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Capture-Recapture Estimation for Elusive Events with Two Lists

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Abstract
The application of capture-recapture methods to the estimation of population parameter for epidemiologic and demographic event has been growing recently. This paper presents a robust capture-recapture model for estimating the size of elusive epidemiologic event. The proposed estimator $\widehat{N}_c$, the Petersen estimator $\widehat{N}_s$ and another estimator $\widehat{N}_o$ were compared using the Akaike Information Criterion (AIC) and the Mean Absolute Deviation (MAD) through simulation studies. Both the AIC and MAD revealed that $\widehat{N}_c$ is a better and robust estimator. It was discovered that $\widehat{N}_s$ under estimates the total elusive population *N*, $\widehat{N}_o$ over estimate *N* while $\widehat{N}_c$ was always consistent and performs better than the other two. $\widehat{N}_c$ is particularly better with lower recaptures ('relisting') n_{11} which is the case with elusive events. The Petersen $\widehat{N}_s$ breaks down in the presence of elusiveness. The other estimator, $\widehat{N}_o$ always over estimates, except in few cases. It is therefore recommended that the proposed estimator $\widehat{N}_c$ be used for estimating dual system elusive events.
Keywords: Trap Response; Dual System Estimation; Elusive events; Petersen estimator; capture-recapture.

1. Introduction

Dual System Estimation (DSE) is the nomenclature given to two samples Capture-Recapture Method when applied to populations other than animals (Seber, 1965 and IWGDMF, 1990). The classical method of estimation for this type of experiment was developed and applied to ecological problem by Petersen in 1896, using tagged plaice, and Lincoln in 1930 who used band returns to estimate the size of the North American waterfowl population as reported by Seber (1982a). Thus, the two-sample capture-recapture method is tagged Lincoln-Petersen methods in some literature (Seber, 1982a; Pollock, 1991; Haines et al., 2000). Since the work of Petersen and Lincoln, several authors have applied their model and its modified form under varying situations.

Sekar and Deming (1949) for instance, estimated birth and death rates using two lists: the registrars list (R) and the Interviewers list (I) obtained from a complete house –to – house canvass. They also discussed theoretically that stratification can be used to improve the Petersen estimates when heterogeneity is thought to affect the estimates. Shapiro (1949) used the Petersen estimates to estimate Birth registration completeness in the United States. Birth records on file for infants born during the period December 1, 1939 to March 31, 1940 were matched against infants cards filled by enumerators and a set of death records on the file for children whose birthdates were within the test period but who died before the census data April 1, 1940.

Alho (1990) introduced an estimator of the unknown population size in a dual registration process based on a logistic regression model. His model allows different capture probabilities across individuals and across capture times. The probabilities are estimated from the observed data using conditional maximum likelihood. Alho developed his method because the classical estimator is known to be biased under population heterogeneity (Seber, 1982b; Burnham and Overton, 1978; Alho, 1990).

A Bayesian modification of the Lincoln index was given by Gaskell and George (1972). They observed that when the recaptures (n_{11}) is small the interval between possible values of $\widehat{N}$ for fixed $n_{1.}$ and $n_{.1}$ are large and that the formula (2.2) is at best a poor estimator of the total animal population *N* when n_{11} >10. They incorporated a prior based on the belief that the experimenter does begin with an idea about the value of *N*, this may only be possible in animal experiment. This is not possible in demographic or epidemiologic elusive events. For other applications and modifications of the Petersen estimator for estimating animal population see Schwarz and Seber (1999).

It was discovered that the Petersen Method is sensitive to the marginal totals and that it depends so much on the sampling effort (Jibasen, 2011). That is, if the sampling effort is poor yielding fewer n_{11}, the Petersen estimator gets poorer, in line with the observation given by Gaskell and George (1972).

It is known in capture-recapture experiments that whether an individual is caught or not depends on a variety of circumstances. One of this is trap response in which individual may exhibit an enduring behavioral response to the first capture. That is, after an individual has been captured once, the individual has a long memory of its first-capture experience and the effect lasts for the remainder of the experiment, leading to a higher (trap-happy) or lower (trap-shy) capture probability for all subsequent recaptures. Behavior of individuals towards trap will usually give wrong population estimates; trap-shyness results in over estimates of population size, while trap-happiness results to underestimation. Trap response will generally affects the size of the second catch (marginal total). In applying this method to elusive populations, individuals are always on the hide or run once arrested or even hearing about the presence of law enforcement agents. This is similar to trap-shyness, that is, do everything to avoid a re-arrest. Thus an individual may not be easily re-arrested, leading to a lower recapture ('relisting') probability. In this work therefore, a novel capture-recapture estimation procedure for an elusive population is proposed.

