

Magnetohydrodynamic Flow over an Immersed Axi-Symmetrical Body with Curved Surface and Heat Transfer

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Abstract

In this paper analysis has been carried out on the effects of thermal radiation on unsteady flow of a Newtonian fluid over an immersed axi-symmetrical body with curved surface in the presence of a uniform transverse magnetic field. The governing equations are made dimensionless and solved by finite difference method. The results for the velocity and temperature profiles are presented graphically along with the different range of dimensionless numbers of magnetic parameter M, Peclet number, Reynold number, Eckert number, Prandtl number etc. These results will have major application in designing bodies requiring high maneuverability and less resistance to the motion e.g. aerofoil and cooling fans.

Keywords: Thermal radiation, curved surface, MHD, Heat transfer

1. Introduction

Today heat Exchangers are widely used as an important component in heat transfer area. Depending on the application of heat exchangers in the industry, several methods are used for increasing the heat transfer with a reasonable pressure drop. These are very significant types of flow encountered in several engineering applications, such as in polymer processing, gas turbines, various propulsion device for aircraft, space vehicles, MHD power generators, flight magnetohydrodynamics as well as in the field of planetary magnetosphere, aeronautics and chemical engineering.

The study of temperature distribution and heat transfer is of great importance to all branches of engineering and science for its almost universal occurrence. If the temperature of the surrounding fluid becomes high, then the thermal radiation effect is significant in the case of space technology. G.C. Shit and R. Haldar (2012) studied combined effects of thermal radiation and hall current on MHD free-convective flow and mass transfer over a stretching sheet with Variable Viscosity and concluded that increasing the thermal radiation parameter produces a decrease in the temperature of the fluid. This is because of the fact that the thermal boundary-layer thickness decreases with increasing the thermal radiation parameter. Kishore *et al* (2010) analyzed effects of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past a surface embedded in porous media. Mostafa (2009) analyzed the influence of radiation and temperature dependent viscosity on the problem of unsteady MHD flow and heat transfer past an infinite vertical porous plate.

Barenblatt *et al* (2002) in their study on the model of the turbulent boundary layer with non- zero pressure gradient observed that the turbulent boundary layer at large Reynolds number consist of two separate layers upon which the structure of the vorticity fields is different, although both exhibit similar characteristics. In the first layer, vertical structure is common to all developed bounded shear flows and the mean flows .The influence of viscosity is transmitted to the main body of flow via streaks separating the viscous sub layer. The second layer occupies the remaining part of the intermediate region of the boundary layer. The upper boundary of the boundary layer is covered with statistical regularity by large scale "humps" and the upper layer is influenced by the external flow via the form drag of these humps as well as by the shear stress. In earlier works it is shown that the mean velocity profile is affected by the intermittency of the turbulence and as the humps affects intermittency, the two seeking regions are visible. On the basis of these considerations, the effective Re, which determines the flow structure in the first layer (and is affected in turn by the viscous sub layer), was identified as one set of such parameters. The other parameters that influence the flow in the upper layer include pressure gradient∂_xP; dynamic (friction) viscosityμ; velocity u; fluid's kinematic viscosityϑ and density.

In recent past, many researchers have been attracted to solving the boundary layer equations. Smith *et al* (1970) in one of their papers presented a method for solving the complete incompressible laminar boundary layer equations; both for two dimensional and axi- symmetrical laminar flow, essentially full generality and with speed.

Omboro G.O. (2009) in his study on the convection heat transfer in a fluid flow over a curved surface established that as fluid flows over an immersed curved surface, some work is done against viscous effects and energy spent is converted into heat and also vorticity formed in the boundary layer due to high velocity gradient is swept outwards from the boundary layer. Mugambi K.E (2008) in their research, an investigation of forces produced by fluid motion on a submerged finite curved plate, established the relationship between geometrical shape of the curvature and the variation of drag force of specific velocities of the viscous fluid. Prandtl (1904) studied convective heat transfer in homogenous fluid flow of Reynolds number of order less than 2000 over an immersed axi-symmetrical body with curved surface. In their study, it was noted that when Reynolds number was increased, the dissipation also increased. When the curvature of the surface was increased, the heat dissipation also increased.

