

On random coincidence points and random best approximation in p-normed spaces

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Abstract:

In this paper random coincidence point results are proved for pair of commuting mapping defined on weakly compact separable subset of complete p-normed space. And then , use them to study the random best approximation in p-normed space with separablity condition.

Keywords : p-normed space ,random coincidence point ,random best approximation.

1. Introduction:

The well known best approximation theorems have been of importance in nonlinear functional analysis minmax theory , game theory ,…els. The calassiced approximation problem can be described as :

"Let A be a subspace of a normed linear space X ,$x \in X$,a best approximation to x from the elements of A would be a vector $y \in A$ such that $\|x - y\|$ =dist(x, A)[Singh 1997].

The early problems of best approximation theory like Ky-Fan' s theorem and Prolla's theorem [Singh 1997]depends on convexity properties which involve introducing a mapping with some hypotheses. These deals with Brosowski-Meinardus type [Singh 1997] .

Fixed point and coincidence point theorems have been used at many places in approximation theory. One of them is proving the existence of best approximation. It is well known that random fixed point and (coincidence) theorems are stochastic generalization of classical fixed point and (coincidence)theorems that are known as deterministic results. Initially, the study of random fixed points into being by the prague school of probabilistic in 1950. In 1955 , Spacek proved firstly, existence of random fixed point for contraction mappings in separable complete metric space and then Han's [1957,1961] gave other results. In 1979 , Itohs extended Spacek's and Hans's results to multivalued contraction mappings . on the other hand ,Mukherjce [1966] gave a random version of schadner's fixed point theorem in atomic probability measure space and Bharucha-Reid [1972,1976] generalized these results in general probability measure spaces. Also Beg and Shanzad [1993,1994] proved some theorems about common fixed points and coincidence points for pair of compatiable random mappings which are generalization for commuting random mappings.

This work was extended by Itoh [1979] for the existence of coincidence point of commuting random mappings defined on separable weakly compact subset of banach space. Recently , some papers about random fixed points and random coincidence points have been established such as Jhadd[1]and Salua[2][2014] and Nashine[*][2008].

As applications of random fixed point theorems , Itoh[1979] represented a random solutionof a differential equation in banach space. And ,Nashine [2010] proved random best approximation result.

In this paper , we give some results about random coincidence points which extending the previous work by Itoh[1979],Xu [1990] and Shahzad and Laitf[2000].

2.Preliminaries

We need the following definitions and facts:

Let X be a linear space and $\| \|_p$ be a real valued function on X with $0 < p \leq 1$. The pair (X, $\| \|_p$) is called a p-normed space if for all x , y in X and scalars λ :

i. $\|x\|_p \geq 0$ and $\|x\|_p = 0$ iff $x = 0$

ii. $\|\lambda x\|_p = |\lambda|^p \|x\|_p$

iii. $\|x + y\|_p \leq \|x\|_p + \|y\|_p$

Every p-normed space X is a metric space with $d_p(x, y) = \|x - y\|_p$,for all x , y in X . If $p = 1$,we have the concept of a normed space[Nashine*(2008)]. For examples of p-normed space see [Nashine 2006]. A p-normed space is not necessarily locally convex space .And the continuous dual $X^`$ of p-normed space X need not separate the point of X[Nashine* 2008]. Throughout this paper X will be complete p-normed space whose dual separates the points of X .The following classes are needed

2^X is the classes of all subsets of X ,

CB(X) is the classes of all bounded closed subsets of X ,

CD(X) is the classes of all nonempty closed subsets of X,

K(X) is the classes of all nonempty compact subsets of X ,

$K_w(X)$ is the classes of all nonempty weakly compact subsets of X

$\bar{A}$ is the closure of a set A ,

$w-\overline{(A)}$ is the weakly closure of a set A .

Now , for any A,B in CB(X) the Hausdorff distance defined by

$\mathcal{D}(A,B)=\max\{\sup_{a\in A} d(a, B),\sup_{b\in B} d(A, b)\}$

where $d(a, B)= \inf_{b\in B}\left\{\|a - b\|_p\right\}$[Zeidler 1986].Let A be a subset of X for $x_\circ \in X$, define the set $p_A(x_\circ) = \left\{z \in A: \|x - z\|_p = d_p(x, A)\right\}$,then an element $z \in p_A(x_\circ)$ is called a best approximation of $x_\circ$ in A [Nashine 2006].