2. Methodology

Consider a closed population of size N individuals. Let a sample of size $n_{1.}$ be drawn from N without replacement, marked and returned back into the population. For simplicity, we call this *system* 1 as presented in Table 1 in the appendix. Here, elements of $n_{1.}$ sample may represent a set of drug addicts that were caught by narcotic officials (*system* 1), probably reformed and released back into the population. If at the second time, another sample of size $n_{.1}$ was drawn from the same closed population without replacement using *system* 2, the interest now is to examine the members of the first sample $n_{1.}$ (say, of drug addicts) that were caught the second time (i.e. n_{11}) (recaptured) by *system* 2.

The Lincoln-Petersen experiment is the simplest capture-recapture method, for estimating the size N of a closed population which consists of catching, marking, and releasing a sample (sample 1) of n_1 animals. After allowing the marked to mix with the unmarked, a second sample, $n_{.1}$ is taken from the population. In demography, this is known as dual system estimation (Erickson and Kadame, 1985). Equating the proportion of the marked recovered in the second sample (n_{11}) to the population proportion $n_{1.}/N$ leads to an estimate of N.

Mathematically, we have that

$$\frac{n_{11}}{n_{.1}} = \frac{n_{1.}}{N} \tag{2.1}$$

which simply leads to the estimate,

$$\hat{N} = \frac{n_{1.}n_{.1}}{n_{11}} \tag{2.2}$$

If $n_{1.}$ and $n_{.1}$ are regarded as constants (that is, a random sample without replacement), then from Table 1 n_{11} has a hypergeometric distribution of the form

$$p(n_{11}) = \begin{cases} \frac{\binom{n_{1.}}{n_{11}}\binom{n_{2.}}{n_{21}}}{\binom{N}{n_{.1}}}, & n_{11} = max(0, n_{.1} - n_{1.}), \dots, min(n_{1.}, n_{.1}) \\ 0, & otherwise \end{cases} \tag{2.3}$$

Sometimes sample 2 is taken with replacement, in this case n_{11} has a binomial distribution. According to Seber (1982a), the use of hypergeometric distribution emphasizes the fact that it is basically the activity of the experimenter that brings about randomness. However, another approach in which randomness is related to the activity of the animals considers the N animals in the population as N independent multinomial trials each with the same probability of belonging to a given capture-recapture category. These categories are; caught in the first sample (n_{12}) only, caught in the second sample (n_{21}) only, caught in both sample (n_{11}) and caught in neither sample (n_{22}).

The assumptions of the Petersen estimator are well known (Seber, 1982a; Pollock, 1991). These are;

i.) The population size is closed so that N is constant.
ii.) For each sample, each individual has the same probability of being included in the sample.
iii.) Marking does not affect the catchability of animals.
iv.) Animals do not lose their marks between samples.
v.) All marks (or tags) are reported on recovery in the second sample.
vi.) The animals are independent of one another as far as catching is concerned.

The third assumption above does not hold in the presence of trap response, similar to elusiveness in human population.

The assumptions for the elusive events can be reformulated as follows:

i.) The population size is closed, that is, no addition is allowed at the time of data collection.

ii.) Individuals can be matched from list to list, that is, individuals will not change their names or identities, moving from one system to another.

iii.) Catching (listing) does affect the catchability ("listability") of individuals.

iv.) The listing systems may not be independent.

Solving (2.2) for n_{11} yields, $n_{11} = \frac{n_{1} n_{1}}{\hat{N}}$

Or more generally,

$$E(n_{11}) = \frac{n_{1} n_{1}}{N} \tag{2.4}$$

It can be seen that (2.3) is the expected value of the hypergeometric random variable, intuitively, the Petersen Method (2.2) is at its "best" when the recaptures n_{11} (those re-listed, in terms of epidemiology) follows the hypergeometric distribution.

