

The effect of joule heating, thermal radiation on the MHD convective Heat and Mass Transfer flow of a variable electrically conducting micro polar fluid.

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Abstract

The magneto-hydrodynamic (MHD) and effect of Joule heating, thermal radiation, chemical reaction and dissipation on the convective heat and mass transfer flow of a micro polar fluid past a stretching sheet with variable electric conductivity in the presence of a magnetic field. The non-linear equations governing the flow, heat and mass transfer have been solved numerically by employing fourth order Runge – Kutta shooting method. Results are obtained for a range of Eckert number (Ec), magnetic, Joule heating, micro rotation parameter, heat source parameter, thermal radiation parameter and chemical reaction parameter. The sheet is assumed to be non-isothermal with prescribed heat flux varying with length.

Keywords: Joule heating, magnetic field, micro polar fluid, stretching sheet.

Introduction:

Eringen (1966) [10] proposed a theory of molecular fluids taking into account the inertial characteristics of the substructure particles, which are allowed to undergo rotation. Physically, the micropolar fluid can consist of a suspension of small, rigid, cylindrical elements such as large dumbbell shaped molecules. The theory of micropolar fluids is generating a very much increased interest and many classical flows are being re-examined to determine the effects of the fluid microstructure.

The concept of micropolar fluid deals with a class of fluids that exhibit certain microscopic effects arising from the micro motions of the fluid elements. These fluids contain dilute suspension of rigid macromolecules with individual motions that support stress and body moments and are spin inertia. Micropolar fluids are those which contain micro-constituents that can undergo rotation, the presence of which can affect the hydrodynamics of the flows so that it can be distinctly non-Newtonian. It has many practical applications, for example, analyzing the behavior of exotic lubricants (Khonsari, 1990[20]; Khonsari and Brew, 1989[19]), the flow of colloidal suspensions or polymeric fluids (Hadimoto and Tokioka, 1969[14]), liquid crystals (Lockwood et al., 1987[23]; Lee and Egigen, 1971[22]), additive suspensions, human and animal blood (Ariman et al., 1974[2]), turbulent shear flow and so forth.

Earlier (Sakiadis, 1961[28]) introduced the concept of continuous surfaces such as polymer sheets of filaments continuously drawn from die. He studied the boundary layer behavior on continuum solid and flat surfaces. The boundary layer flows on continuous surfaces is an important type of flows occurring in a number of technical processes, for example, continuous casting, glass fiber production, metal extrusion, hot rolling, cooling and/or dying of paper and textiles, wire drawing, etc (Tadmor and Klein, 1970[30]; Fisher, 1976[12]; Altan et al., 1979[1]). Peddison and McNitt (1970)[25] derived boundary layer theory for micropolar fluid which is important in a number of technical processes and applied this equation to the problems of steady stagnation point flows, steady flows past a semi-infinite flat plate. Eringen (1972) [11] developed the theory of thermomicropolar fluids by extending the theory of micropolar fluids. Ebert (1973) [6] presented a similarity solution for boundary layer flows near a stagnation point for the micropolar fluid. Studies of free or mixed convection in micropolar fluids past flat, curved and/ or wavy surfaces have been focused by a number of workers because of the importance of the heat transfer on the flows field of micropolar fluid for determining the quality of the final products of the process mentioned above (Karwe and Jaluria, 1988a[17]; 1988b[18]). Yucel (1989) [31] studied the mixed convection flow of micropolar fluid over a horizontal plate. Char and Chang (1995) [4] studied the laminar free convection flow of a micropolar fluid past an arbitrary curved surface. Gorla (1992) [13] studied mixed convection in a micropolar fluid from a vertical surface with uniform heat flux. Hossain et al. (1995) [15] studied the mixed convection flow of micropolar fluids with variable spin gradient viscosity along a vertical plate. Desseaux and Kelson (2000) [5] studied the flow of a micropolar fluid bounded by a stretching sheet.

El-Haikem et al. (1999) [9] have studied the Joule heating effects on magneto hydrodynamic free convection flow of a micropolar fluid. El-Amin (2001) [7] has studied the magneto hydrodynamic free convection and mass transfer flow in micropolar fluid with constant suction. Very recently Rahman and Sattar (2006) [26] have studied the magneto hydrodynamics convective flow of a micropolar fluid past a continuously

moving vertical porous plate in the presence of heat generation/absorption. In the above mentioned work they have extended the Md. Ziaul Haque et al. work of El-Arabawy (2003) [8] to a MHD flow taking into account the effect of free convection and micro rotation inertia term which has been neglected by El-Arabawy (2003). However, most of the previous works assume that the plate is at rest. Kim (2001) studied the unsteady MHD free convection flow of micropolar fluid past a vertical moving porous plate in a porous medium. W.A. Aissa, A.A. Mohammadein (2005) [3] has studied the joule heating effects on a micropolar fluid past a stretching sheet with variable electric conductivity. M.M. Rahman (2009) [27] has studied the convective flows of micropolar fluids from radiative isothermal porous surfaces with viscous dissipation and joule heating.
Siddheshwar and pranesh [29] investigated magneto-convection in a micropolar fluid.

The present work investigated the effects of joule heating on a micropolar fluid with variable conductivity past a stretching, continuous sheet in the presence of a magnetic field. The study considers the surface with prescribed wall heat flux varying with length using a numerical technique based on the shooting method.

The effects of the magnetic field parameter, suction parameter, Eckert number and micropolar parameter on the velocity of the fluid, temperature gradient and angular velocity of microstructures as well as the coefficient of heat flux and shearing stress at the plate are investigated at specific values of prandtl number. Different values of physical parameters are tabulated and discussed numerically and graphically.

Formulation of the Problem:

We consider a steady incompressible two-dimensional boundary layer of an electrically conducting micropolar fluid spreading over a permeable plane surface .An applied magnetic field of strength B is applied transverse to the plate in the y-direction and varies in strength as a function of x and is defined as:

$$B=(x, B(x)) \tag{2.1}$$

We consider a Cartesian rectangular co-ordinate system O(x, y) with the plate in the y-direction and x-axis normal to the boundary. The sheet is located at y=0 and its leading edge is the origin of the Cartesian coordinate system.
The external electric field assumed to be zero and the magnetic Reynolds number is assumed to be small. Hence the induced magnetic field is small compared with the external magnetic field. Moreover, the electrical conductivity σ is assumed to have the form

$$\sigma=\sigma_0 u \tag{2.2}$$

The fluid of density (ρ) is at rest and the motion is created by stretching the sheet with a speed proportional to the distance from the fixed origin(x=9).The viscosity coefficient (μ) represents constant, the pressure gradient and the body forces are negligible in the presence of viscous dissipation and internal heat generation. The effects of uniform mass and heat transfer characteristics in stationary surroundings are investigated. Under these assumptions, the governing equations within the boundary layer are given by:

1. Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.3}$$

2. Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(\nu+\frac{k}{\nu})\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma(B(x))^2}{\nu}u-\rho g \tag{2.4}$$

3. Angular momentum:

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{k}{\rho j}(2N+\frac{\partial u}{\partial x}) \tag{2.5}$$

4. Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_f}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{(\mu+k)}{\rho C_p}(\frac{\partial u}{\partial y})^2+\frac{\sigma(B(x))^2}{\rho C_p}u^2-\frac{1}{\rho C_p}\frac{\partial(q_R)}{\partial y} \tag{2.6}$$

5. Equation of diffusion

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_1\frac{\partial^2 C}{\partial y^2}-k_r^{'}(C-C_\infty) \tag{2.7}$$

6. Equation of State:

$$\rho-\rho_\infty=-\beta\rho_\infty(T-T_\infty)--\beta^{\bullet}\rho_\infty(C-C_\infty) \tag{2.8}$$

Where u and v are the velocity components along the x and y axes ,respectively, N is angular velocity, k is vortex viscosity,υ is kinematic viscosity, γ is spin gradient viscosity is the micro inertia per unit mass, T,C are the temperature, concentration in the fluid, T_∞ , C_∞ are ambient temperature, concentration ,respectively. k_f is the thermal conductivity, Cp is the specific heat at constant pressure, kr is the coefficient of chemical reaction,$D_{1i}$is the molecular diffusivity and q_R is the radiative heat flux.
The boundary conditions are

$$u = cx, v = vo, N = -s\frac{\partial u}{\partial y}, -k_f\frac{\partial T}{\partial y} = q_w(x) = bx^m, -D_1\frac{\partial C}{\partial y} = m_w(x) = ax^n \qquad on \quad y = 0$$

$$u = 0\ ,\ N = 0\ , T = T_\infty\ ,\ C = C_\infty \qquad as \quad y \to \infty \tag{2.8a}$$

Positive and negative values of v_w indicate blowing and suction respectively, while v_w=0 corresponds to an impermeable sheet.
A comment on the boundary conditions used for the micro rotation term will be made here. When s=0,we obtain from the boundary condition(2.8a)for the micro rotation that N(x,0)=0,which represents the case of concentrated particle flows in which the microelements close to the wall are not able to rotate as was stated by Jeena and Mathur(16).The case corresponds to s=1/2 results in the vanishing of the anti symmetric part of the stress tensor and represents weak concentrations and the case corresponding to s=1 is representative of turbulent boundary layer flows.
Following Roseland approximation (*) we take

$$q_R = -\frac{4\sigma^\bullet}{3\beta_R}\frac{\partial (T'^4)}{\partial y} \tag{2.9}$$

$$T'^4 \cong 4T_\infty^3 T - 3T_\infty^4$$

Using (2.9) the equation of energy (2.6) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_f}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{(\mu + k)}{\rho C_p}(\frac{\partial u}{\partial y})^2 + \frac{\sigma (B(x))^2}{\rho C_p}u^2 + \frac{16\sigma^\bullet T_\infty^3}{3\beta_R}\frac{\partial^2 T}{\partial y^2} \tag{2.10}$$

For the flow under study, it is relevant to assume that the applied magnetic field strength B(x) has the form (9)

$$bX) = \frac{B_O}{\sqrt{x}}, B_O\ is\ cons\tan t \tag{2.11}$$

The third term in equation (2.4) taking into account equation (2.2) and (2.11) can be rewritten as

$$\frac{\sigma (B(x))^2}{\rho}u = \frac{\sigma B_o^2}{\rho x}u^2 \tag{2.12}$$

By substituting from equation (2.12) and (2.8) in equation (2.4) the momentum equation can be rewritten as:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (v + \frac{k}{v})\frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho}\frac{\partial N}{\partial y} + \frac{\sigma B_o^2}{\rho x}u^2 + \tag{2.13}$$
$$+ \beta\rho_\infty (T - T_\infty) + \beta^\bullet \rho_\infty (C - C_\infty)$$

In view of the continuity equation (2.3) we define a stream function $\psi(x, y)$ as

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x} \tag{2.14}$$

Using (2.14) we find that

$$u = cxf'(\eta) \quad , \quad v = -\sqrt{c\nu}\, f(\eta) \tag{2.15}$$

Where prime denote differentiation with respect to η .

