Reliability Estimation in Multi-Component Stress-Strength Based on Burr-III Distribution

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Abstract

In this paper, we estimate the multicomponent S out of K stress-strength system reliability for Burr-III distribution with non-identical component strengths which are subjected to a common stress. Both stress and strength are assumed to have Burr-III distribution with common and known scale parameter. The research methodology adopted here is to estimate the parameters by using five method of estimation maximum likelihood, least square, weighted least square, regression and moment estimation. The reliability is estimated using the same methods of estimation and results are compared by Monte-Carlo simulation study using MSE and MAPE criteria, the results show that the Ml was the best between them.

Keywords: Burr-III distribution, stress-strength, reliability estimation, ML, LS, WLS, Rg and MOM estimation.

1. Introduction

Burr 1942 has suggested this family of distributions by solving the following differential equation:-

$$\frac{dy}{dx} = y(1-y)g(x,y), \quad y = F(x),$$

Where the function $g(x,y) > 0$ must be positive for $0 \le y \le 1$, x in the range over which the solution is being satisfied by using different forms of $g(x,y)$ and Pearson systems, Burr in 1941 obtained twelve distribution functions this system is useful for approximating histograms, particularly.[Rodriguez (2004)]

Many researchers studied the Burr-III distribution, like Nahed A. Mokhlis (2005) and Chansoo Kim and Woosuk Kim (2014) (see Kim & Kim (2014) and Mokhlis (2005)).

The CDF of Br3(α,θ) is:-

$$F(x) = (1 + x^{-\theta})^{-\alpha}; \qquad x > 0;\ \alpha, \theta > 0 \qquad (1)$$

Where the parameters $\theta > 0$ and $\alpha > 0$ are the shape parameters of the distribution. Its PDF is:-

$$f(x) = \alpha\theta x^{-(\theta+1)}(1 + x^{-\theta})^{-(\alpha+1)}; \qquad x > 0;\ \alpha, \theta > 0 \qquad (2)$$

The stress-strength model is used in many applications in physics and engineering. In the statistical approach to the stress-strength model, most of the considerations depend on the assumption that the component strengths are independently and identically distributed (iid) and are subjected to a common stress. [Hassan & Basheikh (2012)[1]]

The system reliability of multicomponent based on X and Y two independent and identical random variable.

The system of multicomponent stress-strength studied by Amal S. Hassan and Heba M. Basheikh (2012) [Hassan & Basheikh (2012)[2]] where they expanded the system of multicomponent to be with 2-part multicomponent stress-strength.

The main aim of this article is to discuss the derivation of the mathematical formula of reliability in multicomponent stress-strength $R_{s,k}$ for Burr type III distribution, then estimation $R_{s,k}$ by using ML, LS, WLS, Rg and MOM methods, and comparison among the results of the estimation methods by using mean square error (MSE) and mean absolute percentage error (MAPE), that will get from a simulation study.

2. Experimental Aspect of Reliability in multicomponent stress-strength:-

In this article, the system made up of k non-identical components. Out of these k components, k_1 are of one category and their strengths $Y_1, Y_2, \ldots, Y_{k_1}$ are iid random variables distributed Br3D with parameters (α_1, θ). The remaining components $k_2 = k - k_1$ are of different category and their strengths $Y_{k_1+1}, Y_{k_1+2}, \ldots, Y_k$ are iid random variables distributed Br3D with parameters (α_2,θ). This system is subjected to a common stress X which is independent random variable distributed Br3D with parameters (λ,θ). Let $f_1(y_1;\alpha_1,\theta)$ be a common probability density function (PDF) of strengths $Y_1,Y_2,\dots,Y_{k_1}$, $f_2(y_2;\alpha_2,\theta)$ be a common PDF of strengths $Y_{k_1+1},Y_{k_1+2},\dots,Y_k$ and $g(x;\lambda,\theta)$ be PDF of stress X. The corresponding cumulative distribution functions are given, respectively, by: [Hassan & Basheikh (2012)[2]]

$$\left.\begin{aligned} F_1(y_1;\alpha_1,\theta) &= \left(1+y_1{}^{-\theta}\right)^{-\alpha_1}; && y_1>0;\ \alpha_1 \text{ and } \theta>0,\\ F_2(y_2;\alpha_2,\theta) &= \left(1+y_2{}^{-\theta}\right)^{-\alpha_2}; && y_2>0;\ \alpha_2 \text{ and } \theta>0,\\ G(x;\lambda,\theta) &= \left(1+\mathrm{y}^{-\theta}\right)^{-\lambda}; && y>0;\ \lambda \text{ and } \theta>0\,. \end{aligned}\right\}$$