For demographic and epidemiologic elusive events, one cannot influence the recaptures, since lists form the sampling occasions. Moreover, for such events the recaptures are relatively small, yielding low recapture (re-listing) probabilities. Hence, we seek to estimate *N* via a Horvitz-Thompson method, where the listing probabilities ('listabilities') of all individuals is used. This approach was used by Huggins (1991), who proposed modeling the capture probabilities p_i of animals. Huggins used a form of the Horvitz-Thompson estimator, where the capture probabilities of the animals $\hat{p}_k$ are estimated for which

$$\hat{N} = \sum_{k=1}^{m_{t+1}} \frac{1}{p_k} \tag{2.5}$$

For dual system estimates (single recaptures), this yields exactly the Petersen estimator (see Jibasen, 2011). That is,

$$\hat{p}_k = 1 - \prod_j^2 (1 - \hat{p}_s)$$

$$\rightarrow \hat{p}_k = 1 - (1 - p_{1.})(1 - p_{.1}) \tag{2.6}$$

It can be shown that $\hat{p}_k = \frac{r}{\hat{N}}$ where $\hat{N}$ is the Petersen estimator (which is $\hat{N}_s$ in this work). That is, the Huggins (1991) method is related to the Petersen estimator by

$$\hat{N}_s = \frac{r}{\hat{p}_k}$$

Here we replaced $\hat{p}_k$ with another probability p (which we called 'coverage probability') given as

$$p = \frac{\sum_x^s x f_x}{\sum_x^n s f_x} \tag{2.7}$$

(see Seber, 1982b) where, x = 0,1,.., s, is the number of times an individual is listed, f_x is the frequency of individuals occurring x times and s is the number of systems (sources).

For a two system formulation,

$$\hat{p} = \frac{\sum_x^s x f_x}{\sum_x^n s f_x} = \frac{n}{rs} \tag{2.8}$$

where r is the number of different individuals listed (caught), n is the total number of individuals on both lists and s is the number of systems. Following the multinomial setting, the joint probability density function for this model is given by Jibasen (2011) as;

$$f(n_{1.}, n_{.1}, n_{11}) = \binom{r}{n_{1.}} \binom{r - n_{1.}}{n_{.1} - n_{11}} \binom{n_{1.}}{n_{11}} p^{n_{1.} + n_{.1}} (1 - p)^{sr - n_{1.} - n_{.1}} \tag{2.9}$$

The maximum likelihood estimator (MLE) of p is $\hat{p} = \frac{n}{sr}$ which finally leads to our proposed estimator

$$\hat{N}_c = \sum_{j=1}^{r} \frac{1}{\hat{p}} = \frac{r}{\hat{p}} \qquad (2.10)$$

The listing probabilities ('listabilities') for this model are a random sample of all individuals in the population, that is, $\hat{N}$ is based on all listed individuals.

Another estimator used in this work is the no factor model estimator $\hat{N}_o$ (Jibasen, 2011). This estimator is given as;

$$\hat{N}_o = \frac{n^2}{4n_{11}} \qquad (2.11)$$

Under this model it is assumed that all individuals have equal chance of being listed (caught) regardless of systems, occasion, or even individual heterogeneity. This assumption is purely unrealistic especially for elusive events in which a drug addict caught the first time will always developed measures to avoid being caught the second or subsequent times, thereby having different probabilities (chances) of being caught at different times and by different systems. However, the new proposed estimator $\hat{N}_c$ has largely catered for such a realistic situation.

3. Simulation Studies

Simulation was carried under the hypergeometric setting 2.3 using equation 2.2, where the population size *N* was assumed, the marginal totals $n_{1.}$ and $n_{.1}$ were fixed, but the recaptures n_{11} was generated randomly. The values assumed for *N* are 90, 100, 300, 500, 1000, 2000, 3000 with corresponding values of $n_{1.}$ and $n_{.1}$ fixed at 50, 50, 150, 260, 450, 900, 1500 and 10, 10, 80, 30, 40, 80, 100 respectively. From the above, the values of $n_{2.}$ needed to simulate number of recapture n_{11} are simply obtained by the difference $N - n_{1.}$. For each triplet $(N,\ n_{1.},\ n_{.1})$ as specified above, the simulation scheme was repeated ten times and the total size of the elusive population *N* was estimated using the two estimators $\hat{N}_o$, $\hat{N}_s$ and the newly proposed estimator $\hat{N}_c$ as considered in this study. The performance of each estimator was assessed using Akaike Information Criterion (AIC) and the Mean Absolute Deviation (MAD). All the simulations and data analysis were performed within the environment of R statistical package (www.cran.org).