Proceeding with the analysis, the following transformations are introduced:

$$\eta=\sqrt{\frac{c}{\nu}}y\ ,\quad \psi=\sqrt{c\nu}\,xf(\eta),\quad N=c\sqrt{\frac{c}{\nu}}xg(\eta),\ T-T_{\infty}=\frac{D}{k}\sqrt{\frac{\nu}{c}}(\frac{x}{L})^{2}\theta(\eta),$$
$$C-C_{\infty}=\frac{D_2}{D_1}\sqrt{\frac{\nu}{c}}(\frac{x}{L})^{2}\phi(\eta), \qquad (2.16)$$

The momentum, angular momentum, energy and diffusion equations can be written as:

$$(1+\Delta)f'''-(f')^{2}+ff''-M^{2}(f')^{2}+\Delta g'+\frac{G}{R_{ex}^{2}}(\theta+N\phi) \qquad (2.17)$$

$$\lambda g''-\Delta B_1(2g+f''')-gf'+fg'=0 \qquad (2.18)$$

$$\frac{1}{P_r}\theta''-E(1+\Delta)(f'')^{2}-2\theta f'+f\theta'+E(M-1)(f')^{3}=0 \qquad (2.19)$$

$$\phi''+Sc(f\phi'-2f'\phi)-(Sck_c)\phi=0 \qquad (2.20)$$

Where

$R_{ex}=\frac{cx^2}{\nu}$ (Local Reynolds number)

$G=\frac{\beta g\Delta Tx^3}{\nu^2}$ (Grshof number)

$N_1=\frac{\beta^{\bullet}\Delta C}{\beta\Delta T}$ (Buoyancy number)

$M^2=\sigma B_o^2$ (Magnetic parameter)

$\Delta=\frac{k}{\mu}$ (Micro rotation parameter)

$B_1=\frac{k}{\rho j}$

$\lambda=\frac{\gamma}{\mu j}$ (Spin gradient viscosity)

$Sc=\frac{\nu}{D_1}$ (Schmidt number)

$\Pr=\frac{k_f}{\mu C_p}$ (Prandtl number)

$k_r=\frac{k_r^{'}}{c}$ (Chemical reaction parameter)

The Eckert number E is defined as

$$E=\frac{k}{D}\frac{L^2}{\sqrt{c\nu}}\frac{c^3}{C_p}$$

The transformed boundary conditions may be written as

$$f(0)=-\frac{v_w}{\sqrt{c\nu}}=f_w,\ f'(0)=1,\ g(0)=0\theta'(0)=-\frac{b}{D}L^2x^{m-2},\phi'(0)=-\frac{aL^2}{D_2}x^{n-2} \qquad (2.28)$$

$$f'(\infty)=0,\ g(\infty)=0,\ \theta(\infty)=0,\ \ \phi(\infty)=0$$

But as $\theta'(0)$, $\phi'(0)$ must be equal to -1, this implies (as could be concluded from Equation (2.28) that m=2, n=2 and a &b should satisfy the following relations

$$b = \frac{D}{L^2} \quad , \quad a = \frac{D_2}{L^2} \tag{2.29}$$

The set of transformed equations (2.17)-(2.20) are solved using a fourth order Runge-Kutta method of numerical integration.
In order to start a solution, the value of boundary conditions at η=0 are substituted and integration must be carried out up to some large η to see if the boundary conditions at infinity are satisfied.

The shearing stress at the sheet is given by

$$\tau_w = \{(\mu + K)\frac{\partial u}{\partial y} + KN\}_{y=0} = \mu c\sqrt{\frac{c}{\nu}}x((1+\Delta)f''(0) + \Delta g'(0))$$

It is clear that the wall shear stress will increase with increasing x. The skin friction coefficient C_f takes the form:

$$C_f = \frac{\tau_w}{0.5\rho(cx)^2} = -\frac{2}{(R_{ex})^{1/2}}[(1+\Delta)f''(0) + \Delta g'(0)]$$

where

$$R_{ex} = \frac{cx^2}{\nu} \text{ is the Reynolds number}$$

DISCUSSION OF THE NUMERICAL RESULTS:

In this analysis we discuss the effect of Joule heating, thermal radiation, chemical reaction and dissipation on the convective heat and mass transfer flow of a micro polar fluid past a stretching sheet with variable electric conductivity in the presence of a magnetic field.

The non-linear equations governing the flow, heat and mass transfer have been solved numerically by employing fourth order Runge – Kutta shooting method. Results are obtained for a range of Eckert number (Ec) magnetic, Joule heating, micro rotation parameter, heat source parameter, thermal radiation parameter and chemical reaction parameter. The sheet is assumed to be non-isothermal with prescribed heat flux varying with length.

The Velocity (f'), Micro Rotation (g), Temperature gradient (θ') and Concentration gradient (ϕ') are plotted versus η for various values of Hartmann number M and specific values of the physical parameter (fw = 0.2, E = 0.5,, N=1, Q=0.5, Δ=0.5, Sc=1.3, γ=0.5, Nr=0.5 and Ec=0.01).

In figs (2a – 2d), it is found that the temperature gradient (θ'), The velocity (f') and the micro rotating (g) experiences retardation with increasing M. From figs. (2c & 2d) we find that the temperature gradient enhances and concentration gradient reduces with increasing M.

Figs (3a – 3d) represents f', g, θ', ϕ' with Schmidt number (Sc). It is found that lesser the molecular diffusivity smaller the velocity f' and micro ration (g) and concentration gradient (ϕ') and higher the temperature gradient in the flow fluid.

Figs (4a – 4d) represents in the variation of f', g, θ', ϕ' with buoyancy ration N. It can be seen from the profiles that the molecular buoyancy force dominates over the thermal buoyancy force, the velocity f', micro rotation 'g' and temperature gradient enhances while the concentration gradient reduces when the buoyancy forces are in the same direction and for the forces acting in opposite directions,f' and g reduces while the temperature gradient and concentration gradient enhances in the flow fluid.

Figs (5a – 5d) represents in the variation of f', g, θ', ϕ' with Strength of heat source parameter Q. It can be observed from the presence of f' , g. The velocity f' and micro rotation g enhances with increase in the strength of heat generating source and reduces in the case of heat absorbing source. The temperature gradient and concentration gradient reduces with increase in Q > 0 and enhances with |Q| (5c & 5d).

Figs (6a – 6d) represents in the variation of f', g, θ', ϕ' with micro rotating parameter Δ. It is found that the velocity f' and the micro rotating g enhances with increase inΔ, while the temperature gradient and concentration gradient reduces with Δ. The effect of chemical reaction (γ) parameter on f', g, θ', ϕ' exhibited in the figs (7a – 7d). It is found that the velocity f', the micro rotation g and concentration gradient reduces and while the temperature gradient enhances in degenerating chemical reaction case and in the generating chemical reaction case f' & g enhances in the flow fluid while the temperature and concentration gradients reduces in the flow region.

The effect of thermal reaction and flow characteristic is exhibited in figs (8a – 8d). It is found that higher the radiative heat flux larger f', temperature gradient (θ') and smaller the micro rotating g and concentration gradient (ϕ').

Figs (9a – 9d) represents in the variation of f', g, θ', ϕ' with suction parameter (fw). It can be seen from the profiles that higher the suction velocity (>0) smaller the velocity f', micro rotating g, the temperature and concentration gradients and they experience an enhancement with increase in |fw| (<0).

Figs (10a – 10d) show the variation with respect to Eckert number Ec. It is found that higher dissipation heat larger velocity f' and temperature gradient (θ'), while smaller the micro rotation (g) and concentration gradient (ϕ') in the entire flow fluid.

The skin friction (τ), Couple stress (Cw), rate of heat and mass transfer on the plate η=0 is exhibited in table.1 for different variations. It is found that the skin friction and Couple stress and the rate of mass transfer reduce in magnitude with increase in magnetic parameter M while the rate of heat transfer enhances on the plate with M. The variation with Scmidt number Sc shows that lesser the molecular diffusivity larger |τ| and Cw and larger Nu and Sh on the plate .When the molecular buoyancy force dominates over the thermal buoyancy force |τ| and Cw enhances irrespective of the directions of the buoyancy forces. An increase in the buoyancy ratio N enhances Nu and reduces Sh at η=0 when the buoyancy forces are in the same direction and for the forces acting in opposite directions, |Nu| reduces and |Sh| enhances on the plate.

The variation of τ,Cw, Nu and Sh with heat source parameter Q shows that |τ| and Nu enhances with increase in the strength of the heat generating source and reduce with that of heat absorbing source. An increase in Q>0, reduces Nu and Sh while an increase in |Q|(<0) enhances |Nu| and reduces |Sh| at the plate. With reference to the Eckert number Ec, we find that higher the dissipative heat smaller the stress and couple stress while larger the rate of heat and mass transfer at the plate .Higher the radiative heat flux smaller the stress, couple stress and Sherwood number at η=0.With respect to suction parameter fw, we find that |τ| and |Cw| reduces and |Nu| enhances with increase in |fw| while |Sh| reduces with fw>0 and enhances with |fw| on the plate. The variation ofτ, Cw, Nu and Sh with chemical reaction parameter γ shows that |τ| & |Sh| reduce in the degenerating chemical reaction case and enhances in the generating chemical reaction case. |Cw| and |Nu| reduces at the plate in both the degenerating and generating chemical reaction cases. An increase in the micropolar parameters Δ and α reduces |τ| at the plate. |Cw| and |Sh| enhance with Δ and reduces withα.|Nu| reduces with Δ and enhances with α at the plate.

Conclusion:

In this work, the equations governing the MHD and effect of joule heating on the flow of a micropolar fluid past a stretching, continuous sheet are solved. The sheet is assumed to be non-isothermal with a prescribed wall heat flux varying with length. The effects of the magnetic parameter, suction parameter, Eckert number and micro rotation parameter are investigated. It was found that the velocity decreases with increasing magnetic parameter, and increases with increasing micro rotation parameter. It may be concluded that the increase in the magnetic field has the same influence on the flow field as increasing viscosity. The skin friction coefficient, which is an important physical quantity, increases with increasing both the micro rotation parameter and the magnetic field parameter, which effect is analogous to increasing of viscous effect. It is evident that the temperature gradient increases with an increasing Eckert number and magnetic param

Stress values:

		f' (0)	g'(0)	Nu(0)	Sh(0)
M	0.5	-0.6923	0.21432	0.001114	1.00189
	1.5	-1.0421	0.23461	0.00344	1.00046
	2.5	-1.4845	0.24607	0.00569	0.99850
Sc	0.24	-0.2419	0.18352	0.02945	1.0412
	0.66	-0.6083	0.20975	0.00333	1.0074
	1.3	-0.8424	0.22446	0.00109	1.0012}
N	1	-0.8424	0.22446	0.00109	1.00129
	2	-1.4410	-10.2	0.43028	0.97162
	-0.5	-1.2732	0.23967	-0.00229	0.99987
	-0.8	-1.3667	0.24345	-0.00226	1.00005
Δ	0.2	-0.8424	0.22446	-0.00109	1.00129
	0.5	-0.7622	0.38125	-0.00087	1.00155
	1.5	-0.7048	0.50224	-0.00054	1.00173
Q	0.5	0.8321	-0.0679	0.13125	0.99087
	1.5	1.2919	-0.1051	0.54097	0.98965
	-0.5	-5.020	0.64224	0.79536	1.01598
	1.5	-1.579	0.44852	0.89763	1.00678
Ec	0.01	-0.8424	0.22446	-0.00109	1.00129
	0.03	-0.7724	0.21132	-0.00446	1.0018
	0.05	-0.7115	0.20001	-0.00725	1.0022
Nr	0.5	-0.8424	0.22446	0.001097	1.00129
	1.5	-0.6493	0.18623	0.003142	0.99893
	3.5	-0.5169	0.06265	0.058770	0.99662
fw	0.5	-0.8424	0.22446	0.001097	1.00129
	1.5	-1.1178	0.10018	0.055246	0.99733
	-0.5	0.36113	0.08792	0.120414	0.94594
	-1.5	1.1395	0.02756	0.864692	0.95129
γ	0.5	-0.8424	0.22446	0.001097	1.00129
	1.5	-0.1885	0.11133	0.052464	0.99790
	-0.5	-0.6619	0.23659	0.69412	1.00964
	-1.5	-1.1847	0.19089	0.72340	1.01141
α	0.5	-0.8424	0.2244	0.001097	1.00129
	1.5	-0.1545	0.0422	0.05334	0.99747
	2.5	-0.0821	0.0194	0.54596	0.98956

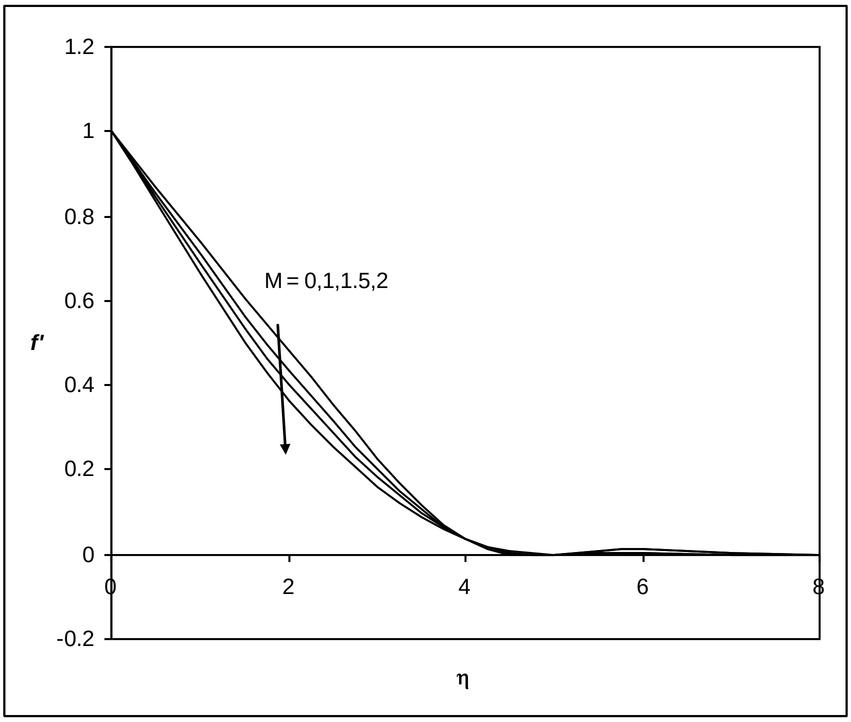

Fig. 2a: Variation of f' with M
Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