(3)

The system operates successfully, if at least s out of k components withstand the stress. According to Johnson, the system reliability with non-identical component strengths $R_{(s,k)}$ is given by

$$R_{(s,k)}=\sum_{i_1=s_1}^{k_1}C_{i_1}^{k_1}\sum_{i_2=s_2}^{k_2}C_{i_2}^{k_2}\int_0^\infty[1-F_1(x)]^{i_1}[F_1(x)]^{k_1-i_1}[1-F_2(x)]^{i_2}[F_2(x)]^{k_2-i_2}dG(X) \qquad (4)$$

Where the summation is over all possible pair (i_1,i_2) with $0\le i_1\le k_1$ and $0\le i_2\le k_2$ such that $s\le i_1+i_2\le k$. It is important to note that the system reliability can be extended to more than two groups of components. The reliability of s-out-of-k system for Br3 can be computed by substituting equations (3) in equation (4) and simplifying:

$$R_{(s,k)}=\sum_{i_1=s_1}^{k_1}C_{i_1}^{k_1}\sum_{i_2=s_2}^{k_2}C_{i_2}^{k_2}\int_0^\infty[1-(1+y^{-\theta})^{-\alpha_1}]^{i_1}[(1+y^{-\theta})^{-\alpha_1}]^{k_1-i_1}[1-(1+y^{-\theta})^{-\alpha_2}]^{i_2}$$

$$[(1+y^{-\theta})^{-\alpha_2}]^{k_2-i_2}\lambda\theta y^{-(\theta+1)}(1+y^{-\theta})^{-(\lambda+1)}\,dy$$

$$R_{(s,k)}=\lambda\sum_{i_1=s_1}^{k_1}C_{i_1}^{k_1}\sum_{i_2=s_2}^{k_2}C_{i_2}^{k_2}\sum_{j_1=0}^{i_1}C_{j_1}^{i_1}(-1)^{j_1}\sum_{j_2=0}^{i_2}C_{j_2}^{i_2}(-1)^{j_2}\int_0^1 u^{[\alpha_1(j+k_1-i_1)+\alpha_2(j_2+k_2-i_2)+\lambda+1]}\,du$$ where

$$u=\left(1+y^{-\theta}\right)^{-1}$$

Then the $R_{(s,k)}$ of Br3D is given by:-

$$R_{(s,k)}=\lambda\sum_{i_1=s_1}^{k_1}C_{i_1}^{k_1}\sum_{i_2=s_2}^{k_2}C_{i_2}^{k_2}\sum_{j_1=0}^{i_1}C_{j_1}^{i_1}(-1)^{j_1}\sum_{j_2=0}^{i_2}C_{j_2}^{i_2}(-1)^{j_2}[\alpha_1(j_1+k_1-i_1)+\alpha_2(j_2+k_2-i_2)+\lambda]^{-1}$$

(5)

Where i, s, k and j are integers.

3. Different method of estimation:

The unknown shape parameters of $R_{(s,k)}$ for Br3D have been estimated by five methods of estimation; ML, LS, WLS, Rg and MOM.

3.1 Maximum likelihood function (MLE):-

Let $X_{1_{i_1}}; i_1=1,2,\dots,n_1$ and $X_{2_{i_2}}; i_2=1,2,\dots,n_2$ strength random samples from $Br3(\alpha_1,\theta)$ and $Br3(\alpha_2,\theta)$ with sample size n_1 *and* n_2, respectively , where α_1 *and* α_2 are unknown parameters, and let Y stress random variable with sample size m is drawn from $Br3(\lambda,\theta)$ where λ is unknown parameter. Then the likelihood function L, using equation (2) as:-[Shawky and AL-Kashkari (2007)]

$$L\left(x_1, x_2, \dots, x_{n_\xi}; \alpha_\xi, \theta\right) = \prod_{i_\xi=1}^{n_\xi}\left[\alpha_\xi \theta x_{\xi_{i_\xi}}{}^{-(\theta+1)}\left(1 + x_{\xi_{i_\xi}}{}^{-\theta}\right)^{-(\alpha_\xi+1)}\right]$$

And $L(y; \lambda, \theta) = \prod_{j=1}^{m}\left[\lambda\theta y_j{}^{-(\theta+1)}\left(1 + y_j{}^{-\theta}\right)^{-(\lambda+1)}\right]$; where ; $\xi = 1,2$