In this study, effects of thermal radiation in a magnetohydrodynamics flow over an immersed axi-symmetrical body with curved surface has been investigated. The study is aimed to determine the velocity distribution, temperature variation within the thermal boundary layer of MHD and the effect of heat generated within the boundary of an immersed axi-symmetrical body with curved surface. The MHD flow in consideration is unsteady and the fluid is assumed to be of constant density. Convective heat transfer is caused by different temperature profiles which bring about temperature gradient. These temperature profiles are brought about by frictional forces on and within the surface of the body when fluid flows over it.

2.0 Mathematical Formulation

Consider convective heat transfer in magnetohydrodynamics over an immersed axi-symmetrical body with curved surface. A stationary curved body is immersed in an ambient fluid with surface temperature which is the same as the surrounding fluid. The flow of an electrically conducting fluid is horizontal along x- axis. There is application of constant magnetic field along the y-axis. Radiation is applied along y axis and neglected anywhere else. The hydrodynamic flow field is axi-symmetrical and the fluid possesses constant thermo physical property with the exceptional of those caused by density changes which generate the buoyancy forces. The energy converted into heat within the boundary layer is transferred from this boundary layer through convection into the rest of the region.

Equations governing the flow of an incompressible, Newtonian fluid over an axi symmetrical body with curved surface are discussed.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} = P_t + \vartheta \frac{\partial^2 u}{\partial y^2} + K_r u^2 - \frac{\sigma u B_0^2}{\rho} \quad (2)$$

$$\frac{\partial u}{\partial t} = P_t + \vartheta \frac{\partial^2 u}{\partial y^2} + \frac{K_r u^2}{h_1} \quad (3)$$

$$C_p \rho \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + K\left(1 - \frac{1}{4}\frac{K_r u}{\left(\frac{\partial u}{\partial y}\right)}\right) A(T_\infty - T_S) + \sigma B_0^2 u^2 - \frac{\partial q_r}{\partial y} \quad (4)$$

where, u, v are the velocity components along x, y coordinates respectively, ρ is the density of the fluid, $k_p$is the permeability of the porous medium, μ is the coefficient of the dynamic viscosity, T is the temperature of the fluid in the boundary layer. By using Rosseland approximation, qr takes the form

$$q_r = -\frac{4\sigma^* \partial T^4}{3k^* \partial y} \quad (5)$$

Where $k^*$is the represents absorption coefficient

The assumption in this study is that the temperature difference within the flow is assumed to be sufficiently small and that the term T^4 equation (5) can be expressed as a linear function of the temperature which is usually truncated Taylor series about the free stream temperature T_∞. The orders of T and T_∞ are assumed to be more or less equal. So that any product of the two respectively temperatures whose order is higher than four is neglected. where $P_t = C^2 m x^{2m-1}$, $dT = (T_\infty - T_s)$ is the temperature difference between the surface and the bulk fluid, K_r *and* h_1 are curvature parameters which are defined as

$$K_r(x) = -\frac{1}{r(x)} \quad (6)$$

$$h_1 = 1 + k_r y \quad (7)$$

Boundary and initial conditions

$$u(t,0) = 0, u(t,\infty) = U_\infty, u(0,y) = 0 \quad (8)$$

Introducing the following non dimensions parameters

$$x = x^* H,\ u = u^* U_\infty, y = y^* H,\ v = v^* U_\infty,\ P = P^* P,\ T^* = \frac{T - T_S}{T_\infty - T_S}\ ,\ t^* = \frac{t}{H} U_\infty \ \text{ or } \ t = \frac{t^* H}{U_\infty} \quad (9)$$