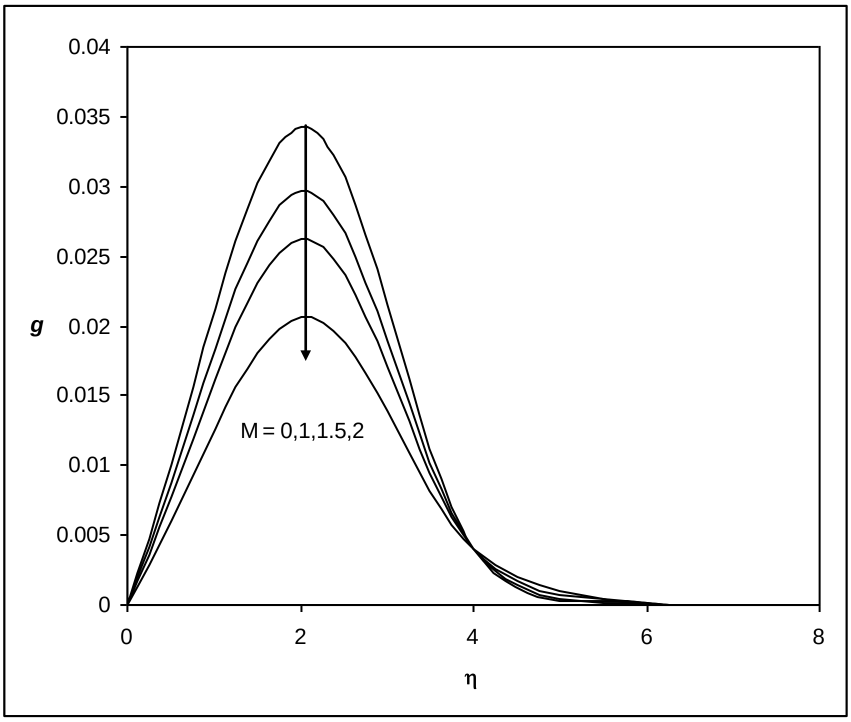

Fig. 2b: Variation of g with M
Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

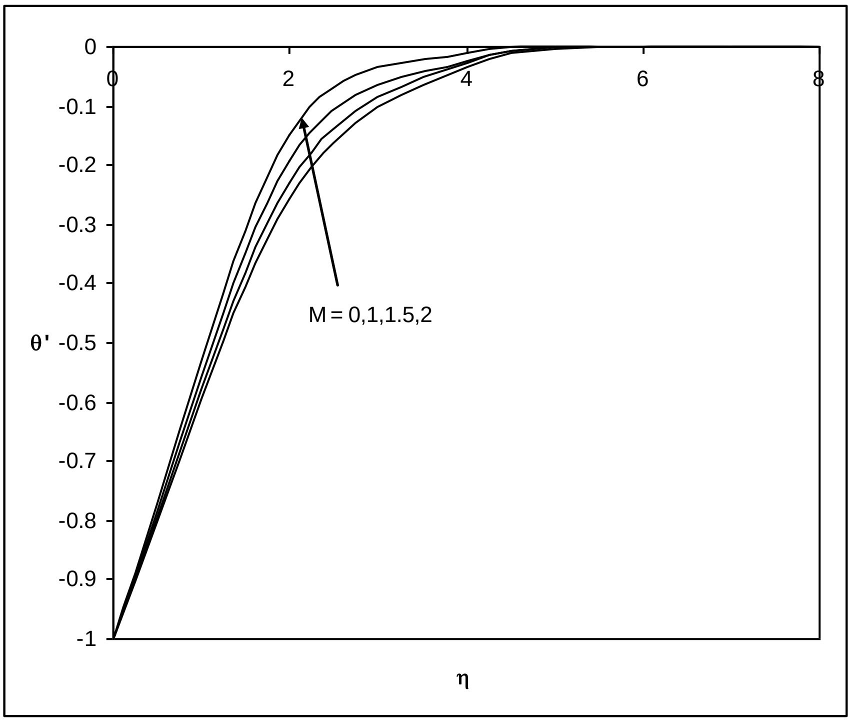

Fig. 2c: Variation of θ ′ with M
Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

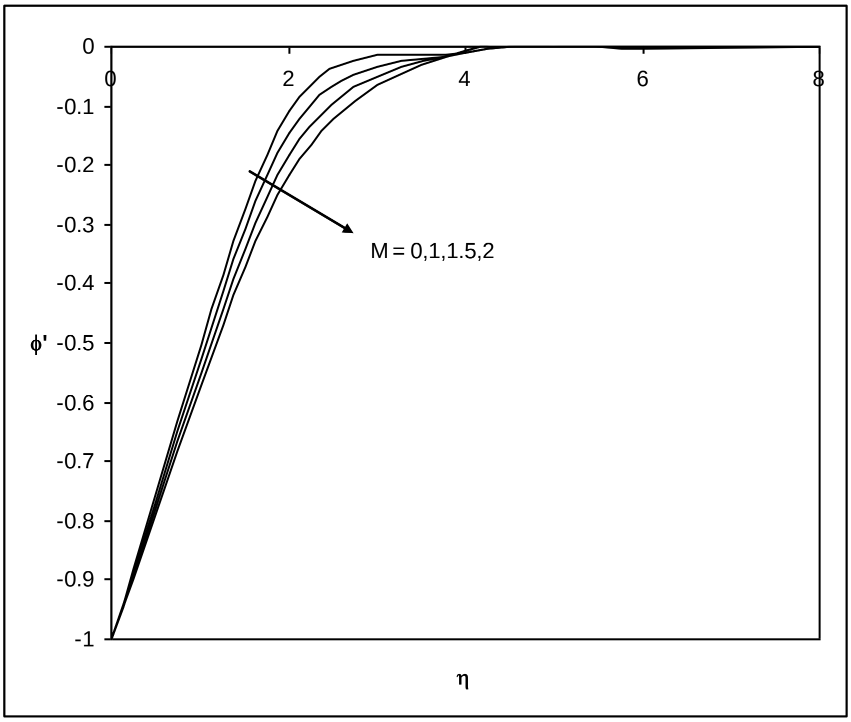

Fig. 2d: Variation of ϕ′ with M
Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

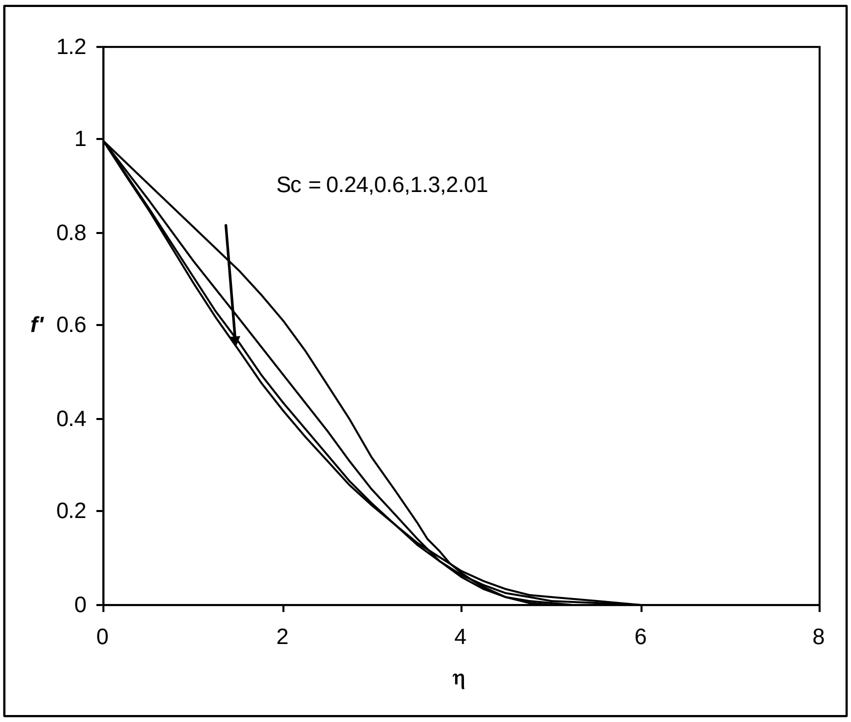

Fig. 3a: Variation of f' with Sc
M=0, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

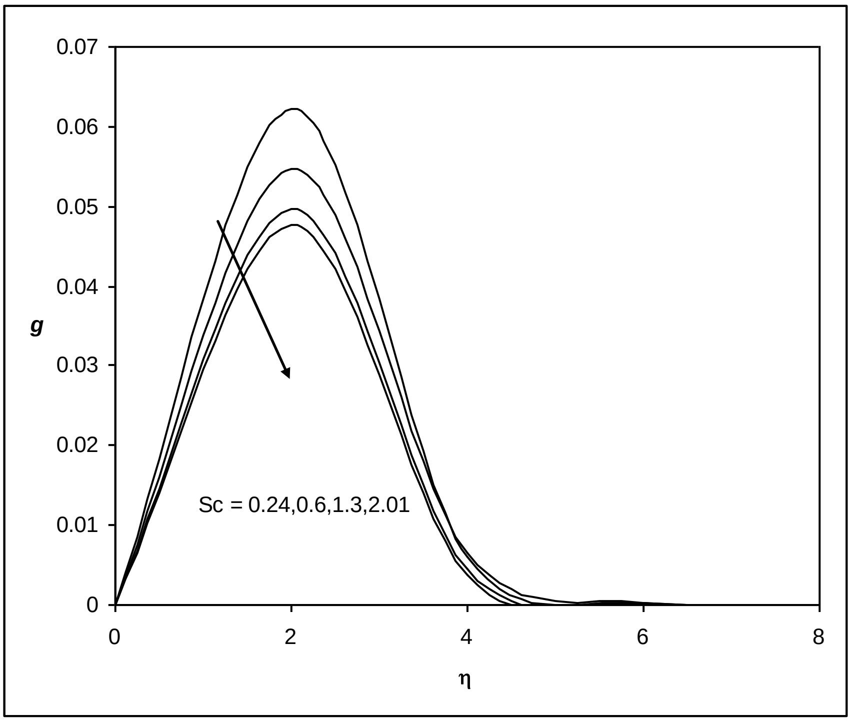

Fig. 3b: Variation of g with Sc
M=0, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

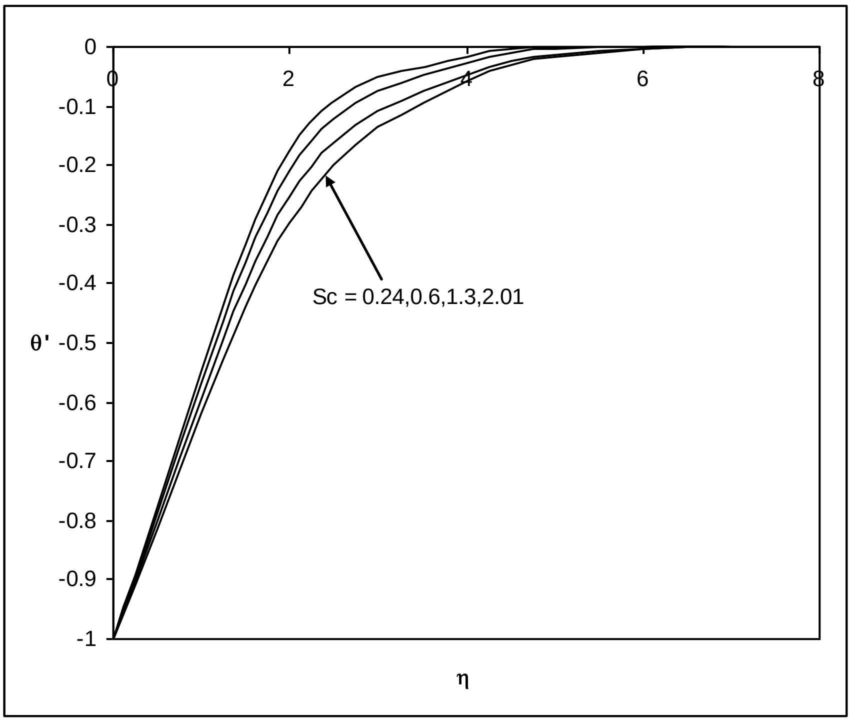

Fig. 3c: Variation of θ ′ with Sc
M=0, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