The partial derivative of log-likelihood function with respect to α_ξ and λ are given by:

$$\left.\begin{aligned}&\frac{\partial \ln L\left(x_1, x_2, \dots, x_{n_\xi}; \alpha_\xi, \theta\right)}{\partial \alpha_\xi} = \frac{n_\xi}{\alpha_\xi} - \sum_{i_\xi=1}^{n_\xi} ln\left(1 + x_{\xi_{i_\xi}}{}^{-\theta}\right) \quad \xi = 1,2\\ &\frac{\partial \ln L(y_1, y_2, \dots, y_n; \lambda, \theta)}{\partial \lambda} = \frac{m}{\lambda} - \sum_{j=1}^{m} \ln\left(1 + y_j{}^{-\theta}\right)\end{aligned}\right\} \tag{6}$$

Then by simplification equations (6), the ML's estimator for the unknown shape parameters α_ξ and λ, $(\hat{\alpha}_{\xi MLE}, \hat{\lambda}_{MLE})$ for $R_{(s,k)}$, are:

$$\hat{\alpha}_{\xi MLE} = \frac{n_\xi}{\sum_{i_\xi=1}^{n_\xi} ln\left(1+x_{\xi_{i_\xi}}{}^{-\theta}\right)}, \xi = 1,2; \ \hat{\lambda}_{MLE} = \frac{m}{\sum_{j=1}^{m} ln\left(1+y_j{}^{-\theta}\right)} \tag{7}$$

Substation the equations (7) in the equations (5), the ML estimator for $R_{(s,k)}$, $(\hat{R}_{BML})$, can be obtained as:

$$\hat{R}_{BML} = \hat{\lambda}_{MLE} \sum_{i_1=s_1}^{k_1} \sum_{i_2=s_2}^{k_2} C_{i_1}^{k_1} C_{i_2}^{k_2} \sum_{j_1=0}^{i_1} C_{j_1}^{i_1} (-1)^{j_1} \sum_{j_2=0}^{i_2} C_{j_2}^{i_2} (-1)^{j_2}$$

$$\left[\hat{\alpha}_{1MLE}(j_1 + k_1 - i_1) + \hat{\alpha}_{2MLE}(j_2 + k_2 - i_2) + \lambda_{MLE}\right]^{-1}$$

3.2 Least Square Method (LS):-

The method of least square is very popular for model fitting, especially in linear and non-linear regression, it can be produce by minimizing the sum of square error between the value and it's expected value. [Ali (2013)]

$$s_1 = \sum_{i_\xi=1}^{n_\xi}\left[F(x_{(i_\xi)}) - E\left(F\left(x_{(i_\xi)}\right)\right)\right]^2 \ S = \sum_{i=1}^{n}\left[F(y_{(i)}) - E\left(F(y_{(i)})\right)\right]^2 \tag{8}$$

Where $E\left(F\left(x_{(i_\xi)}\right)\right)$ and $E\left(F(y_{(i)})\right)$ equal to P_{i_ξ}, P_j the plotting position, where $P_{i_\xi} = \frac{i_\xi}{n_1+1}, i_\xi = 1,2, \dots, n_\xi \ and \ \xi = 1,2$, and $P_j = \frac{j}{m+1}$and$i = 1,2, \dots, m.$ (9)

We obtain the LS estimators to the unknown shape parameters α_1, α_2 to the strength r.v.'s $X_1 \sim Br3(\alpha_1, \theta)$, $X_2 \sim Br3(\alpha_2, \theta)$ with sample size $n_1 \ and \ n_2$, respectively, and λ to the stress r.v. $Y \sim Br3(\lambda, \theta)$ with sample size m. From a distribution function (1), simplification and changing $F\left(x_{\xi_{(i_\xi)}}\right), G\left(y_{(j)}\right)$ by plotting position P_{i_ξ}, P_j (9), and equal to zero, we obtain:-

$$\left.\begin{aligned}&\ln\left(P_{i_\xi}\right)^{-1} - \alpha_\xi \ln\left(1 + x_{\xi_{(i_\xi)}}{}^{-\theta}\right) = 0; \ \xi = 1,2\\ &\ln\left(P_j\right)^{-1} - \lambda \ln\left(1 + y_{(j)}{}^{-\theta}\right) = 0\end{aligned}\right\} \tag{10}$$

Substitution (10) in (8) and taking the first derivative with respect to the unknown shape parameters α_ξ and λ, and equating the result to zero, we get:

$$\left.\begin{aligned}\hat{\alpha}_{\xi LS} &= \frac{\sum_{i_\xi=1}^{n_\xi}\left(\ln\left(P_{i_\xi}\right)^{-1}\ln\left(1+x_{\xi(i_\xi)}{}^{-\theta}\right)\right)}{\sum_{i_\xi=1}^{n_\xi}\left(\ln\left(1+x_{\xi(i_\xi)}{}^{-\theta}\right)\right)^2}, \xi = 1,2 \\ \hat{\lambda}_{LS} &= \frac{\sum_{j=1}^{n}\left(\ln(P_j)^{-1}\ln(1+y_{(j)}{}^{-\theta})\right)}{\sum_{j=1}^{n}\left(\ln(1+y_{(j)}{}^{-\theta})\right)^2}\end{aligned}\right\} \tag{11}$$

Where P_{i_ξ} and P_j as in (9).