Equation (2), (3) and (4) in non-dimensional form

$$\frac{\partial u^*}{\partial t^*} = \frac{HP}{U_\infty{}^2} P_t^* + \frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}} + K_r H u^{*2} - M u^* \quad (10)$$

$$\frac{\partial u^*}{\partial t^*} = \frac{HP}{U_\infty{}^2} P_t^* + \frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}} + K_r H u^{*2} \quad (11)$$

$$\frac{\partial T^*}{\partial t^*} = \frac{1}{Pe}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{Ec}{Re}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{H^2 A}{Pe}\left(1 - \frac{1}{4}\frac{K_r u^* H}{\left(\frac{\partial u^*}{\partial (y^*)}\right)}\right) + R u^{*2} H - \frac{16}{3NPrRe}\left(\frac{\partial^2 T'}{\partial y'^2}\right) \quad (12)$$

Where $M = \frac{\sigma B_0^2 H}{\rho U_\infty}$,$Re = \frac{\rho H U_\infty}{\mu} = \frac{U_\infty H}{\vartheta}$, $Pe = RePr = \frac{\rho U H C p}{K} = \frac{U_\infty H}{\alpha}$, $Pr = \frac{\vartheta}{\alpha} = \frac{\mu\rho}{K\rho Cp} = \frac{Cp\mu}{K}$, $Ec = \frac{U_\infty{}^2}{C_p(T_\infty - T_S)}$ and

$R = \frac{\sigma U_\infty B_0^2}{\rho C_p \Delta t}$, $N = \frac{k^* k_f}{4\sigma^* T_\infty^3}$ is the radiation parameter

boundary and initial conditions in non dimension form

$$u^*(t^*,0) = 0, u^*(t^*,\infty) = 1, \qquad u^*(0,y^*) = 0$$

$$T^*(t^*,0) = 0,\ T^*(t^*,\infty) = 1,\ T^*(0,y^*) = 0 \quad (13)$$

2.1 Method of Solution

The numerical solutions of the problem are obtained by solving the non-linear differential equations (10) and (11) subject to (12) using finite difference method in finite difference form and making $U_{i,j}^{k+1}$ and $T_{i,j}^{k+1}$ subject

$$u^*_{i+1,j} = u^*_{i,j} + \frac{\Delta tHP}{U_\infty{}^2}P_t + \frac{\Delta t}{Re}\left[\frac{u^*_{i,j+1} - 2u^*_{i,j} + u^*_{i,j-1}}{(\Delta y)^2}\right] + \Delta tK_rHu^*_{i,j}{}^2 - \Delta tMu^*_{ij} \quad (14)$$

$$u^*_{i+1,j} = \left\{u^*_{i,j} + \frac{\Delta tHPP_t^*}{U_\infty{}^2} + \frac{\Delta t}{Re}\left[\frac{\{u^*_{i+1,j+1} + u^*_{i+1,j-1}) + u^*_{i,j+1} - 2u^*_{i,j} + u^*_{i,j-1}\}}{2(\Delta y)^2}\right] + \Delta tK_rHu^*_{(i,j)}{}^2 - \Delta tMu^*_{ij}\right\} \div$$

$$\left(1 + \frac{\Delta t}{Re(\Delta y)^2}\right) \quad (15)$$

$$T^*_{i+1,j} = T^*_{i,j} + \frac{\Delta t}{Pe}\left[\frac{T^*_{i+1,j-1} + T^*_{i+1,j+1} + T^*_{i,j+1} - 2T^*_{i,j} + T^*_{i,j-1}}{2(\Delta y)^2}\right] + \frac{\Delta tEc}{Re}\left[\frac{u^*_{i+1,j+1} - u^*_{i+1,j} + u^*_{i,j+1} - u^*_{i,j}}{2\Delta y}\right]^2 + \frac{\Delta tH^2A}{Pe} -$$

$$\left[\frac{K_ru^*_{ij}AH^3\Delta t}{4Pe\left(\frac{u^*_{i+1,j+1} - u^*_{i+1,j} + u^*_{i,j+1} - u^*_{i,j}}{2\Delta y}\right)}\right] + \Delta tRu^*_{ij}{}^2H - \frac{8}{3NPrRe(\Delta y)^2}\left(T^*_{i+1,j-1} + T^*_{i+1,j+1} + T^*_{i,j+1} - 2T^*_{i,j} +\right.$$

$$\left.T^*_{i,j-1}\right) \div \left(1 + \frac{\Delta t}{Pe(\Delta y)^2} + \frac{16}{3NPrRe(\Delta y)^2}\right) \quad (16)$$