The space X is said to be opial if for every sequence $< x_n >$ in X , $x_n \to x$ weakly as $n \to \infty$ and $x \neq y$ implying that

$\lim_{n\to\infty} \inf \|x_n - x\|_p \leq \lim_{n\to\infty} \inf \|x_n - y\|_p$, for all $y \in X$ [Nashine 2006].The mapping $G: A \to CB(X)$ is said to be multivalued demiclosed at $y_\circ$ if the conditions $x_n \in A, x_n \to x$ weakly , $y_n \in Gx_n$,$y_n \to y$ strongly imply The $y \in Gx$ [Shahzad 2005] . A mapping $h:A \to X$ is called affine if

$h(\lambda x + (1 - \lambda)y) = \lambda hx + (1 - \lambda)hy$ for all $x, y \in A$ and $0 < \lambda < 1$[Shahzad 2004] . A mapping $G: A \to CB(X)$ is called h-nonexpansive if $\mathcal{D}(G_x, G_y) \leq d(hx, hy)$for any $x, y \in A$ where $h: A \to X$ [Shahzad 2004].The pair { h, G } is called commuting if $hGx = Ghx$ for all $x \in A$ [Shahzad 2005] .

A set A is called starshaped if there exists at least one point q$\in$ A such that $kx + (1 - k)q \in A$for all $x \in A$ and $0 \leq k \leq 1$. In this case q is called the starcenter of A [Nashine 2006].

Now , let the pair (Ω,Σ) be denote to the measurable space with Σ a sigma algebra of subsets of Ω .

Definition (2.1)[Jhadd[1] and Salua[2] 2014]

A multivalued mapping $F: \Omega\to CB(X)$ is called (Σ-) measurable if, for any open subset B of X , $F^{-1}(B) = \{\omega \in \Omega: F(\omega) \cap B \neq \emptyset\} \in \Sigma$.

Definition (2.2) [Jhadd[1] and Salua[2] 2014]

A single mapping $\delta : \Omega \to X$ is called a measurable selector of a multivalued measurable mapping $F:\Omega \to CB(X)$ if δ measurable and $\delta(\omega) \in F(\omega)$ for each $\omega \in \Omega$.

Definition (2.3) [Jhadd[1] and Salua[2] 2014]

A mapping $h: \Omega \times X \to X$ (or $G: \Omega \times X \to CB(X)$) is called a random operator if for any $x \in X, h(.,x)$ (respectively G(. ,x)) is measurable .

Definition (2.4) [Shahzad and Latif 2000]

A measurable mapping $\delta: \Omega \to A$ is called random fixed point of a random operator $h: \Omega \times X \to X$ (or $G: \Omega \times X \to CB(X)$ if for every $\omega \in \Omega$, $\delta(\omega) = h(\omega, \delta(\omega))$ (respectively $\delta(\omega) \in G(\omega, \delta(\omega))$.

Definition (2.5) [Jhadd[1] and Salua[2] 2014]

A measurable mapping $\delta: \Omega \to X$ is called random coincidence point of a random operator $h: \Omega \times X \to X$ and $G: \Omega \times A \to CB(X)$ if

$h(\omega, \delta(\omega)) \in G(\omega, \delta(\omega))$ for all $\omega \in \Omega$.

Definition (2.6) [Shahzad 2004]

A random operator $h: \Omega \times A \to X$ is called continuous (weakly continuous) if for each $\omega \in \Omega, h(\omega, .)$ is continuous (weakly continuous).

Definition (2.7) [Shahzad and Latif 2000]

A random operator $G: \Omega \times X \to CB(X)$ is said to be demiclosed if for each $\omega \in \Omega$, $G(\omega, .): X \to CB(X)$ is demiclosed.

Definition (2.8) [Shahzad and Latif 2000]

A random operator $h: \Omega \times X \to X$ is said to be affine if for each $\omega \in \Omega$, $h(\omega, .): X \to X$ is affine .

Definition (1.2)

Let X be a p-normed space , $A \subseteq X$ and and $G: \Omega \times X \to CB(X)$ be a multivalued random operator we say that A has property (P_1) if

i. $G : \Omega \times A \to CB(A)$
ii. $(1 - k_n)q + k_n G(\omega, x) \subseteq A$, for some $q \in A$ and a fixed real sequence $< k_n >$ converging to 1 and for each $x \in A$ and for each $\omega \in \Omega$.