Substitution the equations (11) in the equation (5), the LS estimator for $R_{(s,k)}$, $(\hat{R}_{BLS})$, approximately can be obtained as:

$$\hat{R}_{BLS} = \hat{\lambda}_{LS}\sum_{i_1=s_1}^{k_1}\sum_{i_2=s_2}^{k_2} C_{i_1}^{k_1} C_{i_2}^{k_2}\sum_{j_1=0}^{i_1} C_{j_1}^{i_1}(-1)^{j_1}\sum_{j_2=0}^{i_2} C_{j_2}^{i_2}(-1)^{j_2}$$

$$\left[\hat{\alpha}_{1LS}(j_1+k_1-i_1)+\hat{\alpha}_{2LS}(j_2+k_2-i_2)+\hat{\lambda}_{LS}\right]^{-1}$$

3.3 Weighted Least Square Method (WLS):-

The estimators weighted least squares can be obtained by minimizing the following equation.[Ali (2013)]

$$s_1 = \sum_{i_\xi=1}^{n_\xi}\omega_{i_\xi}\left[F(x_{(i_\xi)}) - E\left(F\left(x_{(i_\xi)}\right)\right)\right]^2 \text{ and } s_2 = \sum_{j=1}^{m}\omega_j\left[F(y_{(j)}) - E\left(F(y_{(i)})\right)\right]^2 \tag{12}$$

With respect to the unknown parameters α_ξ and λ, Where $E\left(F\left(x_{(i_\xi)}\right)\right)$ and $E\left(F(y_{(i)})\right)$ equal to P_{i_ξ}, P_j the plotting position, where P_{i_ξ}, P_j as in (9)

$$\left.\begin{aligned}\text{And } \omega_{i_\xi} &= \frac{1}{Var\left[F\left(x_{\xi(i_\xi)}\right)\right]} = \frac{(n_\xi+1)^2(n_\xi+2)}{i_\xi(n_\xi-i+1)}, i_\xi = 1,2,\dots,n_\xi \\ and\ \omega_j &= \frac{1}{Var\left[G\left(y_{(j)}\right)\right]} = \frac{(m+1)^2(m+2)}{j(m-j+1)}, j = 1,2,\dots,m\end{aligned}\right\} \tag{13}$$

We obtain the WLS estimators to the unknown shape parameters α_1, α_2 to the strength r.v.'s $X_1 \sim Br3(\alpha_1,\theta)$, $X_2 \sim Br3(\alpha_2,\theta)$ with sample size n_1 and n_2, respectively, and λ to the stress r.v. $Y \sim Br3(\lambda,\theta)$ with sample size m, by substitution (10) in (12), and taking the partial derivative with respect to the unknown shape parameters α_ξ and λ, and simplify the result, then the final formulas can be written as:

$$\left.\begin{aligned}\hat{\alpha}_{\xi WLS} &= \frac{\sum_{i_\xi=1}^{n_\xi}\omega_{i_\xi}\left(\ln\left(P_{i_\xi}\right)^{-1}\ln\left(1+x_{\xi(i_\xi)}{}^{-\theta}\right)\right)}{\sum_{i_\xi=1}^{n_\xi}\omega_{i_\xi}\left(\ln\left(1+x_{\xi(i_\xi)}{}^{-\theta}\right)\right)^2}, \xi = 1,2 \\ \hat{\lambda}_{WLS} &= \frac{\sum_{j=1}^{n}\omega_j\left(\ln(P_j)^{-1}\ln(1+y_{(j)}{}^{-\theta})\right)}{\sum_{j=1}^{n}\omega_j\left(\ln(1+y_{(j)}{}^{-\theta})\right)^2}\end{aligned}\right\} \tag{14}$$

Where the plotting position P_{i_ξ} and $P_j$as in (9), and ω_{i_ξ} and ω_j as in (13).