3.0. Results and Discussion

To study the behavior of the velocity and temperature profiles, curves are drawn for various values of the parameters that describe the flow.

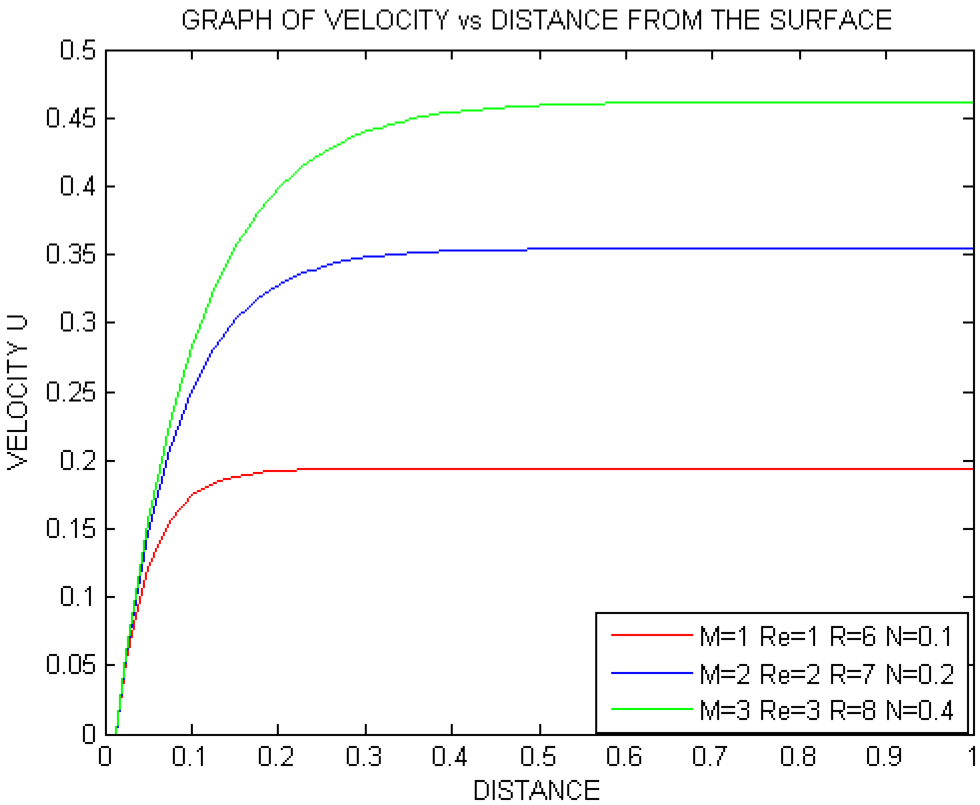

Figure 1: velocity profiles for different values of magnetic number, Reynolds number, joule heating Parameter and Radiation parameter .