4. Results

Various results from the simulations carried out are presented in Table 2 and Tables A1 to A6 in the appendix.

It is observed from Table 2 that our new proposed estimator $\hat{N}_c$ provides better estimates of *N* than either the $\hat{N}_o$ or $\hat{N}_s$ estimator as evident from the results of the AIC and MAD. However, at the seventh iteration of the simulation (Table 2) where $n_{.1} = n_{11}$, the $\hat{N}_o$ yielded a perfect estimate of *N* like $\hat{N}_c$ in many instances. This better performance of $\hat{N}_o$ at this iteration level could be attributed to chance factor since it is just one out of several results obtained. The performances of the three estimators based on their AIC values are presented by the plot of their respective AIC estimates versus the number of times the simulation scheme was repeated as shown in Figure 1. The superiority of $\hat{N}_c$ over the other two estimators is clearly shown by this plot.

When the size of the elusive population *N* increases from 90 to 100, results of AIC and MAD in Table 3 in the Appendix indicated that the new estimator $\hat{N}_c$ consistently performed better than both the Petersen estimator $\hat{N}_s$ and $\hat{N}_o$. More specifically, it is observed that as the number of recapture (relisting) n_{11} gets fewer, both the $\hat{N}_s$ and $\hat{N}_o$ estimators get poorer while the $\hat{N}_c$ estimator continues to yield better estimates of the intended size of the elusive population.

Without loss of generality, various results obtained in this work showed that the new proposed estimator $\hat{N}_c$ of the size of elusive population *N* is more efficient and robust than the other two ($\hat{N}_s$ and $\hat{N}_o$) based on the AIC and MAD criteria employed as evident from various tables of results (Tables 2 to 8 in the appendix).

For better understanding of the performances of the three estimators $\hat{N}_s$, $\hat{N}_o$ and $\hat{N}_c$ under different simulation schemes as defined by the triplet $(N,\ n_{1.},\ n_{.1})$, we plotted their respective AIC values against the number of times the simulation was performed (iterations) as presented in Figure 2 in the appendix. Only the graphs for (300, 150, 80), (500, 260, 30), (1000, 450, 40) and (3000, 1500, 100) schemes based on the results in Tables 4, 5, 6 and 8 respectively are presented in Figure 2 due to space. It is easily observed from the four plots in Figure 2 that the AIC values of our new estimator $\hat{N}_c$ is relatively smaller and more stable (less variable) than that of $\hat{N}_s$ and $\hat{N}_o$ across all the ten repetitions irrespective the size of the elusive population.

5. Discussions and Conclusion

The work here presents a novel estimator of capture-recapture events in elusive population. A thorough comparison of our proposed estimator with two of the existing ones through simulation studies clearly shows the superiority of our estimator over others in terms of better performance at estimating the size of the elusive population *N*. Various results from Table 2 and Tables 3 to 8 in the Appendix show that the estimator $\widehat{N}_o$ always overestimates the size of the elusive population *N*, $\widehat{N}_s$ underestimates *N* while our new proposed estimator $\widehat{N}_c$ always performs better in term of the closeness of its estimates to the target population of elusiveness. Further, the results show that $\widehat{N}_c$ is a better estimator of the targeted elusive population irrespective the population size. Also, our proposed estimator performs better with fewer 'relistings' n_{11}, this is the case of elusiveness.

In other words, the Petersen is at its best when the expected value of n_{11} follows the hyper geometric distribution; but the use of hypergeometric distribution emphasizes the fact that it is basically the activity of the experimenter that brings about randomness. Elusive events are such that the experimenter cannot influence randomness. Thus, Petersen performs very poor with fewer 'relistings' n_{11} which is the case of elusive population.

Results from this work suggest that both $\widehat{N}_o$ and $\widehat{N}_s$ cannot be used to estimate the size of elusive populations. The work established that the new proposed $\widehat{N}_c$ is a better model compared to $\widehat{N}_o$ and $\widehat{N}_s$ (the Petersen's estimator). This turns to suggest that for elusive epidemiologic populations, the popular Petersen's model cannot be used. In such cases our proposed estimator $\widehat{N}_c$ is a better estimator.

Estimation of these types of Events ($\widehat{N}_o$ and $\widehat{N}_s$) has been focused towards multiple recaptures in literature, but some demographic events are such that multiple recaptures may not be possible. This is the case with elusive populations. Hence, the need for this proposed method for estimating the size of such events.