Substitution the equations (14) in the equation (5), the WLS estimator for $R_{(s,k)}$, $(\hat{R}_{BWL})$, can be obtained as:

$$\hat{R}_{BWL} = \hat{\lambda}_{WLS} \sum_{i_1=s_1}^{k_1} \sum_{i_2=s_2}^{k_2} C_{i_1}^{k_1} C_{i_2}^{k_2} \sum_{j_1=0}^{i_1} C_{j_1}^{i_1} (-1)^{j_1} \sum_{j_2=0}^{i_2} C_{j_2}^{i_2} (-1)^{j_2}$$

$$[\hat{\alpha}_{1WLS}(j_1 + k_1 - i_1) + \hat{\alpha}_{2WLS}(j_2 + k_2 - i_2) + \hat{\lambda}_{WLS}]^{-1}$$

3.4 Regression Method (Rg):-

Regression is one of the important procedures that use auxiliary information to construct estimators with good efficiency. [Park (2012)]
The standard regression equation:-

$$z_i = a + bu_i + e_i \quad (15)$$

Where z_i is dependent variable (response variable), u_i is independent variable (Explanatory Variable) and e_i is the error r.v. independent identically Normal distributed with $(0, \sigma^2)$.
Taking natural logarithm to (1), then changing $F(x_{(i)})$ and $G(y_{(j)})$ by plotting position P_{i_ξ} and P_j (9), then simplification the result, we obtain:

$$\left.\begin{aligned} \ln P_{i_\xi} &= -\alpha_\xi \ln\left(1 + x_{\xi_{(i_\xi)}}{}^{-\theta}\right); i_\xi = 1,2, \dots, n_\xi; \text{ Where } \xi = 1,2 \\ \ln P_j &= -\lambda \ln\left(1 + y_{(j)}{}^{-\theta}\right); j = 1,2, \dots, m \end{aligned}\right\} \quad (16)$$

Comparing the equation (16) with the equation (15), we get:

$$\left.\begin{aligned} z_i &= \ln P_{i_\xi}, a = 0, b = \alpha_\xi, u_i = -\ln\left(1 + x_{\xi_{(i_\xi)}}{}^{-\theta}\right) \\ z_i &= \ln P_j, a = 0, b = \lambda, u_i = -\ln\left(1 + y_{(j)}{}^{-\theta}\right) \end{aligned}\right\} \quad (17)$$

Where b can be estimated by minimizing the summation of squared error with respect to b, then we get:

$$\hat{b} = \frac{n\sum_{i=1}^{n} z_i u_i - \sum_{i=1}^{n} z_i \sum_{i=1}^{n} u_i}{n\sum_{i=1}^{n} [u_i]^2 - \left[\sum_{i=1}^{n} u_i\right]^2} \quad (18)$$

And by substitution (17) in (18) we obtain the estimate for unknown shape parameters α_ξ and λ for $R_{s,k}$ then the result will be:

$$\left.\begin{aligned} \hat{\alpha}_{\xi Rg} &= \frac{n_\xi \sum_{i_\xi=1}^{n_\xi} \ln P_{i_\xi} \ln\left(1 + x_{\xi_{(i_\xi)}}{}^{-\theta}\right) - \sum_{i_\xi=1}^{n_\xi} \ln P_{i_\xi} \sum_{i_\xi=1}^{n_\xi} \ln\left(1 + x_{\xi_{(i_\xi)}}{}^{-\theta}\right)}{n_\xi \sum_{i_\xi=1}^{n_\xi} \left[\ln\left(1 + x_{\xi_{(i_\xi)}}{}^{-\theta}\right)\right]^2 - \left[\sum_{i_\xi=1}^{n_\xi} \ln\left(1 + x_{\xi_{(i_\xi)}}{}^{-\theta}\right)\right]^2}; \text{ Where } \xi = 1,2 \\ \hat{\lambda}_{Rg} &= \frac{m\sum_{j=1}^{m} \ln P_j \ln\left(1 + y_{(j)}{}^{-\theta}\right) - \sum_{j=1}^{m} \ln P_j \sum_{j=1}^{m} \ln\left(1 + y_{(j)}{}^{-\theta}\right)}{m\sum_{j=1}^{m} \left[\ln\left(1 + y_{(j)}{}^{-\theta}\right)\right]^2 - \left[\sum_{j=1}^{m} \ln\left(1 + y_{(j)}{}^{-\theta}\right)\right]^2} \end{aligned}\right\} \quad (19)$$