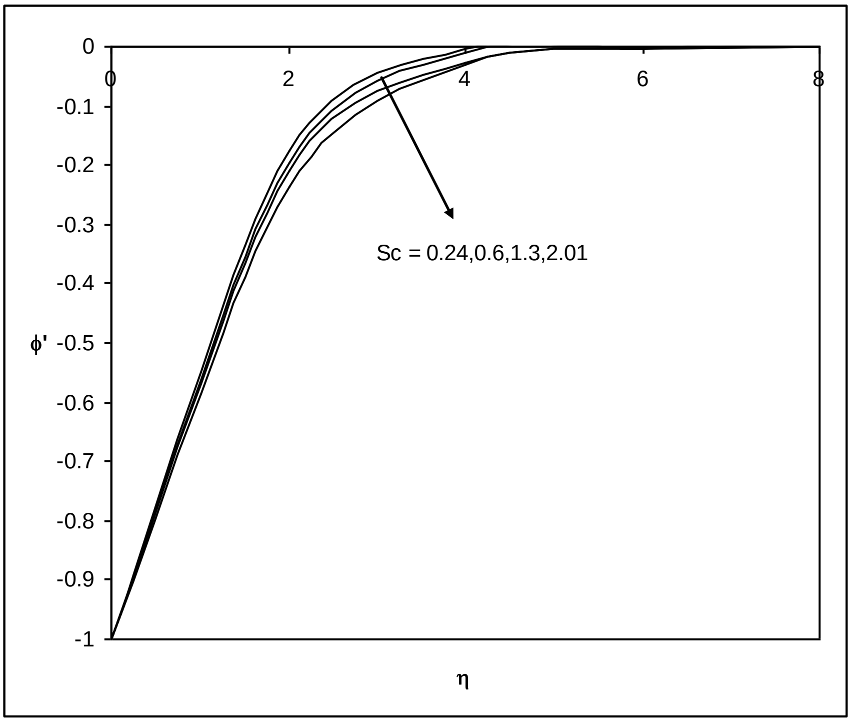

Fig. 3d: Variation of ϕ′ with Sc
M=0, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

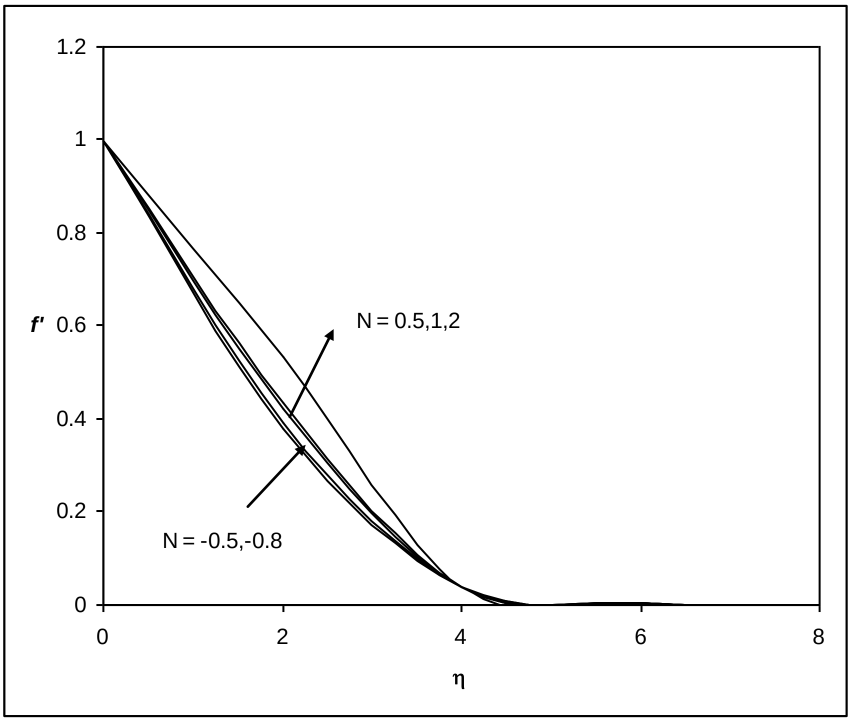

Fig. 4a: Variation of f' with N
M=0, Sc = 1.3, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

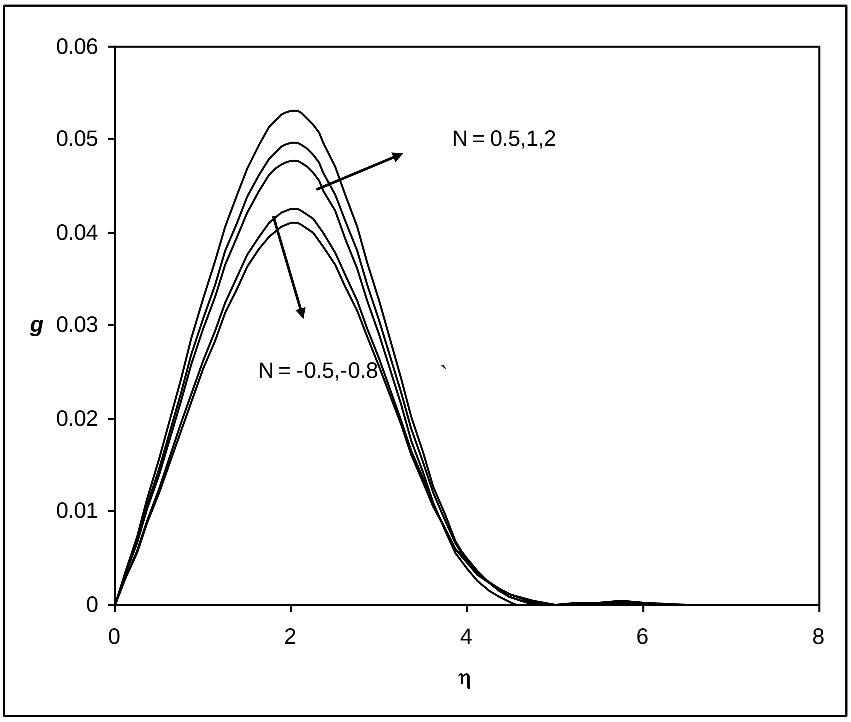

Fig. 4b: Variation of g with N
M=0, Sc = 1.3, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

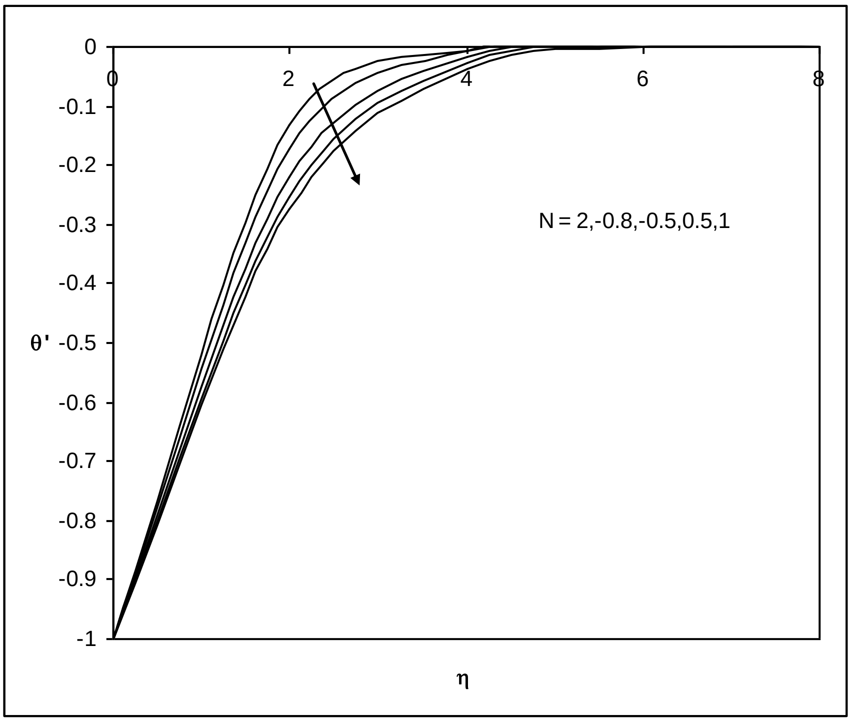

Fig. 4c: Variation of θ ′ with N
M=0, Sc = 1.3, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

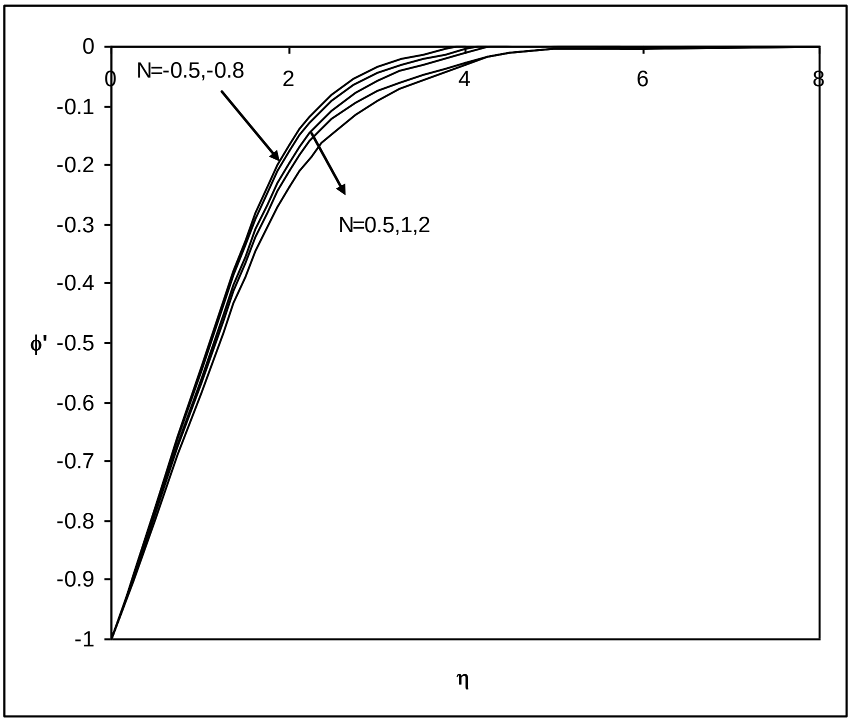

Fig. 4d: Variation of ϕ′ with N
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

