

Hydromagnetic Turbulent Flow

Past A Semi-Infinite Vertical Plate Subjected To Heat Flux

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Abstract

In this study we have investigated a turbulent flow of a rotating fluid past a semi-infinite vertical porous plate subjected to a constant heat flux. A variable magnetic field is applied transversely in the direction normal to the plate. An induced electric current known as Hall current exists due to the presence of both electric field and magnetic field. The differential equations governing this problem are solved numerically using a finite difference scheme. Further we have investigated the effects of various parameters on the velocity, temperature, magnetic field and concentration profile. The skin friction the rate of heat transfer and the rate of mass transfer is calculated using Newton's interpolation formula We noted that the Hall current, rotation parameter, Eckert number, injection and Schmidt number affect the velocity, temperature magnetic field and concentration profiles.

Keywords: Free convection, Turbulent flow, rotation, Magnetic field, Hall current, Heat flux Heat Transfer, Mass transfer

1 Introduction

In recent years the theoretical study of MHD flows has been a subject of great interest due to its widely spread application on separation of matter from fluids, designing of cooling systems with liquid metals, petroleum industry, purification of crude oil, and many other applications.

The study of fluids on a rotating system has received considerable interest due to its application in practical situations like meteorology, geophysical fluids dynamics, gaseous and nuclear reactions. Debnath et.al. [1] Investigated the unsteady rotating flow of a viscous fluid in the presence of Hall Effect. Hayat et.al. [2] Discussed effect of Hall current and heat transfer on rotating flow on a second grade fluid through a porous medium. Katagiri [3] discussed the effect of Hall current on the MHD boundary layer flow past a semi-infinite plate. Kinyanjui et al [4] studied the MHD Stokes problem for a vertical infinite plate in a dissipative rotating fluid with Hall current. Kinyanjui et al [5] presented their work in MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption.

Kwanza J,K, et al. [6] studied Hydromagnetic Free Convection Flow Past a Semi – Infinite Vertical Porous Plate Subjected to Constant Heat Flux With Radiation Absorption. MHD Stokes problem of convective flow from a vertical infinite plate in a rotating fluid. was studied by Ram [7]. Soundalgekar, Singh and Takhar [8] study of MHD free convection flow past a semi-infinite vertical plate with suction and injection.

The early works on fluid dynamics is mostly on laminar flows with very little devotion to turbulent flows but most flows of engineering importance are turbulent and hence the reason for this study. In this study the problem of MHD free convection turbulent flow of a rotating fluid past a semi-infinite plate is considered.

2. Mathematical Analysis.

We consider a hydromagnetic turbulent flow of incompressible viscous rotating fluid past an impulsively started semi-infinite porous plate. A variable magnetic field H is applied in a direction normal to the plate. The choice of co-ordinate is such that the y -axis is taken along the plate in the vertical direction and the x -axis is taken normal to the plate.

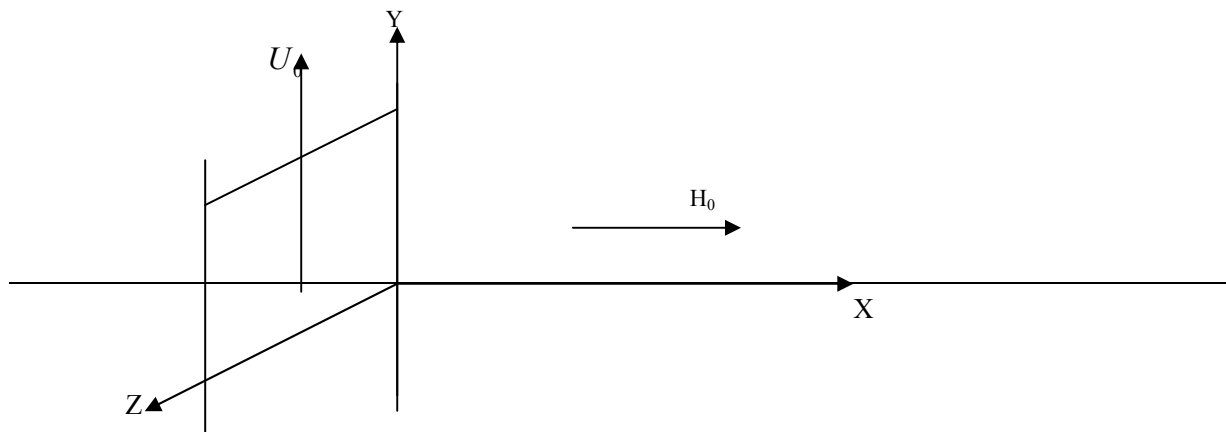


Figure 1 Geometry of the Problem

In this case we consider the turbulent flow when the plate is subjected to a constant heat flux. Turbulent flows are highly irregular and there are rapid fluctuations of velocity in the flow variable with respect to time and location. Mean value provides a basis for studying the spatial variation. The instantaneous value for a general flow say v for a turbulent fluid motion can be given as $v = \bar{v} + v'$ where \bar{v} is the mean value and v' the fluctuating component.