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Appendix

Table 1: A 2×2 contingency table illustrating capture-recapture scheme from a closed population of size *N* individuals using two systems.

		System 2		**Total**
		Caught	Uncaught	
System 1	Caught	n_{11}	n_{12}	$n_{1.}$
	Uncaught	n_{21}	n_{22}	$n_{2.}$
Total		$n_{.1}$	$n_{.2}$	N

Table 2: Table of results of the three estimators ($\hat{N}_o$, $\hat{N}_s$, $\hat{N}_c$) of elusive population *N* for simulation scheme with $N = 90$, $n_{1.} = 50$ and $n_{.1} = 10$. The AIC values of the estimators at each simulation (iteration) as well as their respective computed MADs are equally reported in the table.

Iteration	n_{11}	$\hat{N}_o$	**AIC**	$\hat{N}_s$	**AIC**	$\hat{N}_c$	**AIC**
1	7	129	7.141	71	5.366	94	4.272
2	7	129	7.141	71	5.366	94	4.272
3	6	150	9.137	83	4.490	97	4.543
4	8	113	5.760	63	6.112	90	4.010
5	8	113	5.760	63	6.112	90	4.010
6	8	113	5.760	63	6.112	90	4.010
7	10	90	4.000	50	7.124	83	4.490
8	7	129	7.141	71	5.366	94	4.272
9	8	113	5.760	63	6.112	90	4.010
10	5	180	12.197	100	4.759	101	4.824
MAD		44		28		5	

Table 3 : Table of results of the three estimators ($\hat{N}_o$, $\hat{N}_s$, $\hat{N}_c$) of elusive population N for simulation scheme with N = 100, $n_{1.}$= 50 and $n_{.1}$ = 10. The AIC values of the estimators at each simulation (iteration) as well as their respective computed MADs are equally reported in the table.

Iteration	n_{11}	$\hat{N}_o$	**AIC**	$\hat{N}_s$	**AIC**	$\hat{N}_c$	**AIC**
1	4	225	16.29	125	6.018	105	4.348
2	6	150	8.281	83	5.225	97	4.211
3	6	150	8.281	83	5.225	97	4.211
4	7	129	6.327	71	6.101	94	4.476
5	7	129	6.327	71	6.101	94	4.476
6	7	129	6.327	71	6.101	94	4.476
7	7	129	6.327	71	6.101	94	4.476
8	8	113	4.978	63	6.880	90	4.733
9	6	150	8.281	83	5.225	97	4.211
10	5	180	11.29	100	4.000	101	4.063
MAD		48		23		5	

Table 4: Table of results of the three estimators ($\hat{N}_o$, $\hat{N}_s$, $\hat{N}_c$) of elusive population N for simulation scheme with N= 300, $n_{1.}$ = 150 and $n_{.1}$= 80. The AIC values of the estimators at each simulation (iteration) as well as their respective computed MADs are equally reported in the table.

Iteration	n_{11}	$\hat{N}_o$	**AIC**	$\hat{N}_s$	**AIC**	$\hat{N}_c$	**AIC**
1	52	254	6.698	231	8.106	276	5.456
2	55	240	7.519	218	8.906	266	5.996
3	54	245	7.253	222	8.643	269	5.818
4	49	270	5.786	245	7.254	285	4.904
5	51	259	6.405	235	7.830	279	5.274
6	53	250	6.980	226	8.377	272	5.638
7	57	232	8.030	211	9.424	260	6.351
8	53	250	6.980	226	8.377	272	5.638
9	55	240	7.519	218	8.906	266	5.996
10	55	240	7.519	218	8.906	266	5.996
MAD		52		75		29	

Table 5: Table of results of the three estimators ($\hat{N}_o$, $\hat{N}_s$, $\hat{N}_c$) of elusive population N for simulation scheme with N = 500, $n_{1.}$ = 260 and $n_{.1}$ = 30. The AIC values of the estimators at each simulation (iteration) as well as their respective computed MADs are equally reported in the table.

