Study of Average Losses Caused by Ill-Processing in a Production Line with Immediate Feedback and Multi Server Facility at Each of the Processing Units

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Abstract
In this paper, we have modeled a production line consisting of an arbitrary number of processing units arranged in a series. Each of the processing units has multi-server facility. Arrivals at the first processing unit are according to Poisson distribution and service times at each of the processing units are exponentially distributed. At each of the processing units, the authors have taken into account immediate feedback and the rejection possibility. Taking into account the stationary behavior of queues in series, the solution for infinite queuing space have been found in the product form. Considering the processing cost at each of the processing units, the average loss to the system due to rejection, caused by ill processing at various processing units, is obtained.
Keywords: Queuing Network, Processing Units, Production Line, Multi-Server, Immediate Feedback, Stationary behavior.

1. Introduction

A production line is a sequence of a finite number of processing units arranged in a specific order. At each of the processing units, service may be provided by one person or one machine that is called single- server facility, or it can be provided by more than one persons or more than one machines that is called multi-server facility at the respective processing unit. In this paper we have considered multi-server facility at each of the processing units. At each of the processing units a specific type processing is performed i.e. at different processing units material is processed differently. At a processing unit the processing times of different jobs or materials are independent and are exponentially distributed around a certain value, called mean processing time. To estimate the required measures, we represent a production line by a serial network of queues with multi- server facility at each of the node.

Several researches have been considered the queues in series having infinite queuing space before each servicing unit. Specifically, Jackson had considered finite and infinite queuing space with phase type service taking two queues in series. In [7] has found that the steady state distribution of queue length taking two queues in the system, where each of the two non-serial servers is separately in service. O.P. Sharma [1973] studied the stationary behavior of a finite space queuing model consisting of queues in series with multi-server service facility at each node.

In an production line the processing of raw material starts at the first processing unit. It is processed for a certain time interval at the first processing unit and then it is transferred to the second processing unit for other type of processing, if its processing is done correctly at the first processing unit. This sequence is followed til the processing at the last processing unit is over.

End of processing at each of the processing units give rise to the following three possibilities:

(a) Processing at a unit is done correctly and the job or material is transferred to the next processing unit for other type of processing.
(b) Processing at a unit is not done correctly but can be reprocessed once more at the same processing unit.
(c) Processing at a unit is neither done correctly nor it can be reprocessed at the same processing unit i.e. this job or material is lost, in this situation the job or material is rejected and put into the scrap.

2. Modeling

Let us consider an assembl line consisting of *an* arbitrary number(r) processing units arranged in a series in a specific order. Each of the processing units has multi- server facility.

Let λ = Mean arrival rate to the first processing unit from an infinite source, following Poisson's rule.

μ_i = Mean service rate of an individual server at the i^{th} processing unit having exponentially distributed service times.

s_i = Number of servers at the i^{th} processing unit.

n_i = Number of unprocessed jobs before the i^{th} processing unit waiting for service, including one in service, if any, at any time t.

$p_{i,i+1}$ = Probability that the processing of a job or material at the i^{th} processing unit is done correctly and it is transferred to the $(i+1)^{st}$ processing unit.

$p_{i,i}$ = Probability that the processing of a job or material at the i^{th} processing unit is not done correctly but it can be reprocessed once more, so, it is transferred to the same processing unit for processing once more.

$p_{i,o}$ = Probability that the processing of a job or material at the i^{th} processing unit is neither done correctly nor it remains suitable for reprocessing.

C_i= Processing cost per unit at i^{th} processing unit.

L = Average loss to the system due to rejection of items at various processing units.

$P(n_1, n_2, ...n_r, t)$ = Probability that there are n_1 jobs for processing before the first processing unit, n_2 jobs before the second processing unit, and so on, n_r jobs before the r^{th} processing unit at time t, with $n_i \geq 0 (1 \leq i \leq r)$ and $P(n_1, n_2, ...n_r, t)$=0, if some n_i <0 (because number of jobs cannot be negative).

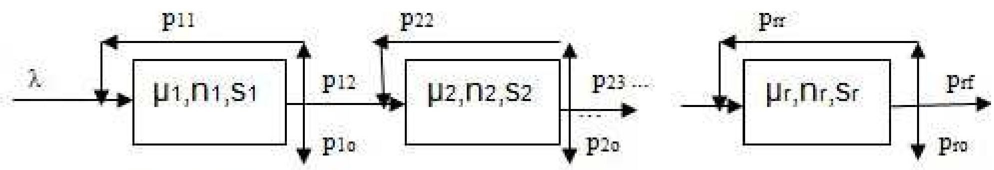

Fig. 1. A Production line

The above production line can be represented by a serial network of queues in which each processing unit is equivalent to a queue with the same number of similar servers and the same numbers of jobs waiting for service. In the above serial network of queues, each queue has immediate feedback. To analyze this serial network of queues firstly we remove the immediate feedback. After the removal of immediate feedback the above serial network of queues is replaced by one as follows:

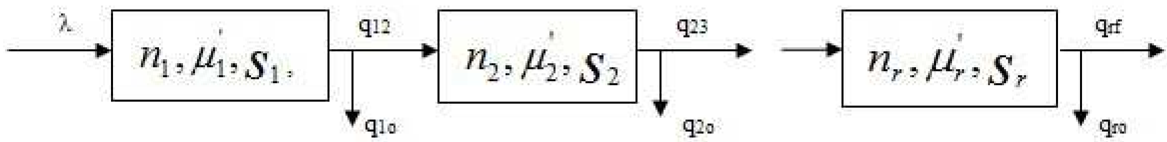

Fig.2 Equivalent Serial Network of Queues

Here $\mu_i' = \mu_i(1-p_{i,i})$, where μ_i' is the effective service at the i^{th} processing unit after the removal of the immediate feedback as given by J.Warland (1988). We define the respective probabilities as follows

$$q_{io} = \frac{p_{io}}{(1-p_{ii})} \quad , \quad q_{ii+1} = \frac{p_{ii+1}}{(1-p_{ii})} \qquad (1)$$

3. Equations Governing the Queuing System

Under the steady state conditions, we have,

$$\left[\lambda + n_1\mu_1' + n_2\mu_2' + \ldots + n_r\mu_r'\right].\mathrm{P}(n_1,n_2,\ldots..n_r)$$

$$= \lambda \;\mathrm{P}\;(n_1-1,n_2n_3,\ldots...n_r) + \sum_{i=1}^{r} n_i\mu_i'.q_{i,i+1}.P(n_1,n_2,\ldots.n_i+1,n_{i+1}-1,\ldots\ldots.n_r) +$$

$$\sum_{i=1}^{r} n_i\mu_i'.q_{i,o}.P(n_1,n_2,\ldots..n_i+1.n_{i+1},\ldots.n_r) \qquad (2)$$

Dividing the above steady state equation by the factor $\left[\lambda + \mu_1' + \mu_2' + \ldots.. + \mu_r'\right]$the above equation is reduced to P.Q=P, where P is the row vector of the steady state probability matrix and Q is the stochastic transition matrix.

4. Solution for Infinite Queuing System

Under the steady state conditions all the queues behave independently and thus the solution of steady state equation in product form is given by

$$\mathrm{P}(n_1,n_2,\ldots\ldots.n_r) = \prod_{i=1}^{r}(1-\rho_i)\rho_i^{n_i} , \qquad (3)$$

where

$n_i \geq 0(1 \leq i \leq r)$ and $\rho_i < 1\;(1 \leq i \leq r)$

If any ρ_i $(1 \leq i \leq r)$>1 then the stability is disturbed and the behavior of the system will not remain stationary consequently solution will not remain valid

Here, we have

$$\rho_i = \frac{\lambda_i}{n_i\mu_i'},$$

Where $\lambda_i = \lambda \prod_{k=1}^{i} \frac{p_{k-1,k}}{(1-p_{k-1,k-1})}, \quad p_{0,0} = 0$

Thus

$$\rho_i = \frac{\lambda}{n_i\mu_i}\prod_{k=1}^{i}\frac{p_{k-1,k}}{(1-p_{k,k})}, \quad \text{With} \quad p_{0,1} = 1 \qquad (4)$$

It can be seen that $\sum_{i=1}^{r}\lambda_i.q_{i,o} + \lambda_r.q_{r,f} = \lambda$ (5)

5. Evaluation of Average Loss

Let c_1 be the processing cost at the first processing unit, c_2 the processing cost at the second processing unit and so on … c_r, the processing cost at the r^{th} processing unit.

If an item is rejected just after its processing at the first processing unit is over, then it causes a loss c_1 to the system. If an item is rejected just after its processing at the second processing unit is over, then it causes a loss (c_1+c_2) to the system. Thus, in general if an item is rejected just after its processing at the r^{th} processing unit is over, then it causes a loss ($c_1+c_2+c_3+\ldots+c_r$) to the system.

L, the average loss per unit time to the system due to rejection of items just after the processing at various processing units, due to ill-processing (processing of an item is neither done correctly nor it can be reprocessed) is

$$L = c_1 \lambda q_{1,o} + (c_1 + c_2) \lambda q_{1,2} q_{2,o} + \ldots + (c_1 + c_2 + \ldots c_r) \lambda q_{1,2} q_{2,3} \ldots q_{r-1,r} q_{r,o}$$

$$= \sum_{i=1}^{r} (c_1 + c_2 + \ldots c_i) \lambda q_{1,2}.q_{2,3}..q_{i-1,i} q_{i,o} ,$$

With $q_{0,1} = 1$

$$= \sum_{i=1}^{r} (c_1 + c_2 + \ldots c_i) \lambda \frac{p_{1,2}}{1-p_{1,1}} \frac{p_{2,3}}{1-p_{2,2}} \ldots \frac{p_{i-1,i}}{1-p_{i-1,i-1}} \frac{p_{i,o}}{1-p_{i,i}} ,$$

$$= \lambda \sum_{i=1}^{r} (c_1 + c_2 + .. c_i) \prod_{k=1}^{i} \left(\frac{p_{k-1,k}}{1-p_{k-1,k-1}} \right) . \frac{p_{i,o}}{(1-p_{i,i})} \quad \textbf{(6)}$$

With $p_{0,0} = 0, and \quad p_{0,1} = 1$

6. Conclusion

The work can be used to find the approximate loss in a manufacturing system and can be extended to make decision policies.

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8. Biography

Abhimanu Singh born on 4th may 1969, got his M.Sc. Degree in mathematics from Ch. Charan Singh University, Meerut, U. P., India, in 1996.

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