The Reynolds averaging rules is used to transform equations governing laminar flow to govern turbulent flows. Initially temperature of the fluid and the plate are assumed to be the same. At $t > 0$, the plate starts moving impulsively on its plane at a velocity U_0 . The plate is maintained at a constant temperature T_w higher than the constant temperature T_∞ of the surrounding fluid and the concentration C_w greater than the constant concentration C_∞ of the surrounding fluid. The flow is turbulent therefore there is a large magnetic fields this implies that Hall currents significantly affect the flow. The fluid and the plate are at a state of rigid rotation with uniform angular velocity Ω about the x-axis taken normal to the plate. For a non-zero rotation rate the velocity vector is of the form $q = u(x, y)\tilde{i} + v(x, y)\tilde{j} + w(x, y)\tilde{k}$. u is the axial velocity while v and w represent the primary and secondary flows. As the rotation is around \tilde{i} it is of the form $\Omega = \Omega\tilde{i}$.

As the plate is semi-infinite in extent and the flow is unsteady the physical variables are functions of x^*, y^* and t^*

2.1 Governing equations

The problem is governed by the following set of equations

$$\frac{\partial \bar{v}^*}{\partial t^*} + \bar{u}_0 \frac{\partial \bar{v}^*}{\partial x^*} + v \frac{\partial \bar{v}^*}{\partial y^*} + 2\Omega w^* = v \frac{\partial^2 \bar{v}^*}{\partial x^{*2}} + \beta g (T - T_\infty) + \beta_c (C^* - C_\infty^*) - \frac{\partial \overline{u^* v^*}}{\partial x^*} + \frac{\mu_0 H_0}{\rho} J_{z^*} \quad (1)$$

$$\frac{\partial \bar{w}^*}{\partial t^*} + \bar{u}_0 \frac{\partial \bar{w}^*}{\partial x^*} + v \frac{\partial \bar{w}^*}{\partial y^*} - 2\Omega v^* = v \frac{\partial^2 \bar{w}^*}{\partial x^{*2}} - \frac{\partial \overline{u^* w^*}}{\partial x^*} - \frac{\mu_0 H_0}{\rho} J_{y^*} \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} + \bar{u}_0 \frac{\partial T^*}{\partial x^*} + \bar{v}^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T^*}{\partial x^{*2}} + \frac{Q^*}{\rho c_p} + \frac{\nu}{c_p} \left[\left(\frac{\partial \bar{v}^*}{\partial x^*} \right)^2 + \left(\frac{\partial \bar{w}^*}{\partial x^*} \right)^2 \right] \quad (3)$$

$$\frac{\partial H^*}{\partial t^*} = \frac{1}{\mu_e \sigma} \left(\frac{\partial^2 H^*}{\partial x^{*2}} \right) - \nu^* \frac{\partial H^*}{\partial y^*} - H^* \frac{\partial \nu^*}{\partial y^*} \quad (4)$$

$$\frac{\partial C^*}{\partial t^*} + u^* \frac{\partial C^*}{\partial x^*} + \nu^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial x^{*2}} \quad (5)$$

The generalized Ohm's law including the effect of Hall current is written as

$$\hat{j} + \frac{\omega_e \tau_e}{H} (\hat{j} \times \hat{H}) = \sigma \left(\hat{E} + \mu_e \hat{q} \times \hat{H} + \frac{1}{e \eta_e} \nabla P_e \right) \quad (6)$$

Solving the above equation for the current density components j_{y^*} and j_{z^*}

$$\hat{j}_{y^*} = \frac{\sigma \mu_e H_0 (m \bar{v}^* + \bar{w}^*)}{1 + m^2} \quad (7)$$

$$\hat{j}_{z^*} = -\frac{\sigma \mu_e H_0 (\bar{v}^* - m \bar{w}^*)}{1 + m^2} \quad (8)$$

Where $m = \omega_e \tau_e$ is the Hall parameter

Substituting equations (7) and (8) in (2) and (3) respectively we get

$$\frac{\partial \bar{v}^*}{\partial t^*} + \bar{u}_0 \frac{\partial \bar{v}^*}{\partial x^*} + \nu \frac{\partial \bar{v}^*}{\partial y^*} + 2\Omega \bar{w}^* = \nu \frac{\partial^2 \bar{v}^*}{\partial x^{*2}} + \beta g (T - T_\infty) + \beta_c g (C^* - C_\infty^*) - \frac{\partial \bar{u}^* \bar{v}^*}{\partial x^*} - \frac{\sigma \mu_e^2 H^{*2} (\bar{v}^* - m \bar{w}^*)}{\rho (1 + m^2)} \quad (9)$$

$$\frac{\partial \bar{w}^*}{\partial t^*} + \bar{u}_0 \frac{\partial \bar{w}^*}{\partial x^*} + \nu \frac{\partial \bar{w}^*}{\partial y^*} - 2\Omega \bar{v}^* = \nu \frac{\partial^2 \bar{w}^*}{\partial x^{*2}} - \frac{\partial \bar{u}^* \bar{w}^*}{\partial x^*} - \frac{\sigma \mu_e^2 H^{*2} (m \bar{v}^* + \bar{w}^*)}{\rho (1 + m^2)} \quad (10)$$