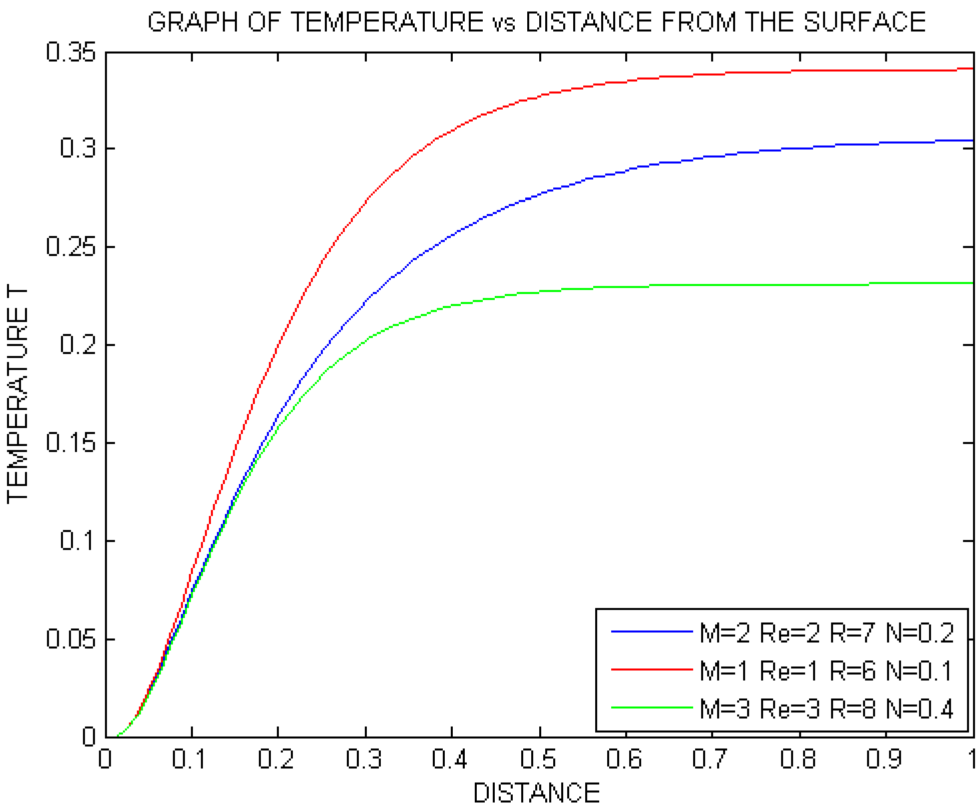

Figure 2: Temperature profiles for different values of magnetic number, Reynolds number, joule heating Parameter and Radiation parameter

Figure 1 and figure 2 shows that an increase in magnetic field parameter M causes a decrease in magnitude of temperature and velocity profiles. This implies that increase in M has a tendency to slow down the velocity of the fluid. Decreasing the velocity of the fluid slows down the movement of the species. Application of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive type of force called Lorentz force. This force has a tendency to slow down the motion of the fluid in the boundary layer. The temperature profiles decrease with increase in M. The reduced velocity by the frictional drag due to the Lorenz force is responsible for reducing thermal viscous dissipation in the fluid leading to a thinner thermal boundary layer. Magnetic field therefore can be used to control the velocity and temperature boundary layer characteristics.

Increase in Reynolds number causes a decrease in the magnitude of velocity and temperature profiles respectively. Re represents the ratio of inertial to viscosity forces. Increase in Re results into a larger inertia that in turn translates to lower velocities.

From the figure 2 above, increase in the joule heating parameter R leads to increase temperature profiles. Increase in joule heating parameter leads to the heating of the fluid thereby boosting the velocity of the convection currents on the surface of the sheet. Radiation parameter has no effect on velocity profile however an increase in N causes a decrease in temperature profile.