Where the plotting position P_{i_ξ} and P_j as in (9).
Substitution the equations (19) in the equation (5), the RM estimator for $R_{(s,k)}$, $(\hat{R}_{BRg})$, can be obtained as:

$$\hat{R}_{BRg} = \hat{\lambda}_{Rg} \sum_{i_1=s_1}^{k_1} \sum_{i_2=s_2}^{k_2} C_{i_1}^{k_1} C_{i_2}^{k_2} \sum_{j_1=0}^{i_1} C_{j_1}^{i_1} (-1)^{j_1} \sum_{j_2=0}^{i_2} C_{j_2}^{i_2} (-1)^{j_2}$$

$$\left[\hat{\alpha}_{1Rg}(j_1 + k_1 - i_1) + \hat{\alpha}_{2Rg}(j_2 + k_2 - i_2) + \lambda_{Rg}\right]^{-1}$$

3.5 Moment method (MOM):-

The MOM was one of the first methods used to estimate the society parameter θ, it was introduced by Pearson (1894). [Ali (2013)]

To derive the method of moment estimators of the parameters of a $Br3D$, let $X_{1_{i_1}}; i_1 = 1, 2, \ldots, n_1$ and $X_{2_{i_2}}; i_2 = 1, 2, \ldots, n_2$ strength random samples from $Br3(\alpha_1, \theta)$ and $Br3(\alpha_2, \theta)$ with sample size n_1 *and* n_2, respectively , where α_1 *and* α_2 are unknown parameters, and let Y stress random variable with sample size m is drawn from $Br3(\lambda, \theta)$ where λ is unknown parameter, , we need the population mean:

$$E(x_\xi) = \alpha_\xi B\left(1 - \frac{1}{\theta}, \alpha_\xi + \frac{1}{\theta}\right); \; where\ \theta > 2$$

$$E(y) = \lambda B\left(1 - \frac{1}{\theta}, \lambda + \frac{1}{\theta}\right); \quad where\ \theta > 2$$

For θ is known, equating the sample mean with corresponding populations mean, we get the shape parameters moment estimators.

$$\frac{\sum_{i_\xi=1}^{n_\xi} x_{\xi i_\xi}}{n_\xi} = \alpha_\xi B\left(1 - \frac{1}{\theta}, \alpha_\xi + \frac{1}{\theta}\right); \qquad \frac{\sum_{j=1}^{m} y_j}{m} = \lambda B\left(1 - \frac{1}{\theta}, \lambda + \frac{1}{\theta}\right)$$

By simplification we obtain the estimators to the unknown shape parameters α_ξ and λ for $R_{(s,k)}$ ($\hat{\alpha}_{\xi MOM}$ and $\hat{\lambda}_{MOM}$), then the result will be:

$$\left.\begin{aligned} \hat{\alpha}_{\xi MOM} &= \frac{\overline{x_\xi}}{B\left(1 - \frac{1}{\theta}, \alpha_{\xi_0} + \frac{1}{\theta}\right)}; \quad where\ \xi = 1,2;\ \theta > 2 \\ \hat{\lambda}_{MOM} &= \frac{\bar{y}}{B\left(1 - \frac{1}{\theta}, \lambda_0 + \frac{1}{\theta}\right)}; \quad where\ \theta > 2 \end{aligned}\right\} \qquad (19)$$

Substitution the equations (19) in the equation (5), the MOM estimator for $R_{(s,k)}$, ($\hat{R}_{BMOM}$), can be obtained as:

$$\hat{R}_{BMOM} = \hat{\lambda}_{MOM} \sum_{i_1=s_1}^{k_1} \sum_{i_2=s_2}^{k_2} C_{i_1}^{k_1} C_{i_2}^{k_2} \sum_{j_1=0}^{i_1} C_{j_1}^{i_1} (-1)^{j_1} \sum_{j_2=0}^{i_2} C_{j_2}^{i_2} (-1)^{j_2}$$

$$\left[\hat{\alpha}_{1MOM}(j_1 + k_1 - i_1) + \hat{\alpha}_{2MOM}(j_2 + k_2 - i_2) + \lambda_{MOM}\right]^{-1}$$

4. Simulation study:

Results based on Monte Carlo simulations to compare the performance of the $R_{(s,k)}$ using different sample sizes are presented. 1500 random sample of size 10,20,35,50.75 and 100 each from stress population, strength population were generated $(\alpha_1, \alpha_2, \lambda, \theta)$ = (1.5, 2, 0.8, 1.2), $(\alpha_1, \alpha_2, \lambda, \theta)$ = (1.5, 2, 2.3, 1.2) for (s_1, k_1, s_2, k_2) = (2, 3, 3, 4) and (s_1, k_1, s_2, k_2) = (1, 2, 3, 3) for $R_{(s,k)}$. The Mean square error (MSE) and Mean Absolute Percentage Error (MAPE) of the reliability estimates over the 1500 replications are given in Tables 2 and 3.