$$\frac{\partial T^*}{\partial t^*} + \bar{u}_0 \frac{\partial T^*}{\partial x^*} + \bar{v}^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T^*}{\partial x^{*2}} + \frac{Q^*}{\rho c_p} + \frac{\nu}{c_p} \left[\left(\frac{\partial \bar{v}^*}{\partial x^*} \right)^2 + \left(\frac{\partial \bar{w}^*}{\partial x^*} \right)^2 \right] \quad (11)$$

$$\frac{\partial H^*}{\partial t^*} = \frac{1}{\mu_e \sigma} \left(\frac{\partial^2 H^*}{\partial x^{*2}} \right) - \nu^* \frac{\partial H^*}{\partial y^*} - H^* \frac{\partial \nu^*}{\partial y^*} \quad (12)$$

$$\frac{\partial C^*}{\partial t^*} + u^* \frac{\partial C^*}{\partial x^*} + v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial x^{*2}} \quad (13)$$

2.2 Initial and Boundary conditions.

Since heat is constantly being supplied to the plate, we apply Fourier's law at $x^* = 0$

$$\frac{\partial T^*}{\partial x^*} = -\frac{q^*}{\kappa} \quad (14)$$

$$t^* \leq 0: \quad v^*(x^*, y^*, t^*) = 0 \quad w^*(x^*, y^*, t^*) = 0 \quad T^*(x^*, y^*, t^*) = T_\infty^* \quad (15a)$$

$$C^*(x^*, y^*, t^*) = C_\infty^*$$

$$t^* > 0: \quad v^*(0, y^*, t^*) = 0 \quad w^*(0, y^*, t^*) = 0 \quad \frac{\partial T^*}{\partial x^*} = -\frac{q^*}{\kappa} \quad C^*(0, y^*, t^*) = C_w^* \quad (15b)$$

$$v^*(\infty, y^*, t^*) = 0 \quad w^*(\infty, y^*, t^*) = 0 \quad T^*(\infty, y^*, t^*) = T_\infty^* \quad (15c)$$

$$C^*(\infty, y^*, t^*) = C_\infty^*$$

In this study non- dimensionalization is based on the following non- dimensional quantities

$$\left. \begin{aligned} t &= \frac{t^* U^2}{\nu} & x &= \frac{x^* U}{\nu} & y &= \frac{y^* U}{\nu} & u_0 &= \frac{u_0^*}{U} & v &= \frac{v^*}{U} & w &= \frac{w^*}{U} \\ \text{Pr} &= \frac{\mu c_p}{\kappa} & Gr &= \frac{\nu g \beta \left(\frac{q^* \nu}{\kappa U} \right)}{U^3} & E_c &= \frac{U^2}{c_p \left(\frac{q^* \nu}{\kappa U} \right)} & H &= \frac{H^*}{H_0} \\ \theta &= \frac{T^* - T_\infty^*}{\left(\frac{q^* \nu}{\kappa U} \right)} & M^2 &= \frac{\sigma \mu_e^2 H_0^2 \nu}{\rho U^2} & \delta &= \frac{Q \nu}{\kappa U^2} & R_m &= \sigma \mu_e L U \\ C &= \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*} & Gc &= \frac{\nu g \beta_c (C^* - C_\infty^*)}{U^3} & E_r &= \frac{\Omega \nu}{U^2} & Sc &= \frac{D}{\nu} \end{aligned} \right\} (16)$$

On introducing the dimensionless variables in equations (9) to (13) and using

The boussinesq's approximations $\tau = -\rho \bar{v} w = A \frac{\partial \bar{v}}{\partial z}$ which gives

$$\left. \begin{aligned} \overline{uv} &= -k^2 x^2 \left(\frac{\partial \overline{v}}{\partial x} \right)^2 \\ \overline{uw} &= -k^2 x^2 \left(\frac{\partial \overline{w}}{\partial x} \right)^2 \end{aligned} \right\} \quad (17)$$

We get the final governing equations as

$$\begin{aligned} \frac{\partial v}{\partial t} + u_0 \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} - 2E_r w = & \left(\frac{\partial^2 v}{\partial x^2} \right) + 2k^2 x \left(\frac{\partial v}{\partial x} \right)^2 + 2k^2 x^2 \left(\frac{\partial^2 v}{\partial x^2} \right) \left(\frac{\partial v}{\partial x} \right) \\ & + G_r \theta + G_c C - \frac{M^2 H^2 (v - mw)}{(1 + m^2)} \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\partial w}{\partial t} + u_0 \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + 2E_r v = & \frac{\partial^2 w}{\partial x^2} + 2k^2 x \left(\frac{\partial w}{\partial x} \right)^2 + 2k^2 x^2 \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial w}{\partial x} \right) \\ & - \frac{M^2 H^2 (v + mw)}{(1 + m^2)} \end{aligned} \quad (19)$$

$$\frac{\partial \theta}{\partial t} + u_0 \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial x^2} - \frac{\delta}{Pr} \theta + Q_1 C + E_c \left[\left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial x} \right)^2 \right] \quad (20)$$

$$\frac{\partial H}{\partial t} = \frac{1}{R_m} \frac{U^2}{v^2} \left(\frac{\partial^2 H}{\partial x^2} \right) - v \frac{\partial H}{\partial y} - H \frac{\partial v}{\partial y} \quad (21)$$