Definition (1.3)

Let X be a p-normed space , $A \subseteq X$ and A has property (P_1) with respect to a random operator $G: \Omega \times X \to CB(X)$, $q \in A$ and sequence $< k_n >$ .A random mapping $h: \Omega \times X \to X$ is said to be have property (P_2) on A with property (P_1) if

$$h(\omega, (1 - k_n)q + k_n G(\omega, x)) = (1-k_n) h(\omega, q) + k_n h(\omega, G(\omega, x))$$

for all $x \in A$ and $n \in N$ and $\omega \in \Omega$.

3.Main results

Shahzad and Latif [2000] prove the fixed point theorems for multivalued random mappings .

Theorem (3.1)[Shahzad and Latif 2000]

Let (X ,d) be a separable complete metric space, G : $\Omega\times X \to CB(X)$ a multivalued random operator, and h : $\Omega \times X \to X$ a continuous random operator such that G $(\omega, X) \subset h(\omega, X)$ for each $\omega \in \Omega$ .If G and h commute and for all $x, y \in X$ and all $\omega \in \Omega$, we have $\mathcal{D}(G(\omega,x),(\omega,y)) \leq k\, d(h(\omega,x),h(\omega,y))$, where $k \in (0,1)$ and H is the Hausdorff metric on CB(X) induced by the metric d, then h and G have a random coincidence point .

Theorem (3.2)

Let A be a nonempty separable weakly compact subset of X has property (P_1) with respect to G , h: $\Omega \times A \to A$ be a continuous random mapping and G: $\Omega \times A \to K(A)$ be an h-nonexpansive multivalued random mapping with properties (P_2) . If G and h are commute , $(h-G)(\omega,.)$ is demiclosed at 0 for all $\omega \in \Omega$ and $h(\omega, A) = A$, $h(\omega,q) = q$ for all $\omega \in \Omega$ then h and G have a random coincidence point .

Proof :

Since A has property (P_1) with respect to G, then for some $q \in A$,we get

$(1-k_n)\, q+k_n\, G(\omega,x) \subseteq A$, where $\lim_{n\to\infty} k_n = 1 (0 < k_n < 1)$,for each $x \in A$ and for each $\omega \in \Omega$.

For each n , define the random mapping G_n by

$$G_n(\omega,x) = (1-k_n)q + k_n G(\omega,x) \text{ , for all } x \in A \text{ and all } \omega \in \Omega .$$

We have , $G_n : \Omega \times A \to K(A)$.

since G is h-nonexpansive ,then

$$\mathcal{D}(G_n(\omega,x),G_n(\omega,y)) = \mathcal{D}[(1-k_n)q + k_n G(\omega,x),(1-k_n)q + k_n G(\omega,y)]$$
$$= (k_n)^p \mathcal{D}\big(G(\omega,x),G(\omega,y)\big)$$
$$\leq (k_n)^p d_p(h(\omega,x),h(\omega,y))$$

For all $x , y \in A$ and all $\omega \in \Omega$. Therefore each G_n is a random h-contraction .

Since $G(\omega,A) \subseteq A = h(\omega,A)$

Then for each n , $G_n(\omega,A) \subset A = h(\omega,A)$, for all $\omega \in \Omega$.

Since h commute with G, $h(\omega,q) = q$ and h has property (P_2) , then

$$G_n(\omega,h(\omega,x)) = (1-k_n)q + k_n G(\omega,h(\omega,x))$$
$$= (1\text{-}k_n)h(\omega,q) + h(\omega,G(\omega,x))$$
$$= h(\omega,(1-k_n)q + k_n G(\omega,x))$$
$$= h(\omega,G_n(\omega,x))$$

For all $x \in A$ and all $\omega \in \Omega$ .Hence each G_n commute with h .

Since A is separable and weakly compact ,then the weak topology on A is a metric topology[Dunford 1958 , p.434] which implies that A is hausdorff [Körner 2015,p.25] then A is strongly closed , this implies A is a complete metric space [Kreyszing 1978,p30] .Thus all the conditions of Theorem (3.1) are satisfied .

Therefore there is a measurable mapping $\delta_n: \Omega \to A$ such that $h(\omega, \delta_n(\omega)) \in G_n(\omega, \delta_n(\omega)$ for each $\omega \in \Omega$.

Now ,for each n define $Q_n: \Omega \to k_w(A)$ by $Q_n(\omega) = w - \overline{\{\delta_\imath(\omega): \imath \geq n\}}$.

Define $Q: \Omega \to k_w(A)$ by $Q(\omega) = \cap_{n=1}^{\infty} Q_n(\omega)$.

