By, kt) \ge \max\left\{\begin{matrix} N(Ax, By, t), N(Ax, Ax, t), N(By, By, t), \\ N(Ax, By, t), N(By, Ax, t) \\ \frac{aN(Ax,By,t) + b\,N(By,ASx,t) + c\,N(Ax,By,t)}{a+b+c}, \\ \frac{N(By,By,t) + N(Ax,Ax,t)}{2} \end{matrix}\right\}$$

$M(Ax, By, kt) \le \min\{ M(Ax, By, t), 1, 1, M(Ax, By, t), M(Ax, By, t), M(Ax, By, t), 1\}$

$N(Ax, By, kt) \ge \max\{ N(Ax, By, t), 0, 0, N(Ax, By, t), N(Ax, By, t), N(Ax, By, t), 0\}$

$M(Ax, By, kt) \le M(Ax, By, kt)$ and $N(Ax, By, t) \ge 0$

then by lemma Ax = By.

Suppose that there is another point z such that Az = Sz. Then by inequality (1) and (2) we have Az = Sz = By = Ty so Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S. By lemma w is the only common point of A and S. Similiarly there is a unique point z Є X such that z = Bz = Tz.

Assume that $w \ne z$ then by (1)

$$M(w, z, kt) = M(Aw, Bz, kt) \le \min\left\{\begin{matrix} M(Sw, Tz, t), M(Sw, Aw, t), M(Tz, Bz, t), \\ M(Sw, Bz, t), M(Tz, Aw, t) \\ \frac{aM(Aw,Tz,t) + b\,M(Bz,Sw,t) + c\,M(Sw,Tz,t)}{a+b+c}, \\ \frac{M(Bz,Tz,t) + M(Aw,Sw,t)}{2} \end{matrix}\right\}$$

$$N(w, z, t) = N(Aw, Bz, kt) \ge \max\left\{\begin{matrix} N(Sw, Tz, t), N(Sw, Aw, t), N(Tz, Bz, t), \\ N(Sw, Bz, t), N(Tz, Aw, t) \\ \frac{aN(Aw,Tz,t) + b\,N(Bz,Sw,t) + c\,N(Sw,Tz,t)}{a+b+c}, \\ \frac{N(Bz,Tz,t) + N(Aw,Sw,t)}{2} \end{matrix}\right\}$$

$$M(w, z, kt) \le \min\left\{\begin{matrix} M(w, z, t), M(w, w, t), M(z, z, t), M(w, z, t), \\ M(z, w, t), M(w, z, t), \\ \frac{\{ M(z,z,t) + M(w,w,t)}{2} \end{matrix}\right\}$$

$$N(w, z, kt) \geq \max \left\{ \begin{matrix} N(w, z, t), N(w, w, t), N(z, z, t), N(w, z, t), \\ N(z, w, t), N(w, z, t), \\ \frac{\{ N(z,z,t) + N(w,w,t)}{2} \end{matrix} \right\}$$

$M(w, z, kt) \leq M(w, z, t)$ and $N(w, z, kt) \geq 0$.

Therefore w = z. Z is a common fixed point of A, B, S, T.

Uniqueness: Let u be another common fixed point of A, B, S, T. Then put x = z and y = u in (1) and (2).

$$M(Az, Bu, kt) \leq \min \left\{ \begin{matrix} M(Sz, Tu, t), M(Sz, Az, t), M(Tu, Bu, t), \\ M(Sz, Bu, t), M(Tu, Az, t) \\ \frac{aM(Az,Tu,t) + b\,M(Bu,Sz,t) + c\,M(Sz,Tu,t)}{a+b+c}, \\ \frac{M(Bu,Tu,t) + M(Az,Sz,t)}{2} \end{matrix} \right\}$$

$$N(Az, Bu, kt) \geq \max \left\{ \begin{matrix} N(Sz, Tu, t), N(Sz, Az, t), N(Tu, Bu, t), \\ N(Sz, Bu, t), N(Tu, Az, t) \\ \frac{aN(Az,Tu,t) + b\,N(Bu,Sz,t) + c\,N(Sz,Tu,t)}{a+b+c}, \\ \frac{N(Bu,Tu,t) + N(Az,Sz,t)}{2} \end{matrix} \right\}$$

$M(z, u, kt) \leq M(z, u, t)$ and $N(z, u, kt) \geq 0$.Then by lemma z = u.

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