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} = Sc \frac{\partial^2 C}{\partial x^2} \quad (22)$$

The initial and boundary conditions (15a) to (15c) in non-dimensional form becomes

$$t \leq 0: \quad v(x, y, 0) = 0 \quad w(x, y, 0) = 0 \quad \theta(x, y, 0) = 0 \quad C(x, y, 0) = 0 \quad (23a)$$

$$t > 0: \quad v(0, y, t) = 1 \quad w(0, y, t) = 1 \quad \frac{\partial \theta}{\partial x} = -1 \quad C(0, y, t) = 1 \quad (23b)$$

$$v(\infty, y, t) = 0 \quad w(\infty, y, t) = 0 \quad \theta(\infty, y, t) = 0 \quad C(\infty, y, t) = 0 \quad (23c)$$

3 Method of solution

As it is not possible to get the exact solution for the equations (18) to (22) we generate numerical solutions. The equations will be solved subject to the boundary conditions (23a, b, c). Writing equations (18) to (22) in finite difference form similar to that described by kwanza (2010)

Computer is used to generate numerical solutions.

4 Calculation of the Rate of Heat Transfer and Skin Friction

The rate of heat transfer is calculated from the temperature distribution in terms of the Nusselt number
 The skin friction is calculated from the velocity profiles using the equations

$$\tau_y = -\left. \frac{\partial v}{\partial x} \right|_{x=0} \quad \text{and} \quad \tau_z = -\left. \frac{\partial w}{\partial x} \right|_{x=0} \quad \text{where} \quad \tau = \frac{\tau^*}{\rho U^2} \quad (24)$$

These are calculated by numerical differentiation using Newton's interpolation formula over the first five points

5 Discussions of results

A program was run for various values of velocity, temperature and concentration profiles for the finite difference equations. Results are obtained for various values of the parameters m , Ec , Er , U_0 , and Sc , associated with the governing problem.

From Figure 2, we note that:

- i. An increase in Hall parameter leads to an increase in primary velocity profiles. This is due to the fact that the effective conductivity decreases with the increase in Hall parameter which reduces the magnetic damping force hence the increase in velocity.
- ii. An increase in the Eckert number Ec leads to an increase in primary velocity profiles. Increase in Ec increases viscous dissipation and therefore decreases the velocity gradient and hence slows down the motion.
- iii. An increase in the rotation parameter Er leads to a decrease in primary velocity profiles. The increase in frequency of oscillation decreases the thickness of the boundary layer which in effect increases the velocity gradient and hence the velocity decreases.
- iv. Increase in time increases the primary velocity profiles. With time the flow gets to the free stream and therefore its velocity increases.

From Figure 3, we note that:

- i. An increase in Hall parameter leads to an increase to secondary velocity profiles. This is due to the fact that the effective conductivity decreases with the increase in Hall parameter which reduces the magnetic damping force hence the increase in velocity.
- ii. An increase in the Ec leads to a decrease in secondary velocity profiles. This is due to increase in viscous dissipation.
- iii. An increase in the rotation parameter Er leads to a decrease in secondary velocity profiles. The increase in frequency of oscillation decreases the thickness of the boundary layer and hence the velocity decreases.
- iv. Increase in time decreases the secondary velocity profiles. With time the flow gets to the free stream and therefore its primary velocity diminishes.

From Figure 4 we note that:

- i. An increase in Hall parameter leads to a slight increase in the temperature profiles. Increase in hall parameter increases the thermal boundary layer hence increase the temperature of the flow.
- ii. An increase in the Ec increases the temperature profiles. This is because increase in Ec reduces the temperature gradient thus increasing the temperature boundary layer and therefore the increase in temperature.
- iii. An increase in the rotation parameter Er increases the temperature profiles. The frequency of oscillation is increased thus increase the temperature.
- iv. Increase in time increases the temperature profiles. With time the flow gets to the free stream where the velocity is high the rate of energy transfer is increased and hence increases the temperature.

From Figure 5 we note that:

- i. An increase in Hall current leads to a slight increase in the magnetic field profiles.
- ii. An increase in the Eckert number Ec increases the magnetic field profiles.
- iii. An increase in the rotation parameter Er decreases the magnetic field profiles. Increase in Er increases the frequency of oscillation
- iv. Increase in time increases the magnetic field profiles.

From Figure 6

- i. An increase in mass diffusion parameter Sc leads to an increase in the concentration profiles. This is because increase in Sc increases molecular diffusivity which results in an increase of the concentration boundary layer. Hence the concentration of the species is higher for large values of Sc .
- ii. Removal of injection leads to a decrease in the concentration profiles. This is due to the fact that this reduces the growth of the boundary layers and hence the decrease in the concentration profiles.
- iii. Increase in time increases the concentration profiles. With time the flow gets to the free stream and therefore its concentration increases.