Since A is complete p-normed space ,then by Theorem (4.1) [Himmelberg 1975] Q is weakly measurable so Q has measurable selector δ [Kuratowski and Ryll 1965] .

By weak compactness of A , we have that a subsequence $\{\delta_m(\omega)\}$ of $\{\delta_n(\omega)\}$ converges weakly to $\delta(\omega)$.Also, by definition of $G_m(\omega, x_m)$, there is a $e_m \in G(\omega, x_m)$ such that $h(\omega, \delta_m(\omega)) = (1 - k_m)q + k_m e_m$

$$h(\omega, \delta_m(\omega)) = (1 - k_m)q + k_m e_m - e_m + e_m$$

$$h(\omega, \delta_m(\omega)) - e_m = (1 - k_m)(q - e_m)$$

Therefore ,

$\left\| h(\omega, \delta_m(\omega) - e_m) \right\|_p = (1 - k_m)^p \left\| q - e_m \right\|_p \to 0$ as $n \to \infty$.

Thus,$\{h(\omega, \delta_m(\omega) - e_m\}$ converges to zero .

since $h(\omega, \delta_m(\omega)) - e_m \in (h - G)\big(\omega, \delta_M(\omega)\big) - e_m$ and $(h - G)(\omega, .)$ is demiclosed at zero ,therefore $h(\omega, \delta(\omega)) \in G(\omega, \delta(\omega))$.

Hence h and G have a random coincidence point $\delta: \Omega \to A$ such that for each $\omega \in \Omega$,

$h(\omega, \delta(\omega)) \in G\big(\omega, \delta(\omega)\big)$.■

corollary (3.3):

Suppose that A,G,h and q satisfy all the hypotheses of Theorem (1.5).If h is weakly continuous and X satisfies opial׳s condition ,then h and G have a random coincidence point .

Proof :

By lemma (3.2)[Bano , Khan and Latif 2003] , the weak continuty and opial׳s coindition implies that demiclosness of (h-G) .

So ,by Theorem (1.5) the proof is complete . ■

Corollary (3.4):

Let A be a nonempty separable weakly compact subset of X which is starshaped with respect to $\in A$, $h: \Omega \times A \to A$ be a continuous affine random operator and $G: \Omega \times A \to K(A)$ be an h-nonexpansive multivalued random mapping . If G and h are commute , $(h - G)(\omega, .)$ is demiclosed at 0 for all $\omega \in \Omega$ and $h(\omega, A) = A$, $h(\omega, q) = q$ for all $\omega \in \Omega$ then h and G have a random coincidence point .

Corollary (3.5):

Suppose that A,h,G and q satisfy all the hypotheses of corollary (1.8). If h is weakly continuous and X satisfies opial׳s condition , then h and G have a random coincidence point .

Theorem (3.6)

Suppose that A,h,G and q satisfy all the hypotheses of Theorem (1.5).

If for each $\omega \in \Omega$, $h(\omega,x) \in G(\omega,x)$ implies the existence of $\lim_{n\to\infty} h^n(\omega,x)$, then h and G have a common random fixed point .

Proof

By Theorem (1.5) there is measurable mapping $\delta:\Omega \to A$ such that $h(\omega,\delta(\omega)) \in G(\omega,\delta(\omega))$ for each $\omega \in \Omega$. Now

$h^n(\omega,\delta(\omega)) = h^{n-1}(\omega,h(\omega,\delta(\omega)) \in h^{n-1}(\omega,G(\omega,\delta(\omega)) = G(\omega,h^{n-1}(\omega,\delta(\omega))$ for each $\omega \in \Omega$ .Letting $n\to \infty$,we obtain $\delta(\omega) \in G\big(\omega,\delta(\omega)\big)$, where $\delta(\omega) = \lim_{n\to\infty} h^n(\omega,\delta(\omega))$. Clearly $\delta(\omega) = h(\omega,\delta(\omega))$ for each $\in \Omega$. Hence ,for any $\omega \in \Omega$,

$\delta(\omega) = h(\omega,\delta(\omega)) \in G(\omega,\delta(\omega)$.■

Remark (3.7)

If h = I (identity random mapping) in Theorem (1.5) ,Theorem (1.6) ,Corollary (1.7) and corollary (1.8) then G has a random fixed point .