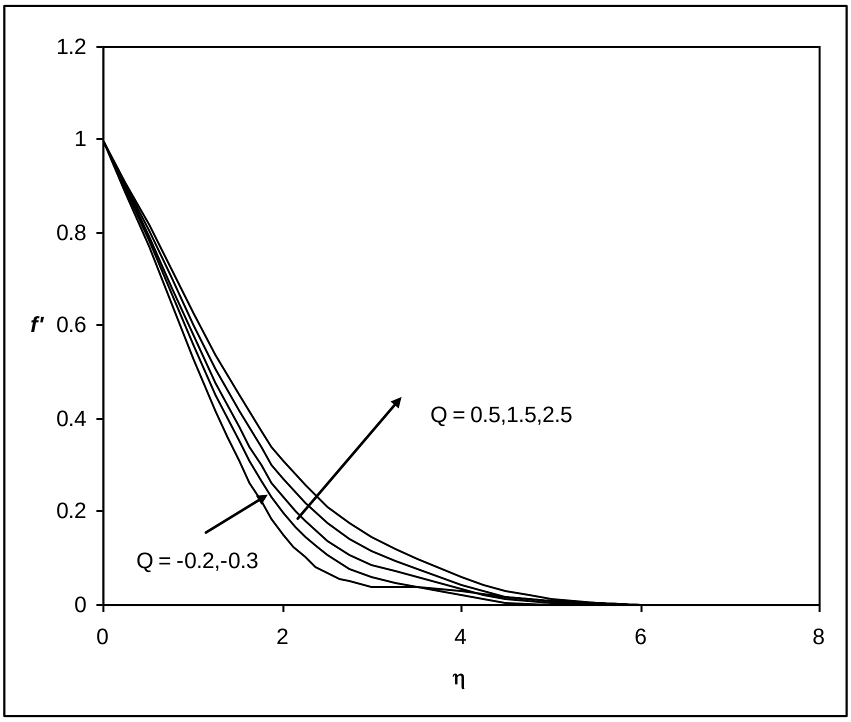

Fig. 5a: Variation of f' with Q
M=0, Sc = 1.3, N= 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

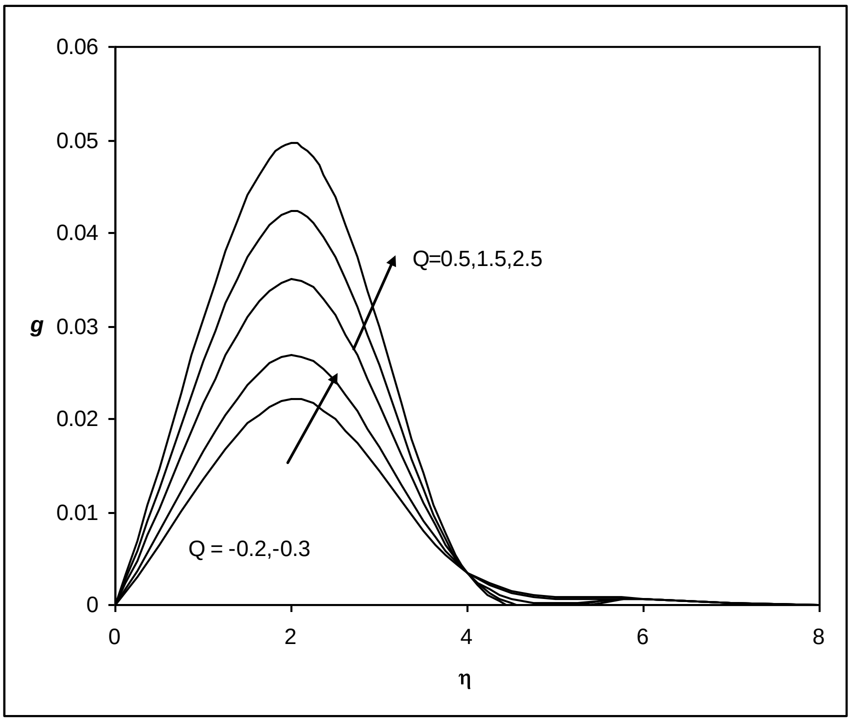

Fig. 5b: Variation of g with Q
M=0, Sc = 1.3, N= 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

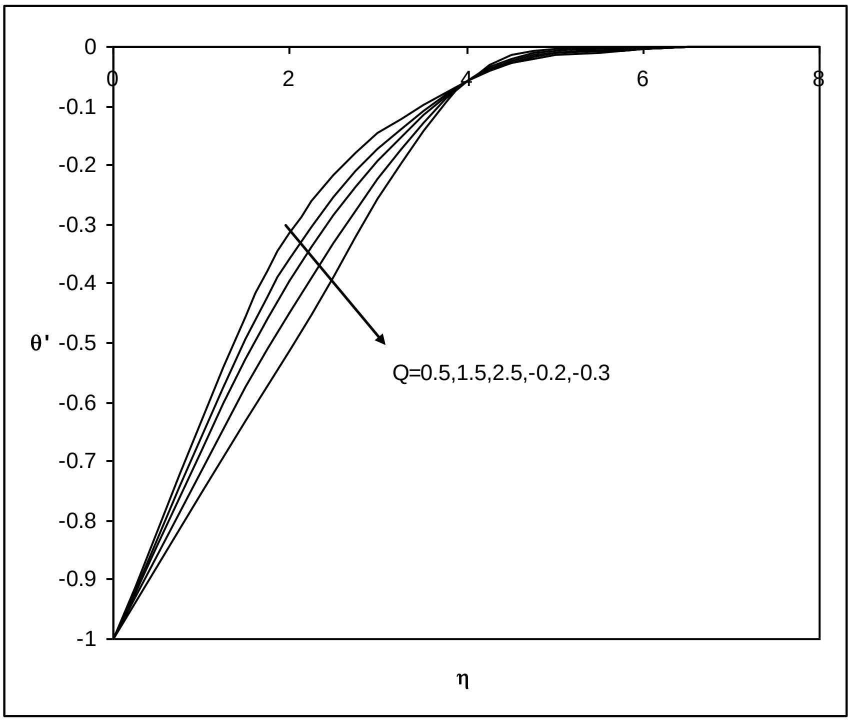

Fig. 5c: Variation of θ ′ with Q
M=0, Sc = 1.3, N= 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

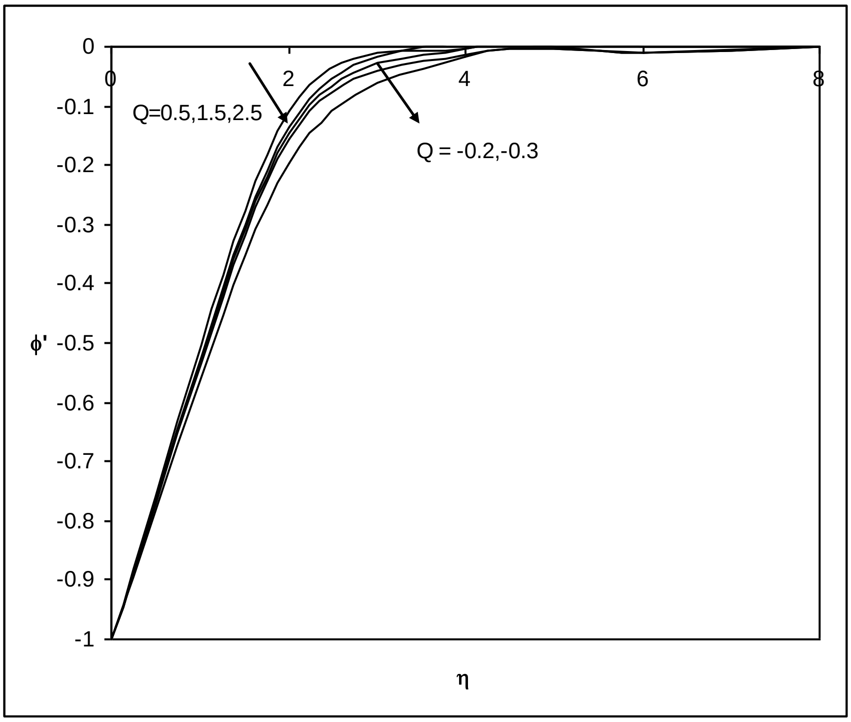

Fig. 5d: Variation of ϕ′ with Q
M=0, Sc = 1.3, N= 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

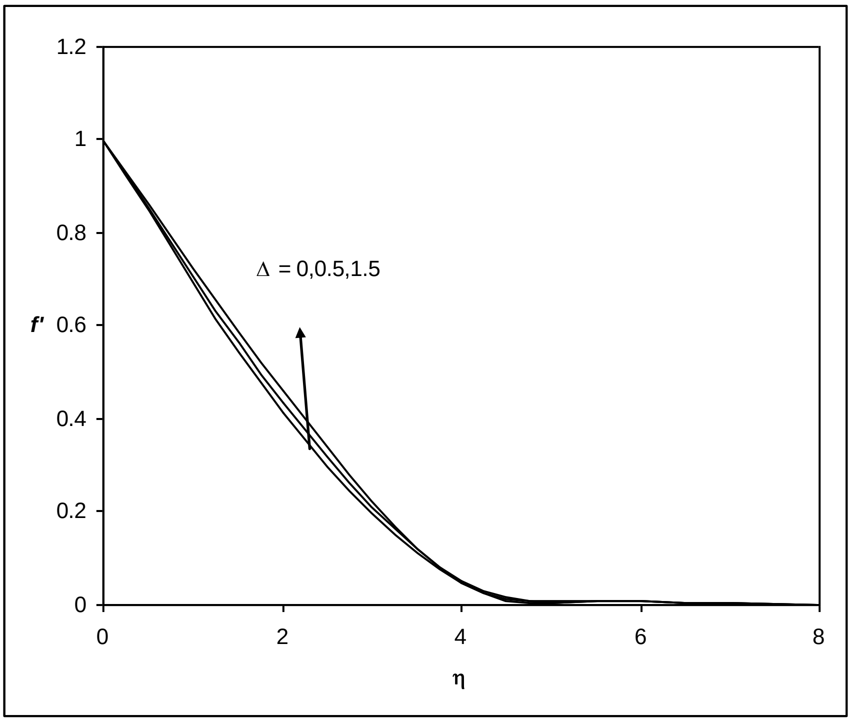

Fig. 6a: Variation of f' with Δ
M=0, Sc = 1.3, N= 0.5, Q = 0.5,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

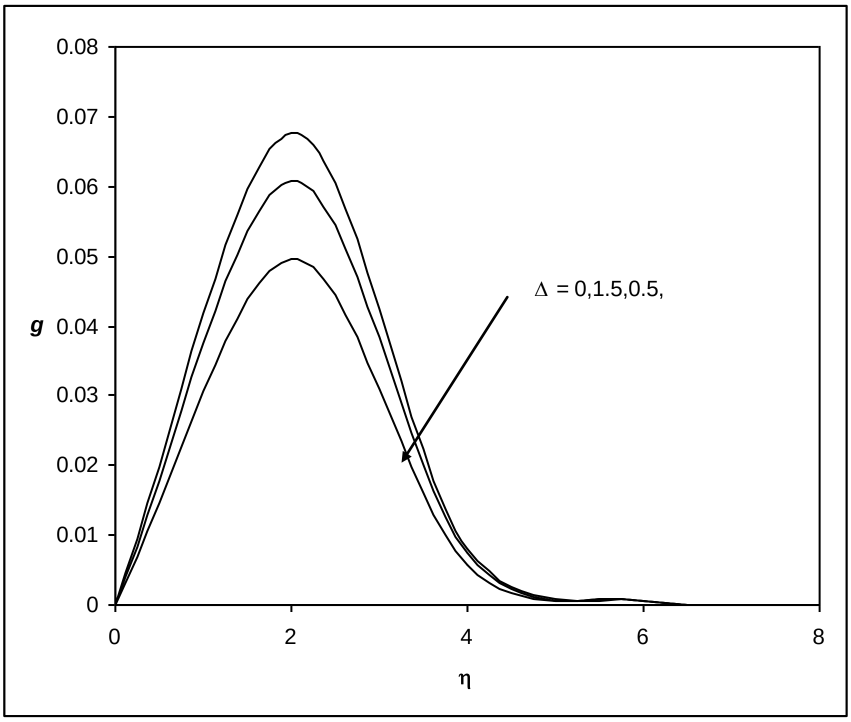

Fig. 6b: Variation of g with Δ
M=0, Sc = 1.3, N= 0.5, Q = 0.5,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