From Table 1 we observe that with $Gr = 5.0$

- i. An increase in Hall parameter leads to a decrease in both τ_y and τ_z . This is due to the fact that increase in Hall parameter increases the velocity and therefore shear resistance is decreased.
 An increase in Eckert number Ec leads to an decrease in τ_y but a slight increase in τ_z . This is due to the effect

of E_c on the velocity which increases the primary velocity but decreases the secondary velocity

An increase in E_r leads to an increase in both τ_y and τ_z . Increase in E_r decreases the velocity and therefore the shear resistance is increased.

An increase in time leads a decrease in τ_y but an increase in τ_z . in free stream the primary velocity is increased while the secondary velocity is decreased.

From table 3 we observe that

- i. An increase in the Hall parameter leads to an increase in the Nusselt number. This is because Hall parameter increases the velocity of the fluid and this has a direct effect of increasing the rate of heat transfer
- ii. An increase in E_c , or the rotation parameter E_r leads to a decrease in the Nusselt number. This is attributed to the fact that E_c and E_r decrease the velocity of the fluid and consequently decrease the rate of heat transfer.
- iii. An increase in time leads to an increase in the Nusselt number. In the free stream the velocity of the fluid is increased and therefore the rate of heat transfer increases.

From table 4 we observe that

- i. Removal of injection leads to an increase in rate of mass transfer and therefore the rate of mass transfer increases.
- ii. Increase in mass diffusion parameter Sc , leads an increase in rate of mass transfer. This is because the diffusion is decreased.
- iii. Increase in time leads to a decrease in rate of mass transfer. With time the flow gets to the free stream where the velocity of the fluid is high and hence the rate of mass transfer.

6 Conclusions

In this paper we analyzed the hydromagnetic flow of a rotating fluid past an impulsively started semi-infinite plate. The magnetic field was taken to be variable and we have considered the turbulent boundary layer.

The equations governing the hydromagnetic flow considered are non-linear there we obtained solutions using the finite difference method with uniform mesh system. Stability of the method was tested by choosing smaller time increments which were found to have no significant difference. A numerical method has been using to calculate the skin friction, the rate of heat transfer and the rate of mass transfer.

We noted that if heat is supplied to the plate at a constant rate, then the flow field is affected. Due to the strong magnetic field the presence of the hall current affected the flow significantly.

In presence of Hall current cooling of the plate by free convection current increases the thermal boundary layer.

7 Validations

In the absence of Turbulent and the magnetic field being constant, results agreed with those of Kinyanjui et al (2000) who investigated the MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption.

References

- Debnath S.C. Ray and Chalterjee. (1979) Effects Of Hall Current On An Unsteady Hydromagnetic Flow Past A Porous Plate In A Rotating Fluid System. *Z. Angew ath Mech* 59 469-471.
- Hayat T., Abbas Z. and Asghar S. (2008) Effect Of Hall Current And Heat Transfer On Rotating Flow On A Second Grade Fluid Through A Porous Medium. *Communication in non-linear Sci and numerical simulation* 13 2177-2192
- Katagiri M. (1969) The Effect of Hall Current On The MHD Boundary Layer Flow Past A Semi-Infinite Plate. *J. Phys Sos Jpn* 27 1051-1059
- Kinyanjui M. Charturvedi N and Uppal S.M (1998) MHD Stokes problem for a vertical infinite plate in a dissipative rotating fluid with Hall current *Energy Conversion & Management*
- Kinyanjui M, Kwanza J.K., and Uppal S.M. (2001)MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption. *Energy conversion & management*
- Kwanza J,K, Marigi E.M and Kinyanjui M. (2010) Hydromagnetic Free Convection Flow Past a Semi – Infinite Vertical Porous Plate Subjected to Constant Heat Flux With Radiation Absorption. *Energy Conversion & Management*.51 1326-1332
- Ram P.C. and Kinyanjui M and Tarkhar S. (1995) MHD Stokes problem of convective flow from a vertical infinite plate in a rotating fluid *Journal of MHD and Plasma research* Vol. 5 No. 2/3 pp 91-105
- Soundalgekar V.M. Singh M. and Takhar H.S. (1983) MHD free convection flow past a semi-infinite vertical plate with suction and injection. *Non-linear analysis methods theory and analysis* vol.7 pp941-944