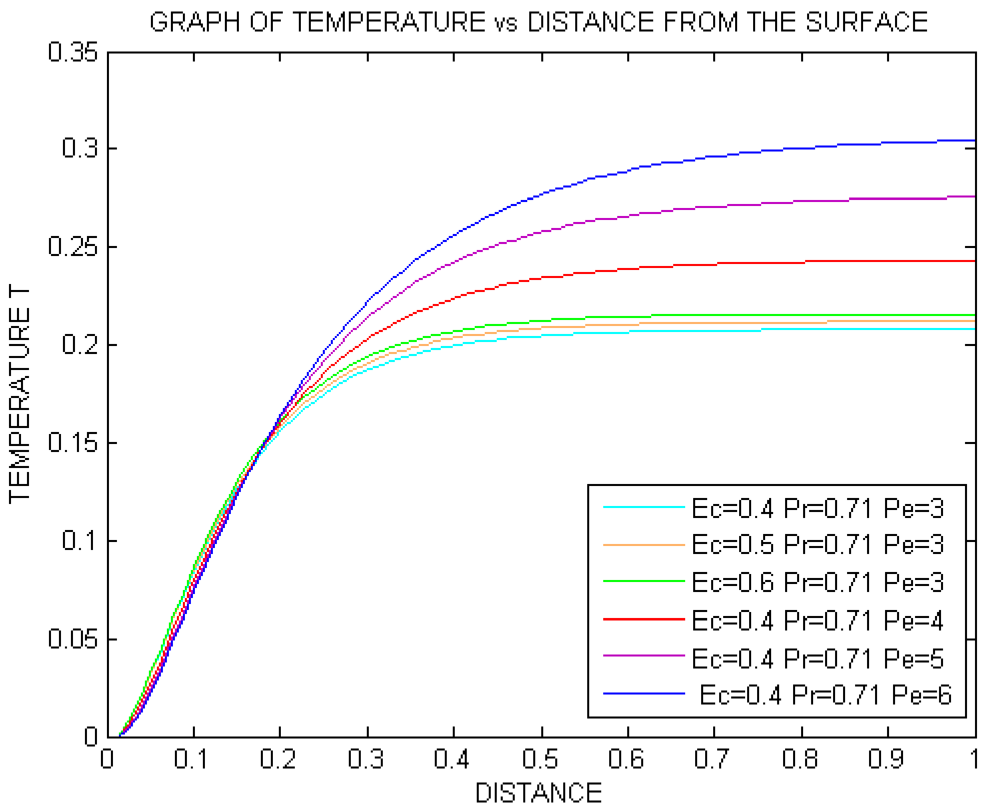

Figure 3: temperature profiles with varying Eckert and peclet numbers

In the figure 3 above, it shows that increase in Eckert number causes an increase in temperature profiles. Increase in Ec leads to an increase in magnitude of velocity profiles. Thus increase in Ec number boosts both the velocity and the temperature of a fluid. The Eckert number expresses the relationship between kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against viscous fluid stresses. A positive Eckert number implies cooling the sheet, implying heating the fluid. This causes a rise in temperature and the velocity of the fluid. Therefore, introducing radiation can be used to control velocity and temperature boundary layers.

In the figure 3 above, increase in Pe leads to an increase in temperature profile and a decrease in Pe leads to a low temperature profile in the boundary layer region. This was attributed by the fact that large Pe lead to an increase in velocities. The increased velocity of the fluid indicates an increase in fluid particles collisions which in turn causes an increase in amount of heat dissipated.

The effect of convective heat transfer on drag and lift is also discussed. The ratio of the shear stress $\tau_{\circ}$ to the quantity $\frac{1}{2}\rho U^2$ is known as local coefficient of drag, or local skin friction, denoted as $C_D = \frac{\tau_{\circ}}{\frac{1}{2}\rho U^2}$. The ratio of total drag force to the quantity is called average coefficient of drag

$\boldsymbol{C_D} = \frac{\boldsymbol{F_D}}{\frac{1}{2}\boldsymbol{\rho A U^2}}$ where $\boldsymbol{\rho}$ is the density of the fluid, A is the area of the surface of the body, U is the free stream velocity. From the above $\boldsymbol{F_D} = \boldsymbol{C_D}\frac{1}{2}\boldsymbol{\rho A U^2}$ as the formula for drag. The formula for lift is given by $\boldsymbol{L} = \boldsymbol{C_L}\frac{1}{2}\boldsymbol{A\rho U^2}$ where L is lift and $\boldsymbol{C_L}$ is the coefficient of lift. For symmetrical bodies the drag coefficient is 0.04 and the lift coefficient is 0.2. The convective heat transfer affects the fluid flowing around the body by varying the velocity of this fluid and hence affects the lift and drag.