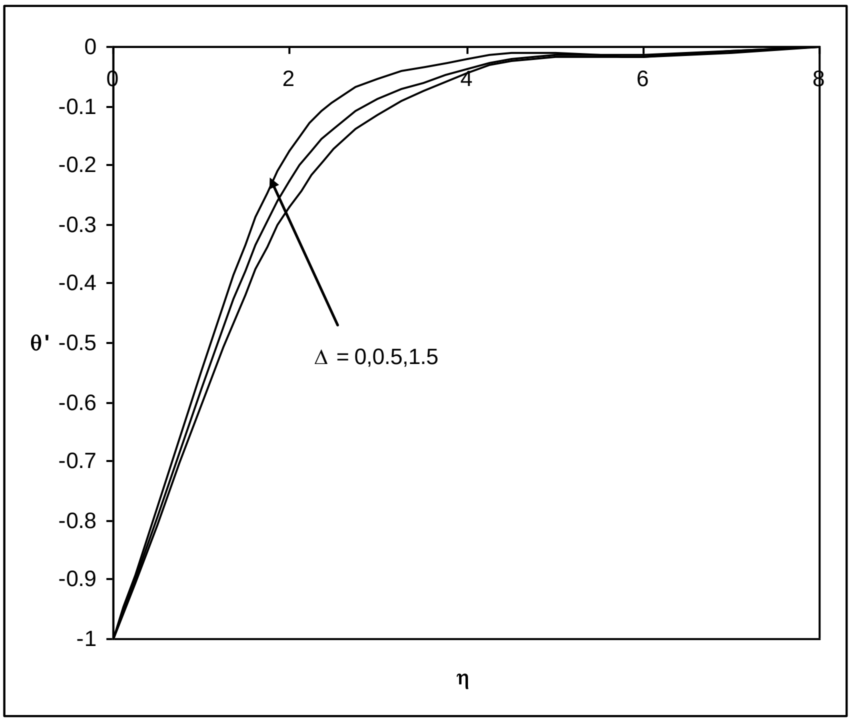

Fig. 6c: Variation of θ ′ with Δ
M=0, Sc = 1.3, N= 0.5, Q = 0.5,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

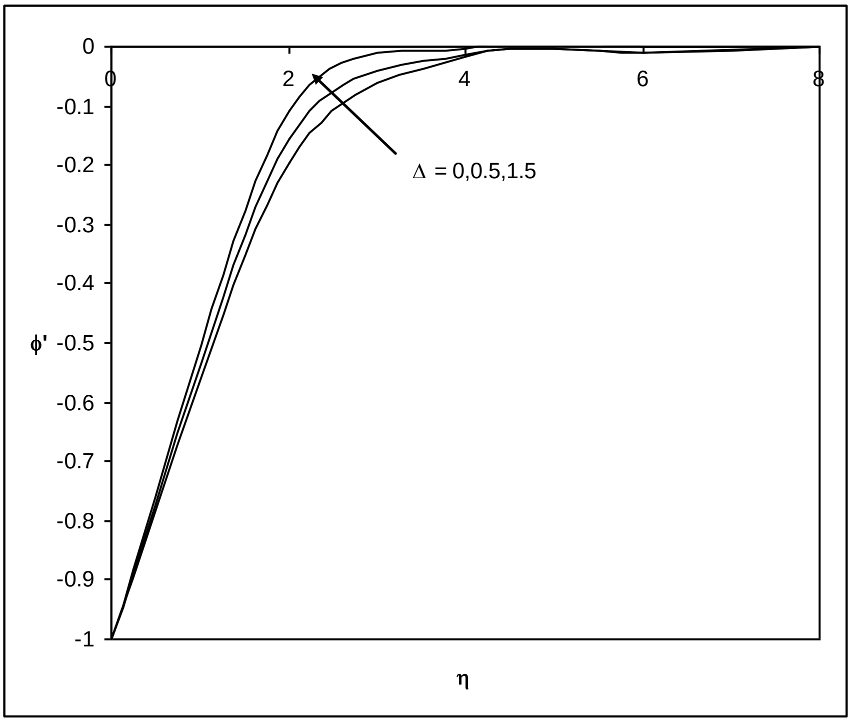

Fig. 6d: Variation of ϕ′ with Δ
M=0, Sc = 1.3, N= 0.5, Q = 0.5,
γ=0.8, Nr=0.5, Ec=0.01, Fw=0.2

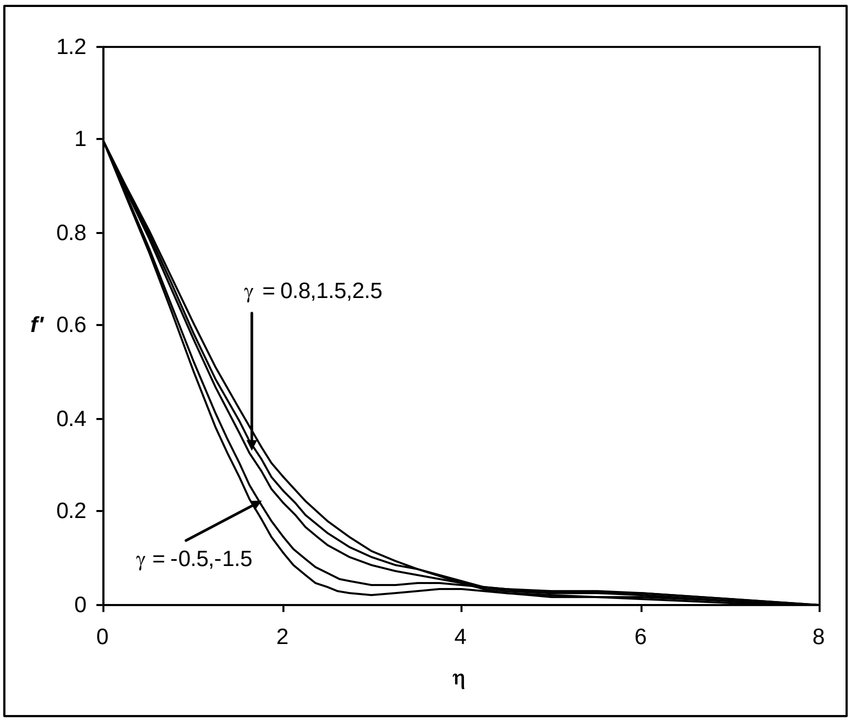

Fig. 7a: Variation of f' with γ
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
Nr=0.5, Ec=0.01, Fw=0.2

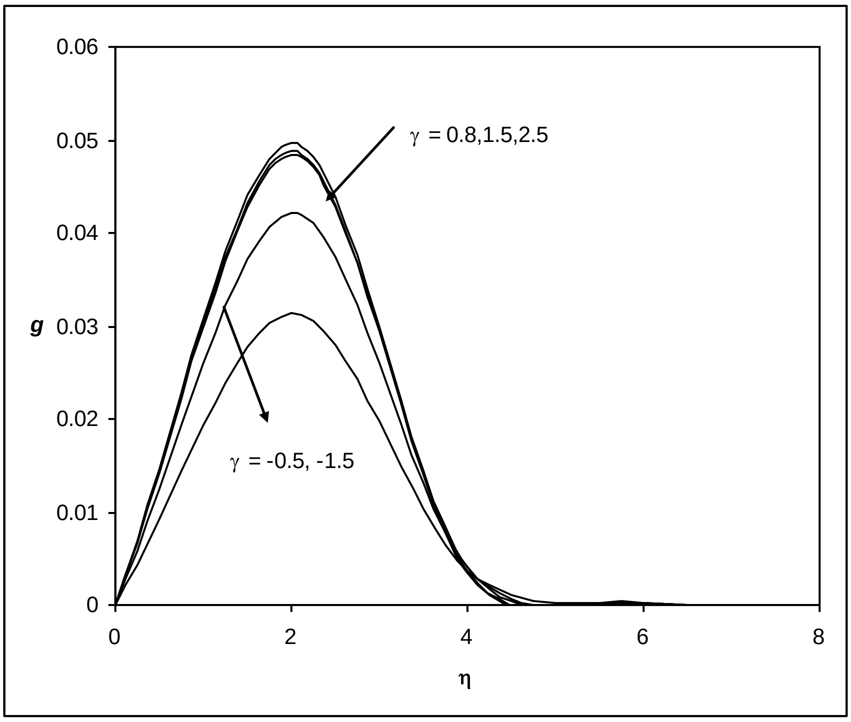

Fig. 7b: Variation of g with γ
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
Nr=0.5, Ec=0.01, Fw=0.2

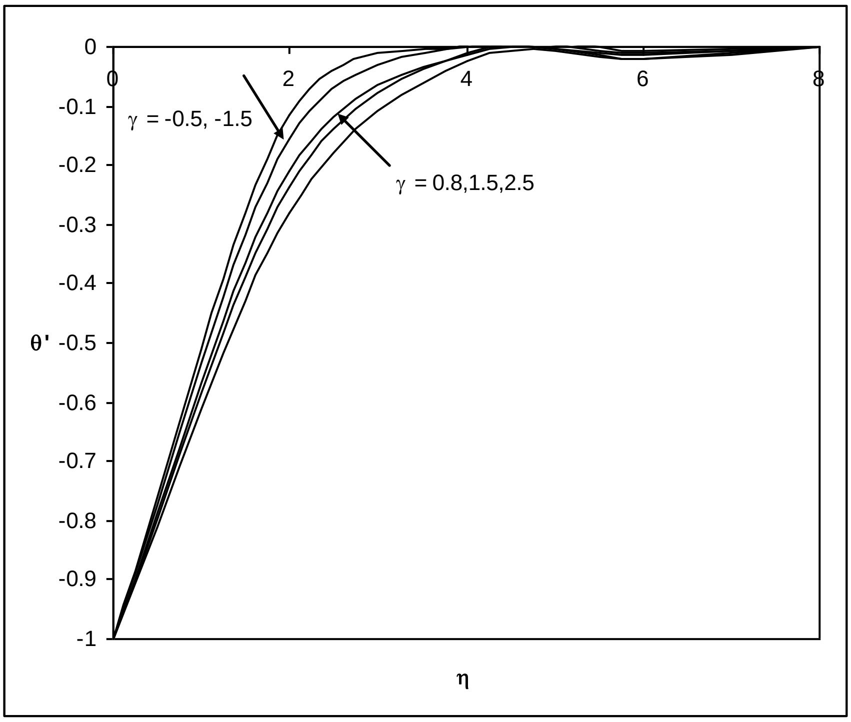

Fig. 7c: Variation of θ ′ with γ
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
Nr=0.5, Ec=0.01, Fw=0.2

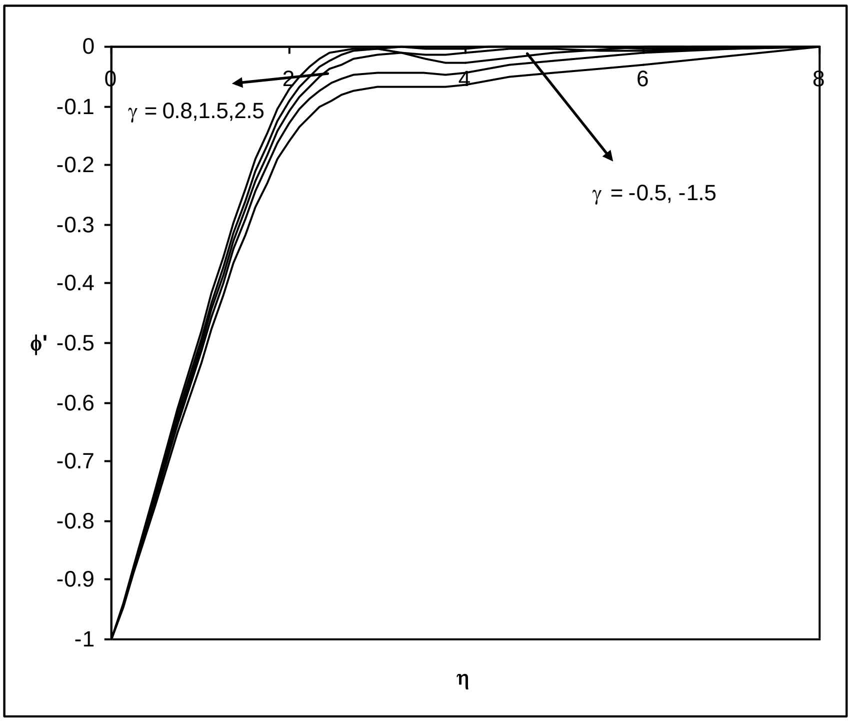

Fig. 7d: Variation of ϕ′ with γ
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
Nr=0.5, Ec=0.01, Fw=0.2