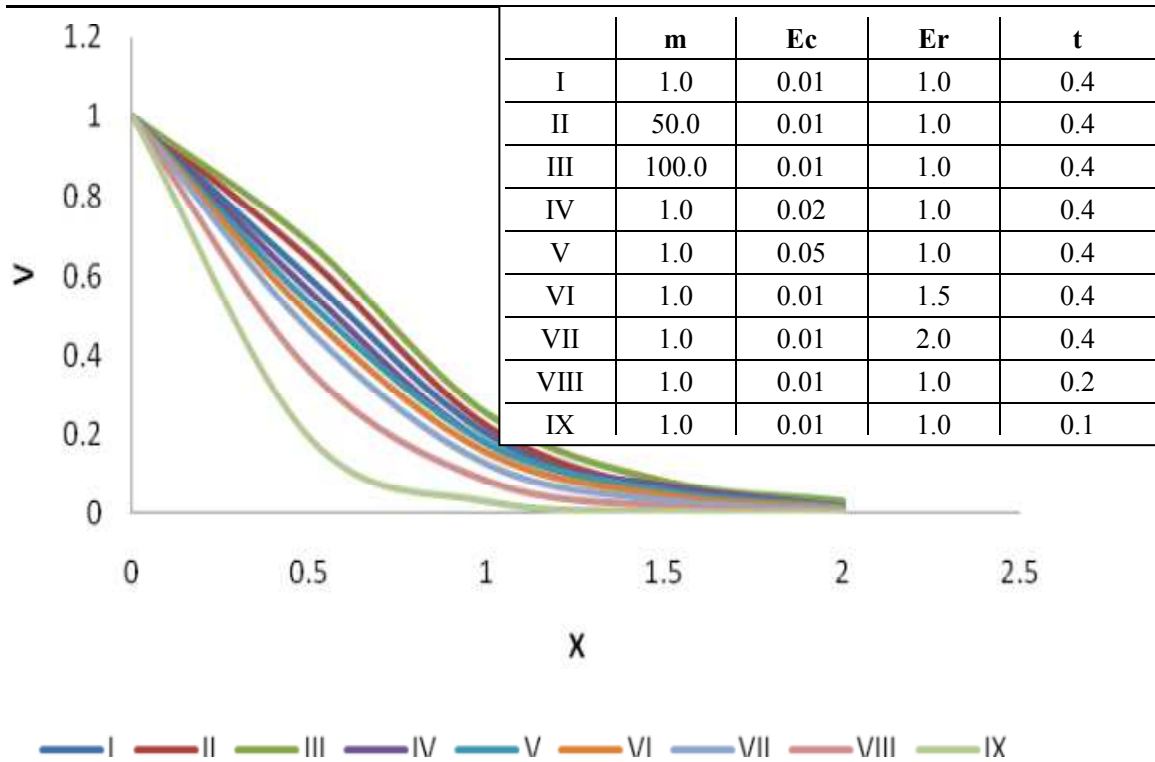


Figure 2: Primary Velocity Profile.