3.1 Conclusion

The analysis of various parameters on unsteady MHD laminar boundary layer flow of an incompressible, electrical conducting fluid past an immersed axi-symmetrical body with curved surface is carried out. The direction of the applied magnetic field is considered to be normal to the direction of the flow. The equations governing the flow are highly non-linear and have been solved by finite difference method. The results obtained show that convective heat transfer in electrical conducting fluid is influenced by magnetic field parameter, Reynolds number, Peckert number, curvature of the body, Eckert number and joule heating parameter.

This study has shown that imposing a transverse magnetic field to a flow slows down the velocity of the fluid. Decreasing the velocity of the fluid slows down the movement of the species thus decrease in frictional drag. Increase in Reynolds number causes a decrease in the magnitude of velocity and temperature profiles respectively. Re represents the ratio of inertial to viscosity forces. Increase in Re results into a larger inertia that in turn translates to lower velocities. When Re is large the inertia forces dominates over the viscous forces. This leads to reduced velocity in the boundary layer. When Re is small it implies that the viscous forces are dominated and hence temperature dissipation in the boundary layer occurs due to increased friction, hence increased drag. Due to this dissipation in the boundary layer, this results to decreased density of the fluid hence reduced lift.

Increase in Pe leads to an increase in temperature profile and a decrease in Pe leads to a low temperature profile in the boundary layer region.

It is realised that increase in Eckert number causes an increase in velocity profiles as well as temperature profiles. Thus increase in Ec number boosts both the velocity and the temperature of a fluid. The Eckert number expresses the relationship between kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against viscous fluid stresses. A positive Eckert number implies cooling the sheet, implying heating the fluid. This causes a rise in temperature and the velocity of the fluid. Therefore, introducing radiation can be used to control velocity and temperature boundary layers.

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Nomenclature

Romans Symbol	**Quantity**
A	Area of the curved surface
B	Magnitude of magnetic field
$\mathbf{C_p}$	Specific heat at constant pressure. Jkg^{-1} K^{-1}
E	External work done
J	Current
L	Reference length, m
P	Pressure, Pa
$\mathbf{E_c}$	Eckert number
Pr	Prandtl number
Pe	Peclet number
Re	Reynolds number
q	Quantity of heat added to the system, Joules (J)

$\mathbf{q_s}$	Local wall heat flux, W/m^2
T	Temperature, K
$\mathbf{T_s}$	Temperature of the body's surface, K
$\mathbf{T_\infty}$	Free stream velocity, m/s
h	Heat transfer coefficient. h= $q_w(T_w-T_\infty)$, W/m^{2}K
u	Outer flow fluid velocity in the x-direction, ms^{-1}
v	Reference fluid velocity in the y-direction, ms^{-1}
w	Fluid velocity in z-direction, ms^{-1}
W	Work done by the system
$\mathbf{F_x, F_y}$	Body forces along the x and y directions respectively
x,y,z	Cartesian co-ordinates
i,j,k	Unit vectors in the x,y and z directions respectively
$\frac{D}{Dt}$	Material derivative $\left(=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)$
$\vec{\nabla}$	Gradient operator $\left(i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right)$
∇^2	Laplacian operator $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$

Greek Symbols

$\boldsymbol{\vartheta}$	Kinematic viscosity, $\vartheta=\frac{\mu}{\rho}$ m^{2}s^{-1}
$\boldsymbol{\mu}$	Absolute viscosity (dynamic viscosity coefficient), kg/ms
$\boldsymbol{\rho}$	Fluid density, kgm^{-3}
$\boldsymbol{\varphi}$	Viscous dissipation function
$\boldsymbol{\delta}$	Boundary layer thickness, m
$\boldsymbol{\sigma_{i,j}}$	Fluid stresses, Nm^{-2}