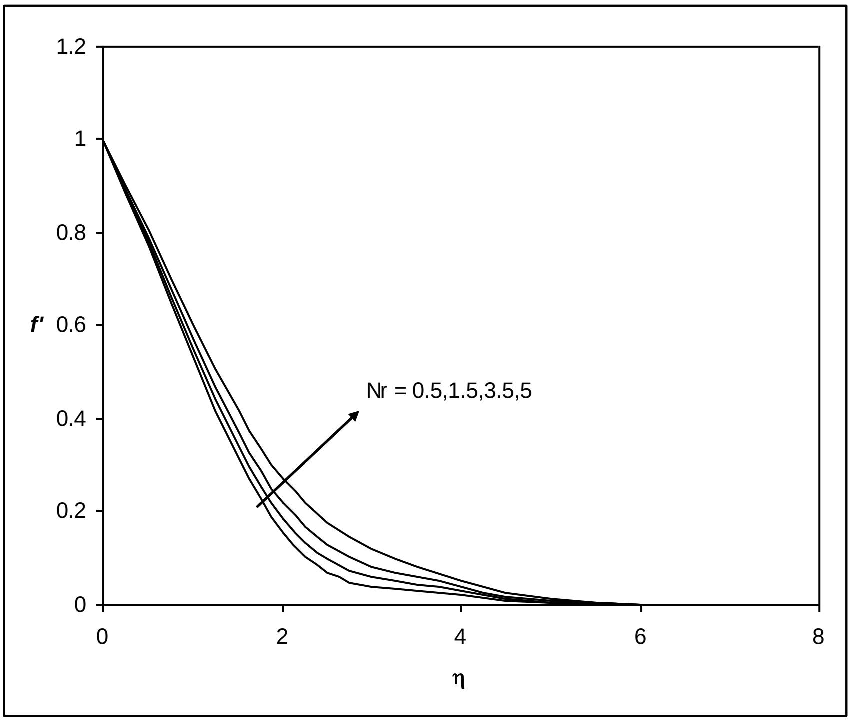

Fig. 8a: Variation of f' with Nr
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Ec=0.01, Fw=0.2

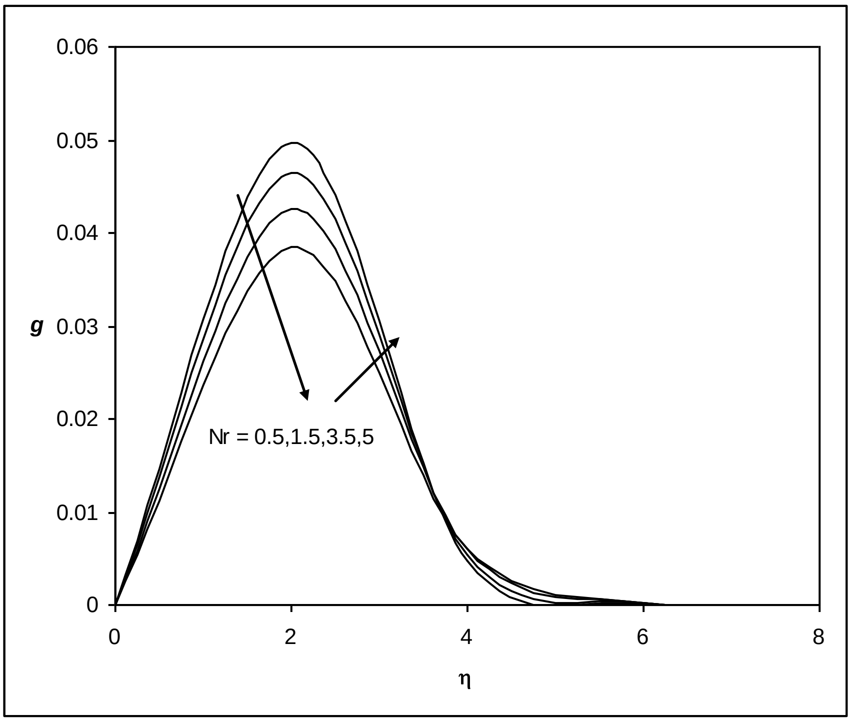

Fig. 8b: Variation of g with Nr
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Ec=0.01, Fw=0.2

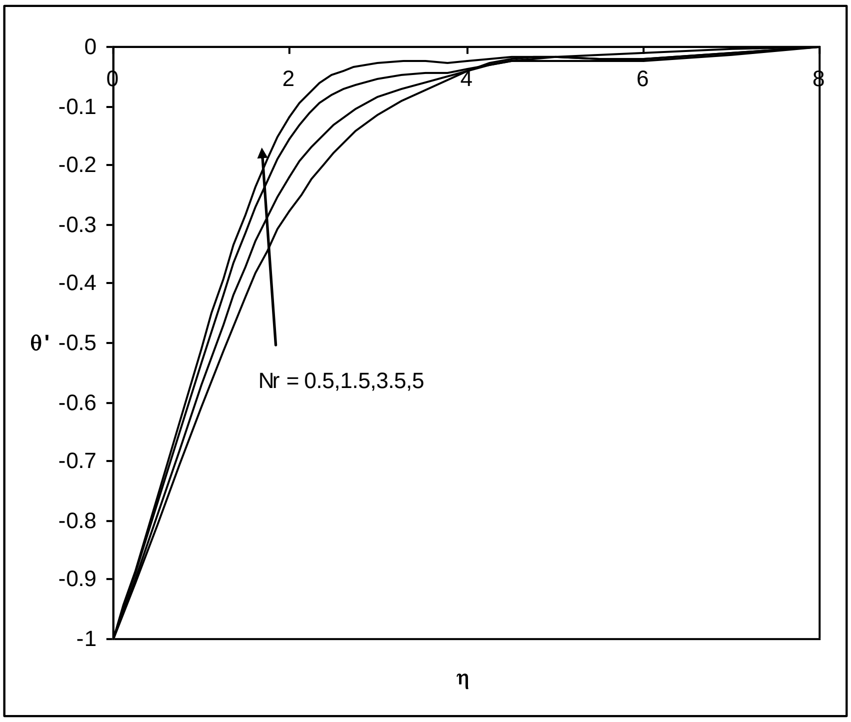

Fig. 8c: Variation of θ ′ with Nr
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Ec=0.01, Fw=0.2

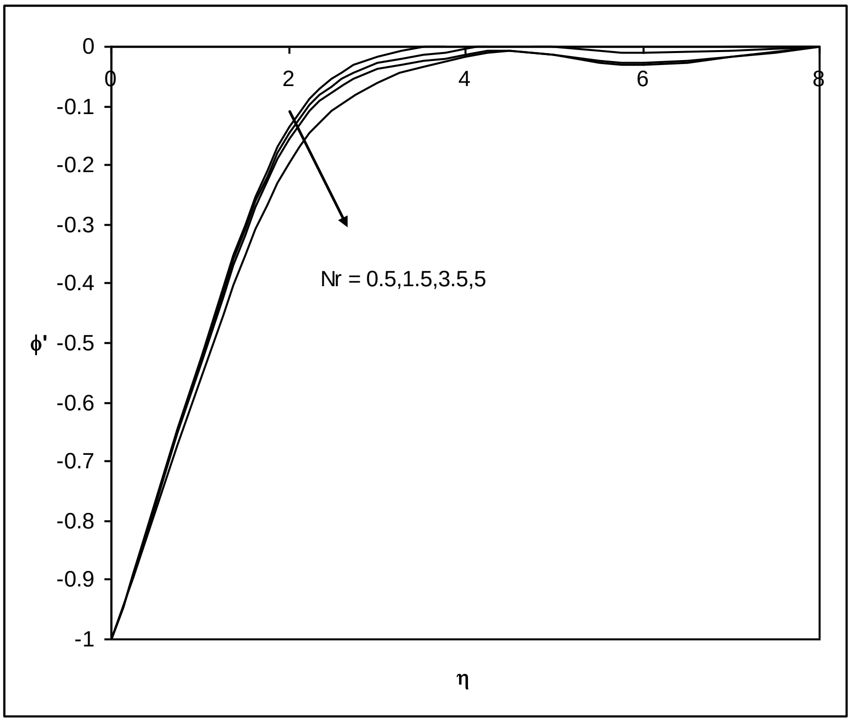

Fig. 8d: Variation of ϕ′ with Nr
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Ec=0.01, Fw=0.2

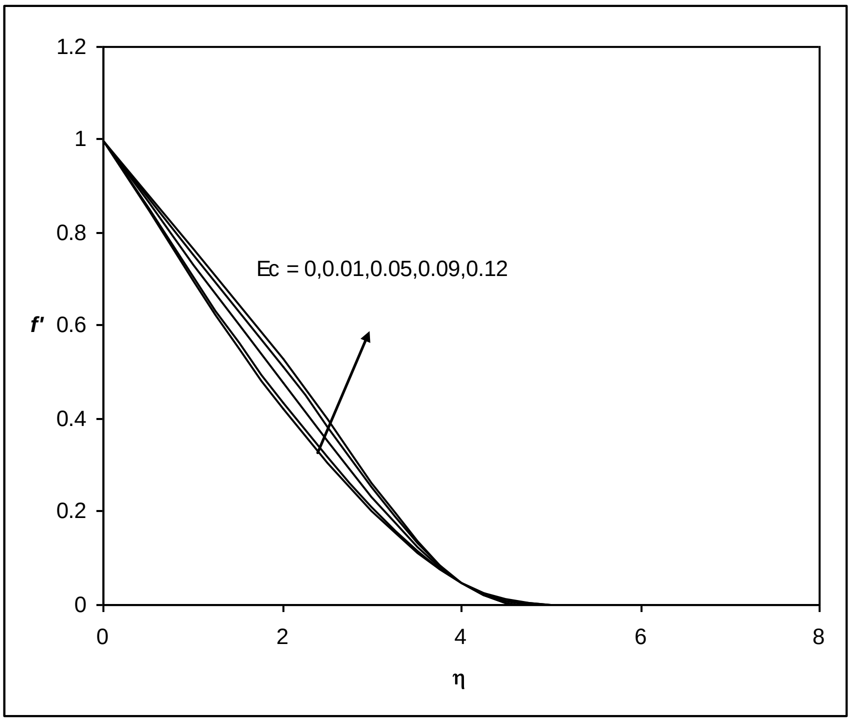

Fig. 9a: Variation of f' with Ec
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Fw=0.2

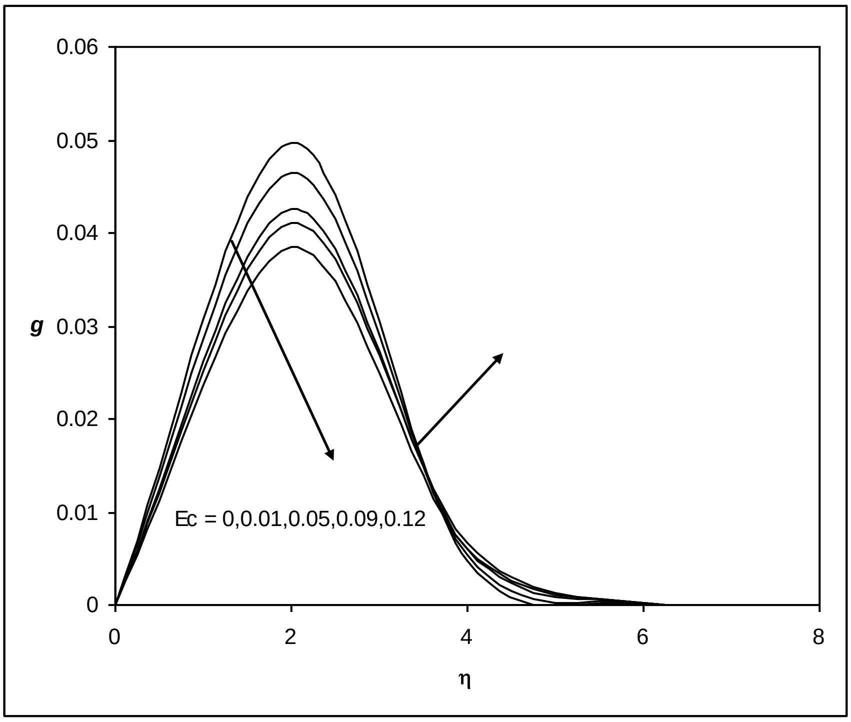

Fig. 9b: Variation of g with Ec
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Fw=0.2