Theorem (3.8)

Let X be a complete p-normed space .Let $h: \Omega\times X \to X$ be a weakly continuous random mapping , and let $G: \Omega\times X \to k(X)$ be an h-nonexpansive multivalued random mapping commute with h, such that $G(\omega,x_\circ) = \{x_\circ\}$ for some $x_\circ \in X$ and for each $\omega \in \Omega$. Let A be a nonempty $G(\omega,.)$-invariant subset of X and $x_\circ = h(\omega,x_\circ) \in G(\omega,x_\circ)$ for each $\omega \in \Omega$. Assume that $p_A(x_\circ)$ is nonempty separable weakly compact and has property (P_1) with respect to G and $h(\omega,q) = q$ for each $\omega \in \Omega$. Further assume that $h(\omega,.)$ is strongly continuous mapping for each

$\omega \in \Omega$ and has property (C) on $p_A(x_\circ)$ with $h\big(\omega,p_A(x_\circ)\big) = p_A(x_\circ)$ for each $\omega \in \Omega$. If (h-G)$(\omega,.)$ is demiclosed on $p_A(x_\circ)$,then there exists measurable map $\delta:\Omega \to p_A(x_\circ)$ such that for each $\omega \in \Omega$,$h\big(\omega,\delta(\omega)\big) \in G\big(\omega,\delta(\omega)\big)$.

Proof

Let $p_A(x_\circ) = M$

Since M has property (P_1) with respect to G, then

$(1-k_n)q+k_n G(\omega,x)\subseteq M$ for some $q\in M$, $\lim k_n = 1 (0 < k_n < 1)$,for each $x \in M$ and for each $\omega \in \Omega$.

For each n , defined the random operator G_n by

$G_n(\omega,x) = (1-k_n)q + k_n G(\omega,x)$,for all $x \in M$ and all $\omega \in \Omega$.

Hence , $G_n:\Omega \times M \to K(M)$. since h is G-nonexpansive ,then

$$\mathcal{D}(G_n(\omega,x),G_n(\omega,y)) = \mathcal{D}\big((1-k_n)q + k_n G(\omega,x),(1-k_n)q + k_n G(\omega,Y)\big)$$

$$= (k_n)^p \mathcal{D}(G(\omega,x),G(\omega,y))$$

$$\leq (k_n)^p d_p(h(\omega,x),h(\omega,y))$$

For each $x,y \in M$ and each $\omega \in \Omega$.

Therefore each G_n is a random h-contraction , and for each n ,$G_n(\omega,M) \subset M = h(\omega,M)$.

Since h commute with G, $h(\omega,q) = q$ for each $\omega \in \Omega$ and h has property (C),then

$G_n(\omega, h(\omega, x)) = (1 - k_n)q + k_n G(\omega, h(\omega, x))$

$=(1\text{-}k_n)h(\omega, q) + h(\omega, G(\omega, x))$

$=h(\omega, (1 - k_n)q + k_n G(\omega, x))$

$=h(\omega, G_n(\omega, x))$

For all $x \in M$ and all $\omega \in \Omega$ .Hence each G_n commute with h .

Since M is separable and weakly compact , then the weak topology on M is a metric topology[Dunford 1958,p.434] which implies that M is hausdorff [Körner 2015,p.25] then M is strongly closed , this implies M is a complete metric space [Kreyszing 1978,,p30] .Thus all the conditions of Theorem (3.1) are satisfied .

Therefore there is a measurable map $\delta_n: \Omega \to M$ such that $h(\omega, \delta_n(\omega)) \in G_n(\omega, \delta_n(\omega)$ for each $\omega \in \Omega$.

For each n define $Q_n: \Omega \to wk(M)$ by $Q_n(\omega) = w - cl\{\delta_i(\omega): i \geq n\}$,

define Q: $\Omega \to wk(M)$ by $Q(\omega) = \cap_{n=1}^{\infty} Q_n(\omega)$.

Since the weakly topology on M is metric topology, then Q is weak measurable and has measurable selector δ.since M is weakly compact , then there is a subsequence $\{\delta_m(\omega)\}$ of $\{\delta_n(\omega)\}$ converges weakly to $\delta(\omega)$.also by definition of $G_m(\omega, x_m)$, there is a $e_m \in G(\omega, x_m)$ such that $h(\omega, \delta_m(\omega))=(1 - k_m)q + k_m e_m$

$=(1 - k_m)q + k_m e_m - e_m + e_m$

$h(\omega, \delta_m(\omega)) - e_m = (1 - k_m)(q - e_m)$

we get $\| h(\omega, \delta_m(\omega) - e_m) \|_p = (1 - k_m)^p \| q - e_m \|_p \to 0$ as $n \to \infty$.