Iteration	n_{11}	$\hat{N}_o$	**AIC**	$\hat{N}_s$	**AIC**	$\hat{N}_c$	**AIC**
1	17	1237	85.62	459	7.46	514	5.20
2	19	1107	68.92	411	11.43	506	4.55
3	23	914	45.76	339	17.64	492	4.71
4	16	1314	95.88	488	5.06	518	5.53
5	22	956	50.59	355	16.19	495	4.40
6	20	1051	62.06	390	13.13	503	4.24
7	22	956	50.59	355	16.19	495	4.40
8	16	1314	95.88	488	5.06	518	5.53
9	18	1168	76.71	433	9.56	510	4.88
10	20	1051	62.06	390	13.13	503	4.24
MAD		607		89		9	

Table 6: Table of results of the three estimators ($\widehat{N}_o$, $\widehat{N}_s$, $\widehat{N}_c$) of elusive population *N* for simulation scheme with *N* = 1000, $n_{1.}$ = 450 and $n_{.1}$ = 40. The AIC values of the estimators at each simulation (iteration) as well as their respective computed MADs are equally reported in the table.

Iteration	n_{11}	$\widehat{N}_o$	**AIC**	$\widehat{N}_s$	**AIC**	$\widehat{N}_c$	**AIC**
1	30	2001	117.7	600	38.61	864	16.29
2	29	2070	126.8	621	36.54	867	15.96
3	29	2070	126.8	621	36.54	867	15.96
4	28	2144	136.6	643	34.44	871	15.63
5	24	2501	186.1	750	25.07	886	14.30
6	33	1819	94.4	545	45.04	852	17.27
7	31	1936	109.3	581	40.68	860	16.62
8	22	2728	219.0	818	19.24	894	13.62
9	27	2223	147.3	667	32.28	875	15.30
10	23	2610	201.7	783	22.28	890	13.96
MAD		1210		337		127	

Table 7: Table of results of the three estimators ($\widehat{N}_o$, $\widehat{N}_s$, $\widehat{N}_c$) of elusive population *N* for simulation scheme with *N* = 2000, $n_{1.}$ = 900 and $n_{.1}$ = 80. The AIC values of the estimators at each simulation (iteration) as well as their respective computed MADs are equally reported in the table.

Iteration	n_{11}	$\widehat{N}_o$	**AIC**	$\widehat{N}_s$	**AIC**	$\widehat{N}_c$	**AIC**
1	49	4900	353.7	1469	49.74	1769	24.93
2	48	5002	368.2	1500	47.13	1773	24.60
3	57	4212	259.1	1263	67.99	1739	27.59
4	63	3811	206.8	1143	80.48	1716	29.56
5	52	4617	314.2	1385	57.05	1758	25.94
6	60	4002	231.4	1200	74.22	1727	28.58
7	59	4069	240.3	1220	72.15	1731	28.25
8	56	4288	269.2	1286	65.88	1742	27.26
9	55	4365	279.7	1309	63.74	1746	26.93
10	60	4002	231.4	1200	74.22	1727	28.58
MAD		2327		702		257	

Table 8: Table of results of the three estimators ($\widehat{N}_o$, $\widehat{N}_s$, $\widehat{N}_c$) of elusive population *N* for simulation scheme with *N* = 3000, $n_{1.}$ = 1500 and $n_{.1}$ = 100. The AIC values of the estimators at each simulation (iteration) as well as their respective computed MADs are equally reported in the table.

Iteration	n_{11}	$\widehat{N}_o$	**AIC**	$\widehat{N}_s$	**AIC**	$\widehat{N}_c$	**AIC**
1	71	9014	784.4	2113	86.04	2922	11.45
2	71	9014	784.4	2113	86.04	2922	11.45
3	65	9846	914.1	2308	67.57	2945	9.26
4	76	8421	694.3	1974	100.21	2903	13.27
5	74	8649	728.6	2027	94.61	2911	12.55
6	67	9552	867.8	2239	73.99	2938	9.99
7	71	9014	784.4	2113	86.04	2922	11.45
8	64	10000	938.5	2344	64.24	2949	8.89
9	55	11636	1205	2727	28.85	2984	5.56
10	64	10000	938.5	2344	64.24	2949	8.89
MAD		6515		770		65	