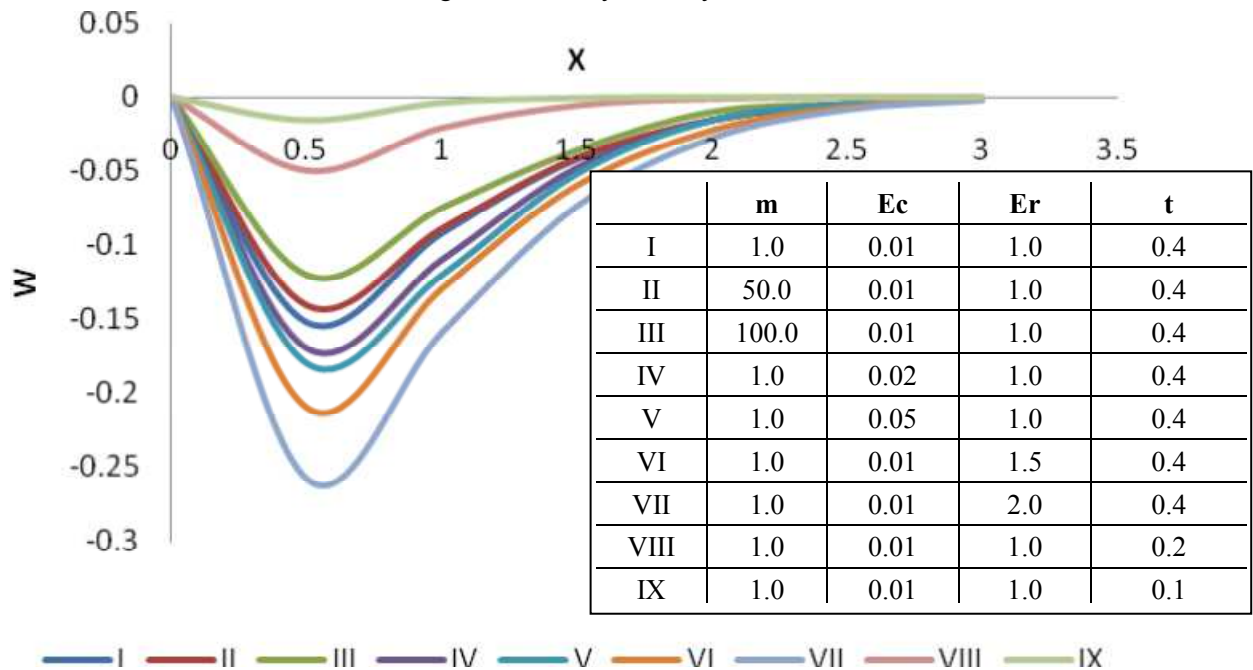


Figure 3: secondary Velocity Profile

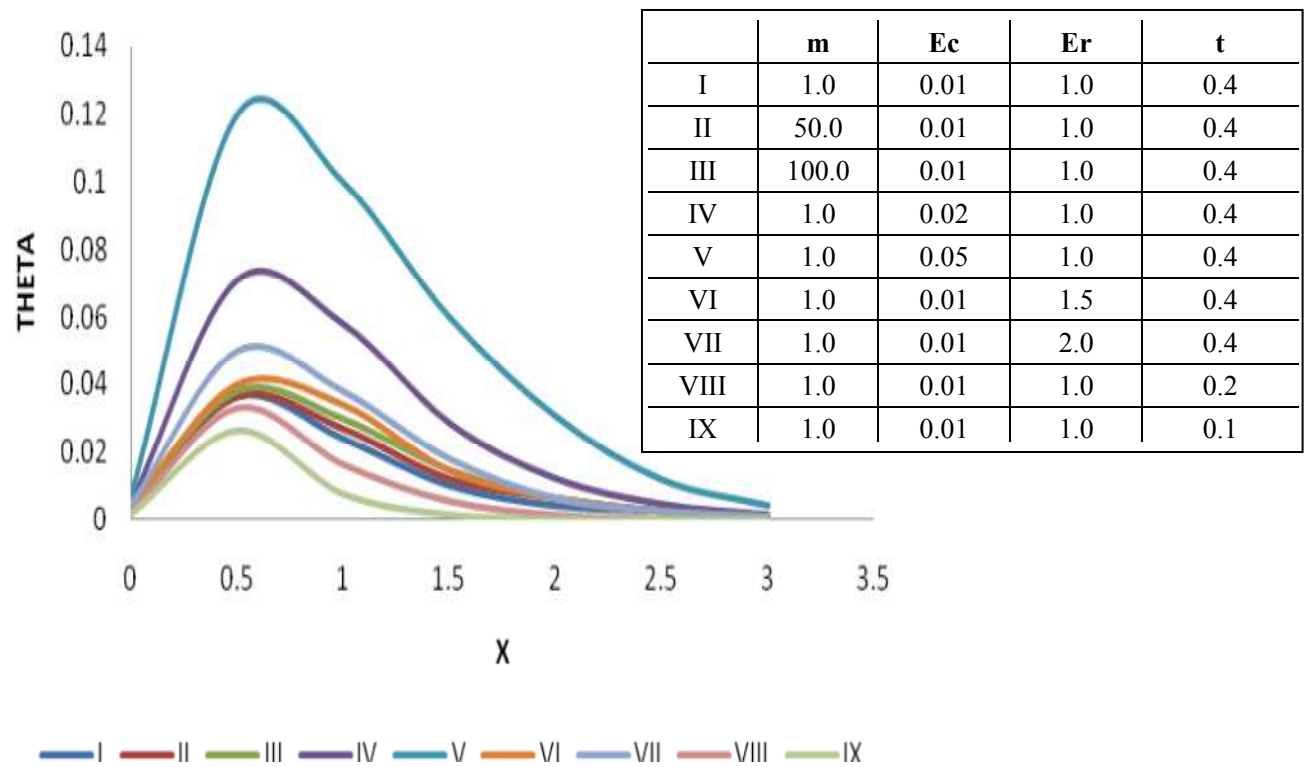


Figure 4: Temperature Profiles

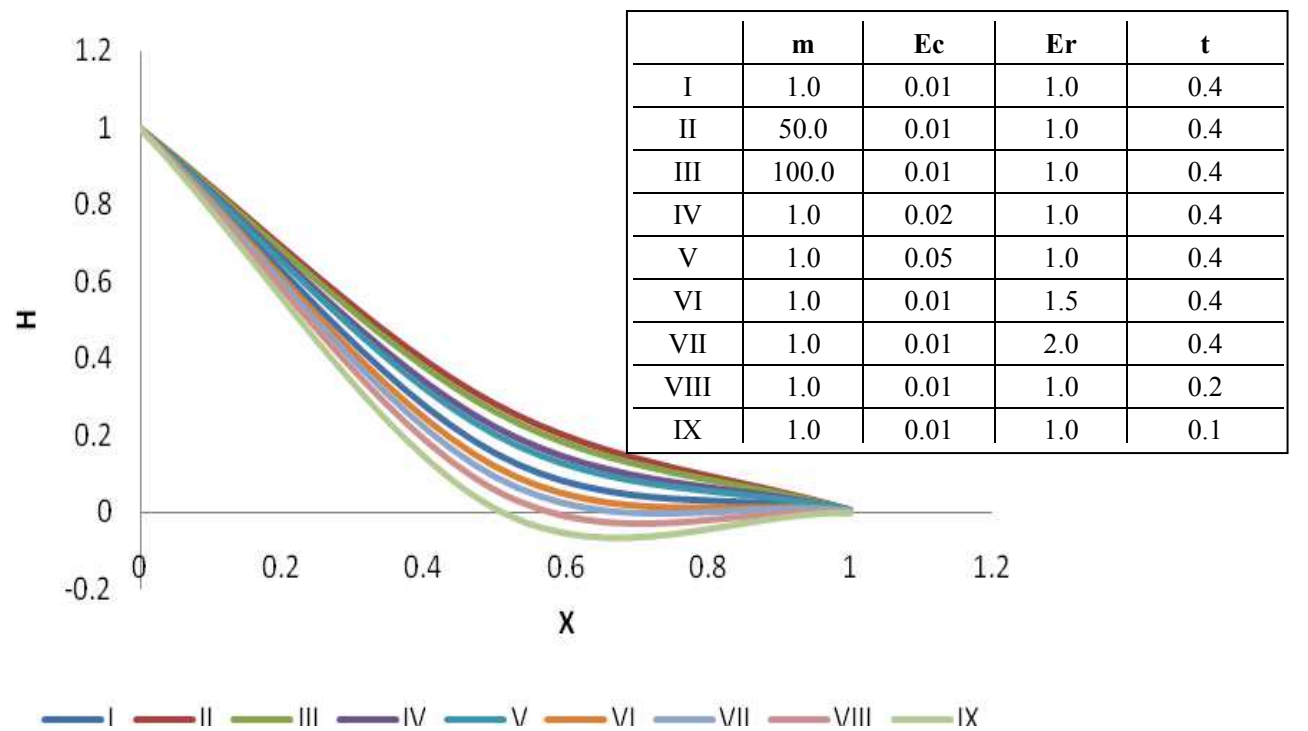


Figure 5: Magnetic field Profile

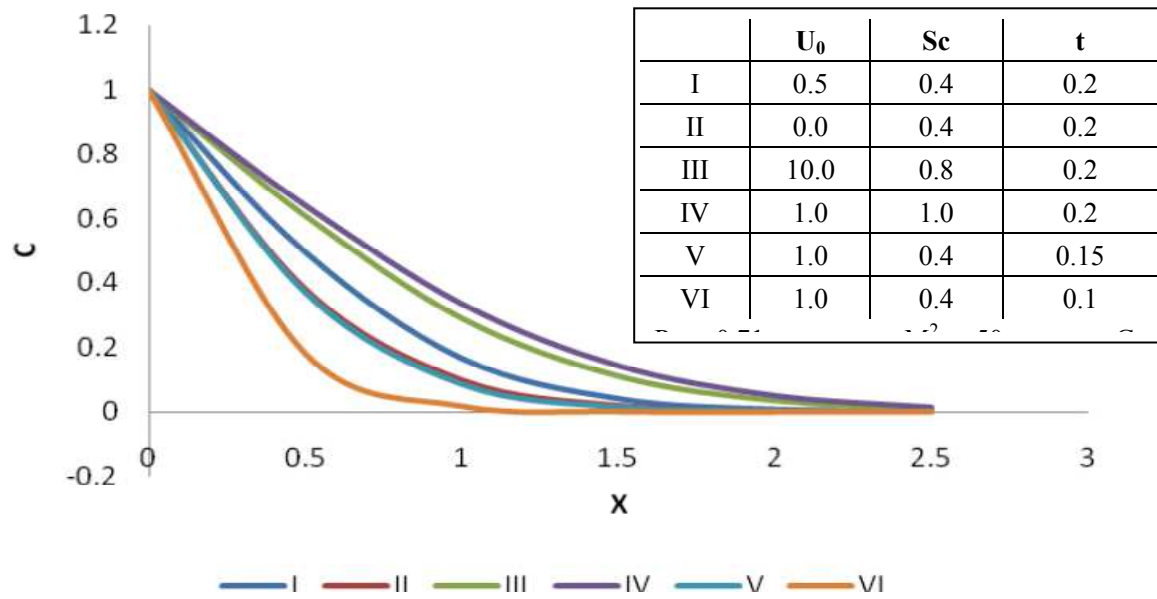


Figure 6: Concentration Profiles.

Table 1: Skin Friction τ_y and τ_z . $Pr = 0.7$, $M^2 = 5.0$ and $Gr = 5.0$ (Cooling of the Plate)

m	Ec	Er	t	τ_y	τ_z
1.0	0.01	1.0	0.4	1.690614	3.829907
50.0	0.01	1.0	0.4	0.55476	3.474412
100.0	0.01	1.0	0.4	0.548994	3.462327
1.0	0.02	1.0	0.4	1.685874	3.830644
1.0	0.05	1.0	0.4	1.671635	3.832856
1.0	0.01	1.5	0.4	2.728611	5.254831
1.0	0.01	2.0	0.4	4.070204	6.468513
1.0	0.01	1.0	0.2	8.672809	1.433269
1.0	0.01	1.0	0.1	14.11346	0.496905

Table 3: Rate of heat transfer, Nusselt number Nu.

m	Ec	Er	t	Nu
1.0	0.01	1.0	0.4	-0.66234
50.0	0.01	1.0	0.4	-0.6545
100.0	0.01	1.0	0.4	-0.65447
1.0	0.02	1.0	0.4	-1.35091
1.0	0.05	1.0	0.4	-3.41602
1.0	0.01	1.5	0.4	-0.70427
1.0	0.01	2.0	0.4	-0.75881
1.0	0.01	1.0	0.2	-0.85075
1.0	0.01	1.0	0.1	-0.81331

Table 4: Rate of mass transfer, Sh

U_0	Sc	t	Sh
0.5	0.4	0.2	5.469969
0.0	0.4	0.2	8.333624
0.5	0.8	0.2	3.767211
0.5	1.0	0.2	3.402
0.5	0.4	0.15	8.421213
0.5	0.4	0.1	14.12577

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