The true value of reliability in multicomponent stress- strength $R_{(s,k)}$ with the given combinations of $(\alpha_1, \alpha_2, \lambda, \theta)$ = (1.5, 2, 0.8, 1.2), $(\alpha_1, \alpha_2, \lambda, \theta)$ = (1.5, 2, 2.3, 1.2) for (s_1, k_1, s_2, k_2) = (2, 3, 3, 4) are $R_{(s,k)}$ =0.5755 and $R_{(s,k)}$ =0.2411 and for (s_1, k_1, s_2, k_2) = (1, 2, 3, 3) are $R_{(s,k)}$ =0.5024 and $R_{(s,k)}$ =0.1863.

From the tables (2) and (3) below, we have observed that:-

1- When (s_1, k_1, s_2, k_2) = (2, 3, 3, 4):-

- ❖ the MSE value decreasing by increasing sample size for MLE, LS, WLS, Rg and MOM estimators. The best MSE value is MLE estimator, followed by LS, WLS, Rg and MOM.
- ❖ The MAPE value decreasing by increasing sample size for MLE, LS, WLS, Rg and MOM estimators. The best MAPE value is MLE estimator, followed by LS, WLS, Rg and MOM.

2- When (s_1, k_1, s_2, k_2) = (1, 2, 3, 3):-

- The MSE value decreasing by increasing sample size for MLE, LS, WLS, Rg and MOM estimators. The best MSE value is MLE estimator, followed by LS, WLS, Rg and MOM.
- The MAPE value decreasing by increasing sample size for MLE, LS, WLS, Rg and MOM estimators. The best MAPE value is MLE estimator, followed by LS, WLS, Rg and MOM.

5. Conclusion:

- The MSE and MAPE value decreases by increasing sample size.
- The performance MLE was the best, followed by LS, WLS, Rg and MOM for all sample sizes, as in the table below.

Table (1): The best estimation method of MSE and MAPE of Br3 for $R_{(s,k)}$.

Method / Sample size	MLE	LS	WLS	Rg	MOM	Best
All sample size	1	2	3	4	5	**MLE**

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Table (2): Results of Mean, MSE and MAPE values for Br3D for $(\alpha_1, \alpha_2, \lambda, \theta)$ = (1.5, 2, 0.8, 1.2).

Methods / (n_1, n_1, m)	(s_1, k_1, s_2, k_2)		MLE	LS	WLS	Rg	MOM	Best
(10,10,10)	(2, 3, 3, 4)	Mean	0.5572	0.5528	0.5512	0.5430	0.5285	-
		MSE	0.0131	0.0154	0.0161	0.0221	0.0535	MLE
		MAPE	0.1562	0.1701	0.1741	0.2058	0.3260	MLE
(20,20,20)		Mean	0.5635	0.5619	0.5567	0.5568	0.5266	-
		MSE	0.0068	0.0082	0.0093	0.0122	0.0441	MLE
		MAPE	0.1127	0.1239	0.1325	0.1531	0.2856	MLE
(35,35,35)		Mean	0.5689	0.5680	0.5595	0.5650	0.5287	-
		MSE	0.0039	0.0048	0.0060	0.0073	0.0377	MLE
		MAPE	0.0859	0.0965	0.1077	0.1190	0.2606	MLE
(50,50,50)		Mean	0.5727	0.5723	0.5633	0.5701	0.5445	-
		MSE	0.0027	0.0031	0.0040	0.0048	0.0315	MLE
		MAPE	0.0726	0.0780	0.0885	0.0961	0.2346	MLE
(75,75,75)		Mean	0.5725	0.5724	0.5615	0.5711	0.5471	-
		MSE	0.0017	0.0022	0.0032	0.0034	0.0258	MLE
		MAPE	0.0578	0.0646	0.0777	0.0808	0.2081	MLE
(100,100,100)		Mean	0.5724	0.5716	0.5600	0.5698	0.5486	-
		MSE	0.0012	0.0016	0.0026	0.0026	0.0250	MLE
		MAPE	0.0487	0.0552	0.0695	0.0707	0.2054	MLE
(10,10,10)	(1, 2, 3, 3)	Mean	0.4940	0.4908	0.4901	0.4848	0.4907	-
		MSE	0.0148	0.0170	0.0179	0.0236	0.0535	MLE
		MAPE	0.1950	0.2093	0.2144	0.2470	0.3847	MLE
(20,20,20)		Mean	0.4943	0.4940	0.4899	0.4916	0.4773	-
		MSE	0.0082	0.0097	0.0110	0.0141	0.0461	MLE
		MAPE	0.1442	0.1566	0.1672	0.1898	0.3498	MLE
(35,35,35)		Mean	0.5013	0.4992	0.4912	0.4958	0.4921	-
		MSE	0.0049	0.0061	0.0077	0.0090	0.0366	MLE
		MAPE	0.1117	0.1257	0.1401	0.1528	0.3079	MLE
(50,50,50)		Mean	0.5021	0.5024	0.4943	0.5016	0.4856	-
		MSE	0.0033	0.0040	0.0054	0.0062	0.0346	MLE
		MAPE	0.0918	0.1014	0.1166	0.1254	0.2915	MLE
(75,75,75)		Mean	0.5004	0.5003	0.4902	0.4997	0.4855	-
		MSE	0.0022	0.0027	0.0040	0.0041	0.0307	MLE
		MAPE	0.0746	0.0830	0.1000	0.1022	0.2740	MLE
(100,100,100)		Mean	0.5010	0.5006	0.4897	0.4998	0.4832	-
		MSE	0.0017	0.0021	0.0033	0.0033	0.0300	MLE
		MAPE	0.0661	0.0728	0.0899	0.0905	0.2663	MLE