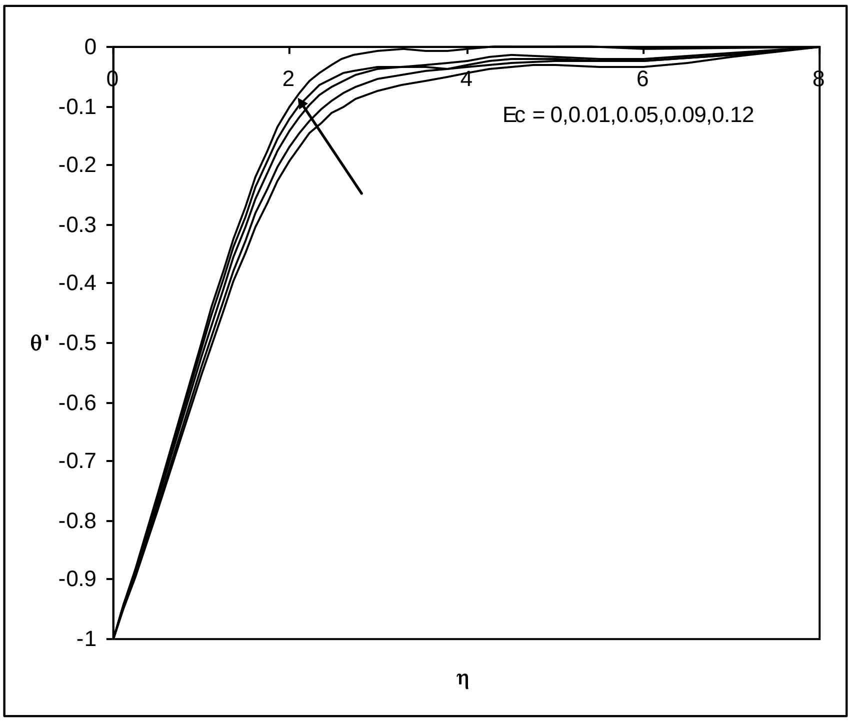

Fig. 9c: Variation of θ ′ with Ec
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Fw=0.2

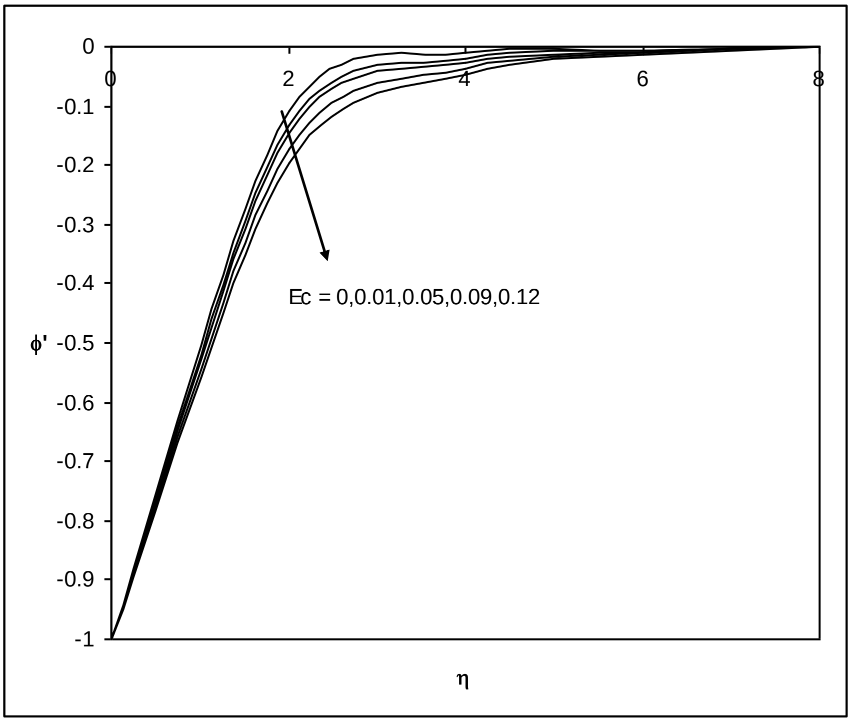

Fig. 9d: Variation of ϕ′ with Ec
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Fw=0.2

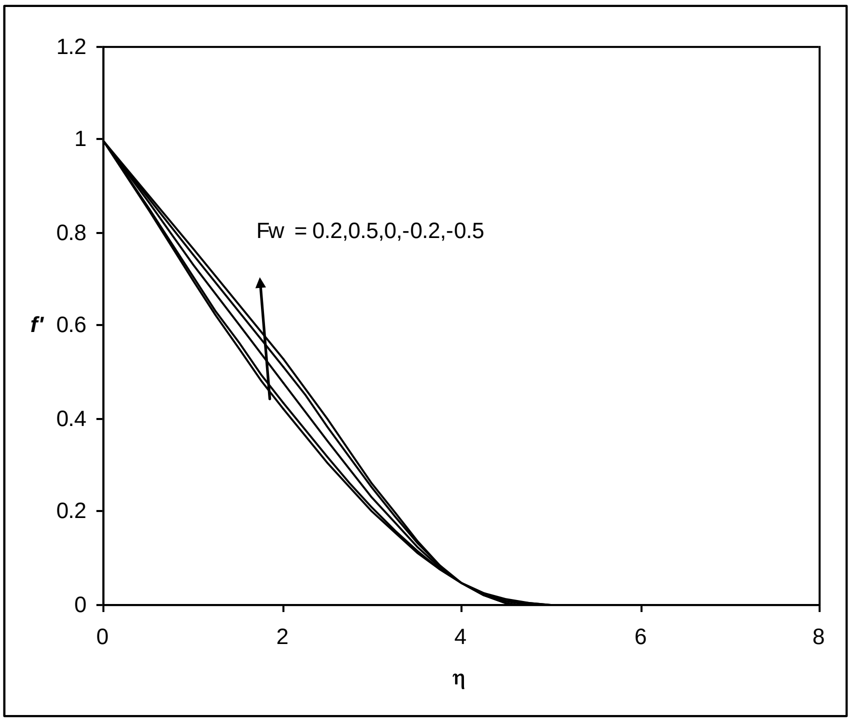

Fig. 10a: Variation of f' with Fw
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01

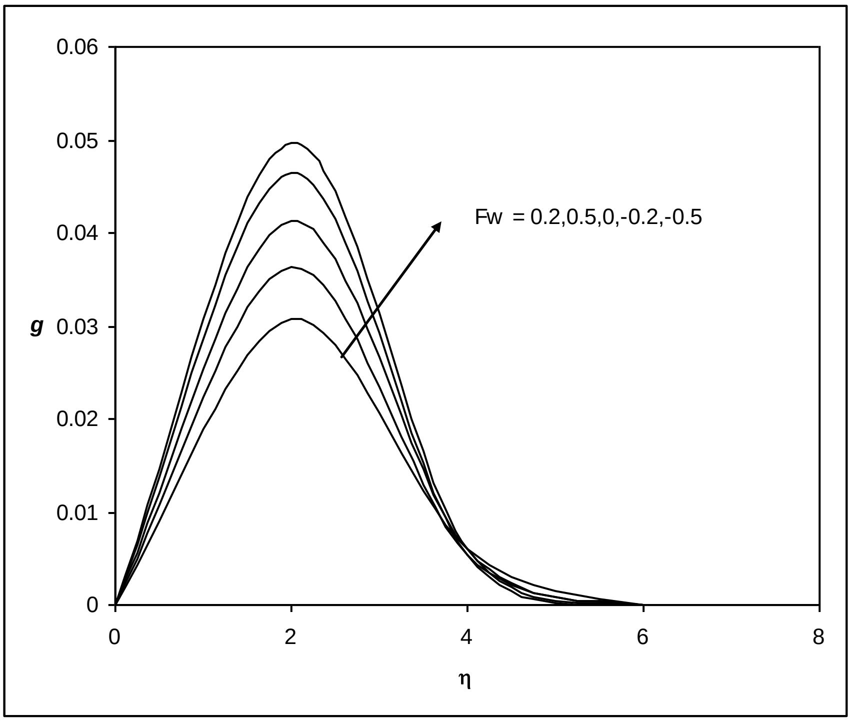

Fig. 10b: Variation of g with Fw
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01

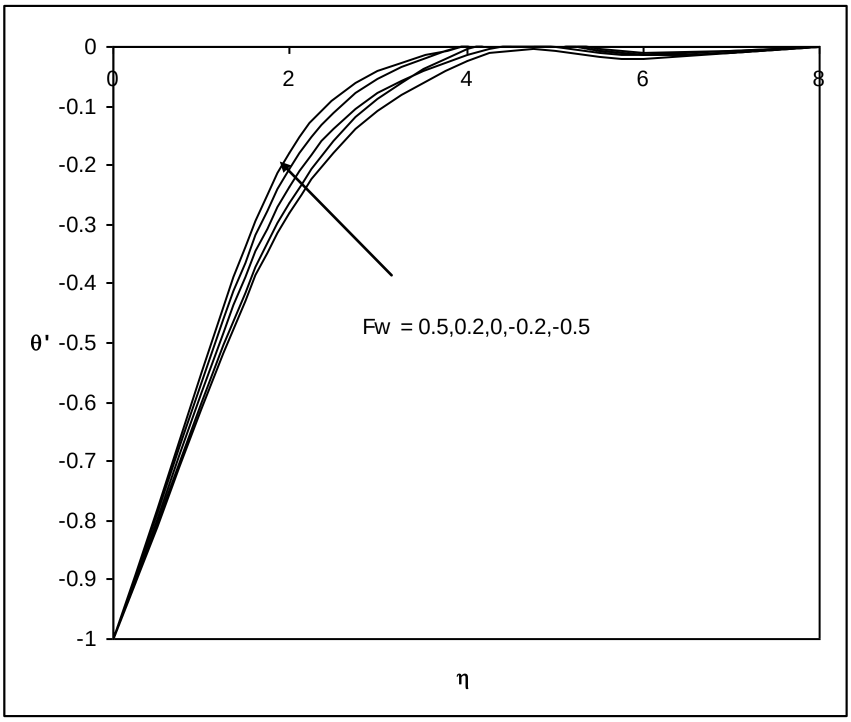

Fig. 10c: Variation of θ ′ with Fw
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01

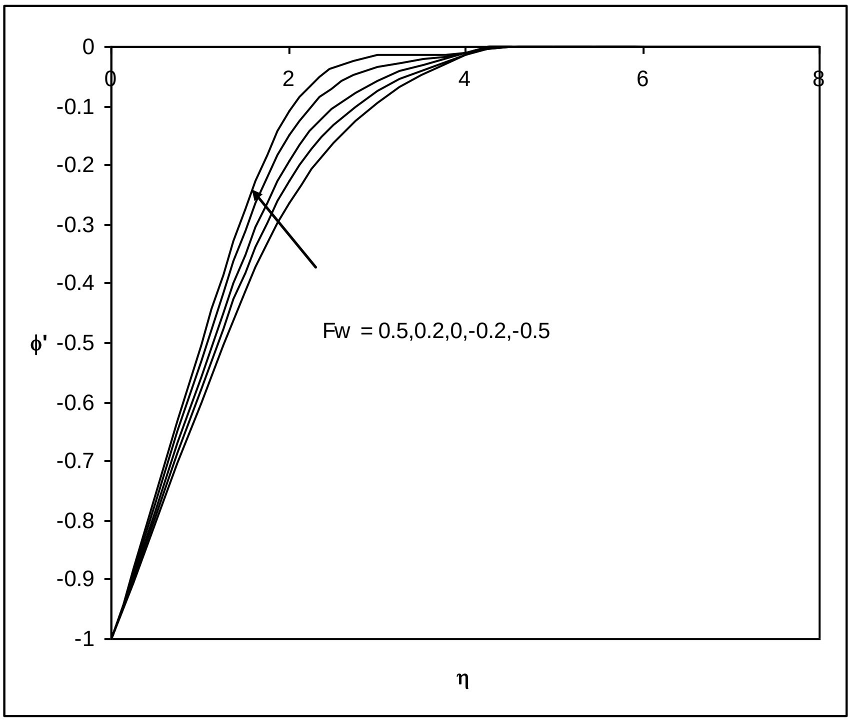

Fig. 10d: Variation of ϕ′ with Fw
M=0, Sc = 1.3, N= 0.5, Q = 0.5, Δ=0,
γ=0.8, Nr=0.5, Ec=0.01

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