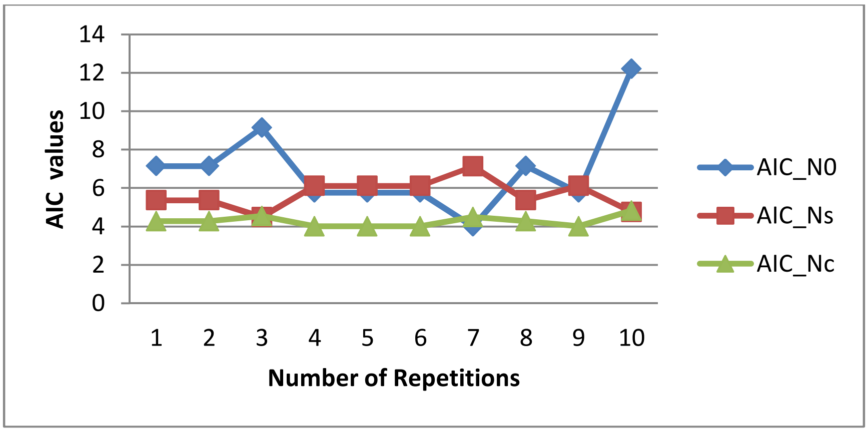
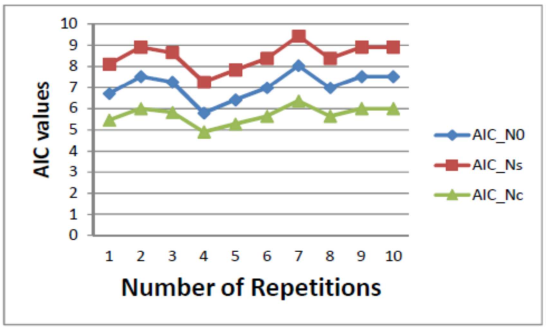
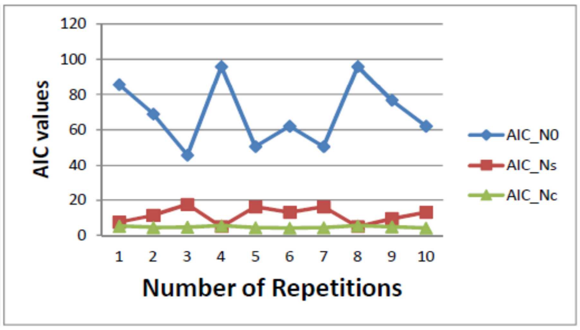
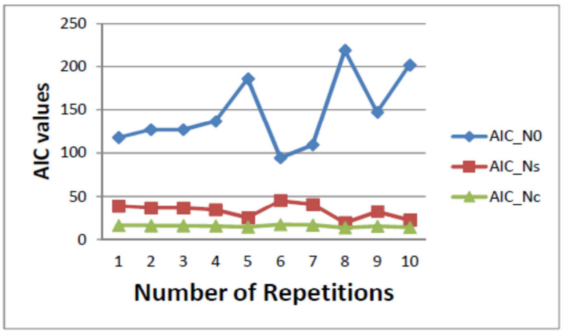


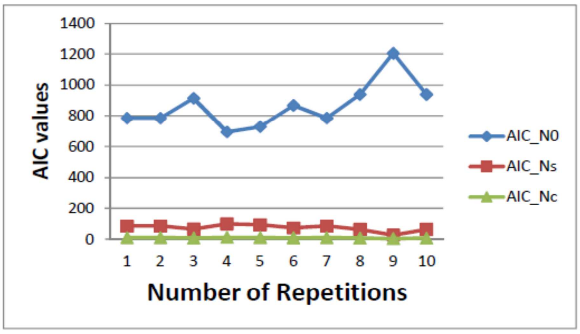
Figure 1: The plot of Akaike Information Criteria (AIC) of the three estimators, $\widehat{N}_s$, $\widehat{N}_o$ and $\widehat{N}_c$ of the elusive population *N*. The plot shows better performance (low AIC values) of our new proposed estimator $\widehat{N}_c$ over others.





$N = 300,\ n_{1.} = 150,\ n_{.1} = 80$ $N = 500,\ n_{1.} = 260,\ n_{.1} = 30$





$N = 1000,\ n_{1.} = 450,\ n_{.1} = 40$ $N = 3000,\ n_{1.} = 1500,\ n_{.1} = 100$

Figure 2: The plot of the AIC values of each of the estimators $\widehat{N}_s$, $\widehat{N}_o$ and $\widehat{N}_c$ against the number of times simulation scheme was repeated for the selected schemes (N, $n_{1.}$, $n_{.1}$) = {(300, 150, 80), (500, 260, 30), (1000, 450, 40), (3000, 1500, 100)} where *N* is the assumed size of the elusive population, $n_{1.}$ and $n_{.1}$ are the total number of persons (drug addicts) caught by systems 1 and 2 respectively.