Table (3): Results of Mean, MSE and MAPE values for Br3D for $(\alpha_1, \alpha_2, \lambda, \theta)$ = (1.5, 2, 2.3, 1.2).

Methods / (n_1, n_1, m)	(s_1, k_1, s_2, k_2)		MLE	LS	WLS	Rg	MOM	Best
(10,10,10)	(2, 3, 3, 4)	Mean MSE MAPE	0.2465 0.0112 0.3551	0.2455 0.0125 0.3717	0.2449 0.0128 0.3775	0.2449 0.0169 0.4305	0.2489 0.0338 0.6234	- MLE MLE
(20,20,20)		Mean MSE MAPE	0.2415 0.0059 0.2554	0.2414 0.0070 0.2812	0.2377 0.0077 0.2953	0.2417 0.0102 0.3372	0.2455 0.0273 0.5575	- MLE MLE
(35,35,35)		Mean MSE MAPE	0.2408 0.0036 0.1975	0.2414 0.0044 0.2214	0.2351 0.0051 0.2404	0.2422 0.0066 0.2709	0.2351 0.0223 0.5055	- MLE MLE
(50,50,50)		Mean MSE MAPE	0.2426 0.0025 0.1685	0.2430 0.0029 0.1807	0.2358 0.0035 0.1986	0.2435 0.0044 0.2210	0.2433 0.0198 0.4672	- MLE MLE
(75,75,75)		Mean MSE MAPE	0.2414 0.0017 0.1389	0.2417 0.0022 0.1557	0.2326 0.0029 0.1798	0.2418 0.0035 0.1942	0.2400 0.0176 0.4482	- MLE MLE
(100,100,100)		Mean MSE MAPE	0.2415 0.0012 0.1147	0.2417 0.0016 0.1333	0.2325 0.0023 0.1616	0.2419 0.0026 0.1697	0.2418 0.0158 0.4182	- MLE MLE
(10,10,10)	(1, 2, 3, 3)	Mean MSE MAPE	0.1954 0.0096 0.4198	0.1973 0.0109 0.4442	0.1973 0.0113 0.4502	0.2016 0.0154 0.5277	0.2177 0.0340 0.7769	- MLE MLE
(20,20,20)		Mean MSE MAPE	0.1874 0.0049 0.2983	0.1882 0.0059 0.3316	0.1861 0.0065 0.3477	0.1911 0.0088 0.4036	0.2128 0.0274 0.6905	- MLE MLE
(35,35,35)		Mean MSE MAPE	0.1904 0.0031 0.2385	0.1905 0.0038 0.2628	0.1860 0.0044 0.2861	0.1919 0.0057 0.3191	0.2125 0.0232 0.6329	- MLE MLE
(50,50,50)		Mean MSE MAPE	0.1901 0.0020 0.1940	0.1918 0.0026 0.2147	0.1868 0.0031 0.2389	0.1943 0.0040 0.2668	0.2021 0.0182 0.5596	- MLE MLE
(75,75,75)		Mean MSE MAPE	0.1882 0.0014 0.1587	0.1899 0.0018 0.1784	0.1839 0.0023 0.2029	0.1919 0.0027 0.2206	0.2014 0.0168 0.5338	- MLE MLE
(100,100,100)		Mean MSE MAPE	0.1880 0.0010 0.1365	0.1881 0.0013 0.1543	0.1808 0.0019 0.1867	0.1887 0.0020 0.1934	0.2052 0.0157 0.5130	- MLE MLE