Thus, $\{h(\omega, \delta_m(\omega) - e_m\}$ converges to zero .since

$h(\omega, \delta_m(\omega)) - e_m \in (h - G)(\omega, \delta_M(\omega)) - e_m$ and $(h - G)(\omega, .)$ is demiclosed at zero ,therefore $h(\omega, \delta(\omega)) \in G(\omega, \delta(\omega))$.

Hence h and G have a random coincidence point $\delta: \Omega \to M$ such that for each $\omega \in \Omega$,

$h(\omega, \delta(\omega)) \in G(\omega, \delta(\omega))$ ■

References

Bano, A.,Khan,A.R., and Latif, A.,"Coincidence points and best approximations in p-normed spaces",Rad.Math.vol.12,(2003),PP.(27-36).

Beg, I.,and Shahzad, N., " Random fixed point theorems for nonexpansive and contractive-type random operators on Banach spaces", J. Appl.Math. Stochastic Anal. Vol.7,(1994), PP.(569-580).

Beg, I.,and Shahzad, N.,"Random fixed points of random multivalued operators on polish spaces", Nonlinear Anal. Vol.20,(1993),PP.(835-847).

Bharucha-Reid, A.T." Fixed point theorems in probabilistic analysis", Bull. Amer .Math .Soc .vol.82(5),(1976) ,PP.(641-657).

Bharucha-Reid, A.T.," Random integral equations" (Academic Press, New York,(1972).

Dunford, A., and Schwartz, J.T,"Linear operators", part I Interscience, New York. (1958).

Hanš , O .," Random operator equations", Proceedings of 4th Berkeley Sympos . Math . Statist. and Prob.,Vol.II. University of California, Part I,(1961), PP.(185-202).

Hanš , O .," Reduzierende zufallige transformationen", Czechoslovak Math .Journal .vol.7(82) ,(1957),.PP.(154-158).

Himmelberg, C.J.,"Measurable relations",Fund.Math.vol.87,(1975),PP. (53-72).

Itoh, S. , "Random fixed point theorems with an applications to random differential equations in Banach spaces", J. Math. Anal .Appl .vol.67,(1979),PP.(261-273) .

Jhadd[1],P. K., and Salua[2],A.S., "On random coincidence and fixed points for a pair of multi-valued and single-valued mappings " J. Anal .Appl.vol.4(1), (2014), PP.(26-35).

Körner ,T.W., "Metric and topological "Janyary 19 ,2015 .

Kreyszig ,E., " Introductory functional analysis with applications "University of Windsor (1978).

Kuratowski, K., and Ryll, N.,"A general theorem on selectors",Bull.Acad.Polon.Sci.Ser.Sci.Math.Astronom.Phys.vol.13,(1965),PP(397-403).

Mukherjee, A . ,"Transformation aleatoives separables theorem all point fixed aleatoire", . C . R. Acad. Sci.Paris Ser .A-B 263 ,(1966),PP.(393-395).

Nashine[*], H.K.,"Random fixed points and invariant random approximation in non-convex domains",Hacettepe J.Math.Stat.vol.37(2),(2008),PP(81-88).

Nashine, H.K.,"Common random fixed point and random best approximation in non-starshaped domain of q-normed spaces",Math.Moravica.vol.14(1),(2010),PP.(89-99).

Nashine, H.K.,"Best approximation for nonconvex set in q-normed space",Arch.Math.(Brno.).vol.42,(2006),PP(51-58).

Shahzad , N., "On random coincidence point theorems "J.Juliusz Schauder Center.Vol.25,(2005), PP.(391-400).

Shahzad, N. , and Latif, A. ," Note a random coincidence point theorem", J .Math. Anal .Appl.vol.245,(2000),PP.(633-638).

Shahzad, N.,"Some general random coincidence point theorems ",New Zealand J.Math.vol.33,(2004),PP(95-103).

Singh S.P., Watson, B. and Srtaveivas , P.,"Fixed point theory and best approximation",The KKM-map Principle, Kluwer Academic,(1997) .

Špaček, A.Zufallige Gleichungen, Czechoslovak Mathematical Journal vol.5(80),(1955),PP.(462-466).

Xu, H. K. , "Some random fixed point theorems for condensing and nonexpansive operators", Pros . Amer .Math .Soc.vol. 110,(1990),PP.(495-500).

Zeidler, E.,"Non linear functional analysis and applications,I.fixed point theorems", Springer Verlage, New York,(1986).