Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.5, No.12, 2015

The hydrodynamic Effect of the Hyaluronic Acid on the Performance Improvement of the Human Synovial Joint

Albert E. Yousif
College of Engineering , Al-Nahrian, Iraq

Ahmad M. Abdulhadi
Baghdad University, Baghdad ,Iraq

Enas Y. Abdullah*
Baghdad University, Baghdad, Iraq
PO box 1178, Toronto M3A 2K7, Canada

Abstract

The aim of this article is to present an the theoretical analysis of the problem is presented through a mathematical model depended on the idea of a Hyaluronic acid (HA) in cartilage and synovial fluid surrounding the joints (hip, knee and ankle) where are a major component of synovial fluid modified Reynolds equation governing the fluid film pressure was derived and solved analytically, and closed form expressions for the squeeze film pressure and load carry capacity were presented. The influence of film thickness and sliding motion on the squeeze film Characteristics were discussed .It has been found that the effect of decreased film thickness tend to increased the load carry capacity , friction force and decreased flow rate, The effect of decreased sliding motion tend to increasing frication force and decreased flow rate and when additives Hyaluronic acid (HA) to bearing material (articular cartilage) .The results indicate to increasing pressure distribution (P) and improve both load carry capacity (W), friction force (F) Compared to the disease synovial hip joint.

Keywords: Hyaluronic acid , Hip joint, load carry capacity, friction force.

1.Introduction

This paper presents mathematical model describe hyaluronic acid of synovial fluid flowed through gap between two articular cartilage in synovial human hip joint[Lin and Liao (2005)] . The present paper gives an analysis of solutions of non-linear , partial ,differential equations for synovial fluid in synovial human hip joint gap Synovial fluid is clear and yellowish substance found in cavity of freely moving joints and interacting with cartilage to provide lubricating action. It occurs in small quantities yet it acts both as a lubricant for the articular surface and as a nutrients for the cartilage . Synovial fluid is secreted by synovial lining cells see figure 1. It plays a very important role in synovial joints. It occupies the joint cavity and lines the synovial joint, providing nutrients and removing catabolic products. The thin film of synovial fluid that covers the surface of the inner layer of the joint capsule and articular cartilage helps to keep the joint surface lubricated and reduces friction as fluid moves in and out of the cartilage as compression is applied, then released .The composition of synovial fluid also contains hyaluronate hyaluronic acid (HA) and glycoprotein called lubricin .The hyaluronate component of synovial fluid is responsible for the viscosity of the fluid and is essential for joint lubrication[Albert and Ali (2013)] .Hyaluronate reduces the friction between the synovial folds of the capsule and the articular surfaces. Changes in the concentration of hyaluronate or lubricin in the synovial fluid will affect the overall lubrication and the amount of friction that is present [Stokes (1966)] . Normal healthy synovial fluid is highly non-Newtonian, such that the viscosity reduces markedly with the increase of shear rate. The marked non-Newtonian features of synovial fluid from normal, healthy joints is evidenced in researches where increase in shear rate of about four orders of magnitude (10^{-1}-10^{3}) sec^{-1} causes a reduction in the effective viscosity of the fluid of no less than three orders of magnitude (10^{2} –10^{-1}) Ns/m^{2}

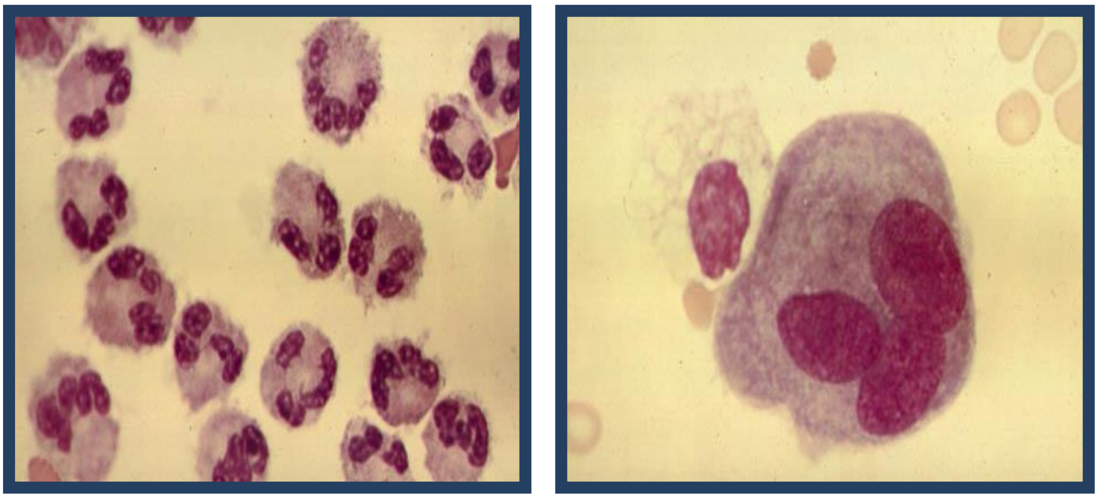

Figure 1. Show that Synovial Lining Cell

1. 1 Model of the hyaluronic acid molecular studies:

It is convenient and permissible to represent synovial fluid flow in the thin human hip joint gap between two bone surfaces , by two planes configuration for the purpose of analysis ; the plane solid is assumed to be rigid and the opposing component consists of a rigid core (representing the bone) covered by a layer of porous ,elastic material (representing the articular), synovial fluid serves as a lubricant (preventing bone-to-bone friction).Because it was studied hydrodynamic effects of the hyluronic acid on bearing so explain sliding motion (u_1, u_2) and squeeze action (v_1, v_2) for each surface .As well as it was founded in the model design mathematical, referring to the length (L) of part hylauronic acid molecular where the length division to four regimes (a, b, c, d) represent small long chain, as shown in figure (2) so clear us pressure inlet (P_i) and pressure outlet (P_0) for important in studied hyluronic acid chain with molecular , in this model requrid to film thickness during flow synovial fluid where (H_i) represent film thickness inlet and $(H_\circ)$ represent film thickness outlet .

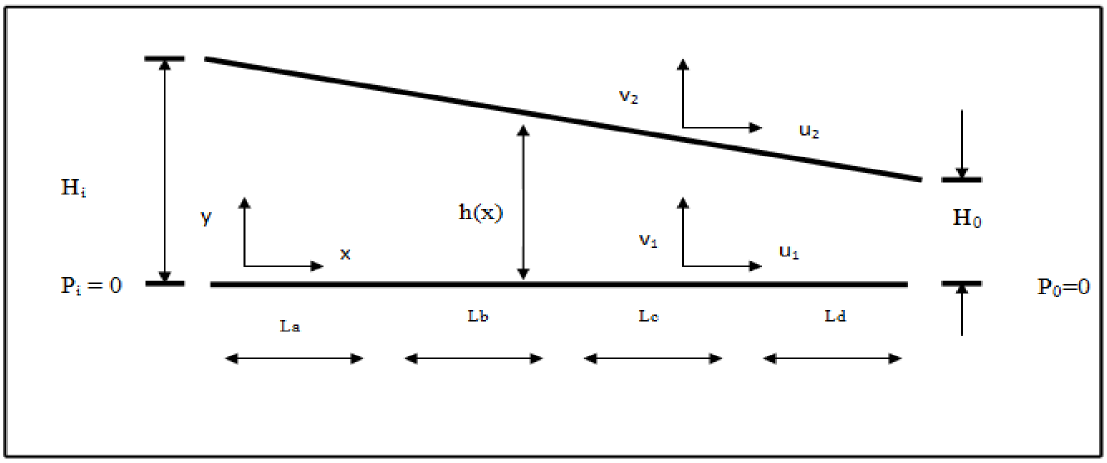

Figure 2. Model of hyluronic acid chain in hydrodynamic

1.1.1 Numerical parameters

The governing equations sliding motion and squeeze action of molecular in synovial fluid is very difficult if not impossible to solve or to study the mechanism estimated in vivo. For this reason the need for a numerical study arises, and the selection of factors values for the best estimation to get good analytics results. The parameters chosen were based on previously measured selected values used in researches, specializing with problem to reach two parameters that are close to reality, such as sliding motion and squeeze of film thickness. The values of film thickness were obtained in chapter four.

Table 1.The estimated values of the parameters involved[Ruggiero and Gomez.,(2013)] and[Higginson and Norman (1974)]

Parameters	Value	Unite
Load (W)	4500	N
Velocity (U)	0.075	m/s
Squeeze (V)	0.035	m/s
Synovial fluid viscosity in normal joint Synovial fluid viscosity in joint	0.01- 0.4 0.0025-0.2	N.s/ m^2
Length of bearing (L)	40	Mm
Film thickness inlet(h_i)	4	μm
Film thickness outlet(h_o)	2	μm
Radians α	0.05	
Length of molecular (L$_p$)	0.5×10^{-3}	Mm
Thickness of molecular (T$_p$)	0.5×10^{-3}	μm

1.12. Basic Equation:

Knowing the velocities of the surfaces and their geometry, we can calculate the pressures in the synovial film with Reynolds Equation[Tawer (1966)].

$$\underbrace{\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}.\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\eta}.\frac{\partial p}{\partial y}\right)}_{Poieuille}=\underbrace{6\rho(U_1-U_2)\frac{\partial(h)}{\partial x}}_{Phyical\quad wedge}+\underbrace{12\rho(V_2-V_1)}_{Squeeze\text{ action}} \qquad (1)$$

The first two terms of Equation(5.1) are the Poiseuille terms and describe the net flow rates due to pressure gradients within the lubricated area; The three foure terms physical wedge and normal squeeze actions are the two major pressure-generating devices in hydrodynamic or self-acting fluid film bearings . The rheological characteristic of the lubricant it is assumed to be non- Newtonian fluid due to the particle concentration effect , as fluid inertia effect neglected, very thin film and no slip between fluid particles and surfaces consequences negligible variations in pressure i.e. ($\frac{\partial p}{\partial y} = 0$) , since incompressible fluid then density negligible .Therefore equation (1) becomes

$$\underbrace{\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}.\frac{\partial p}{\partial x}\right)}_{Poieuille} = \underbrace{6(U_1 - U_2)\frac{\partial(h)}{\partial x}}_{Phyical\ \ wedge} + \underbrace{12(V_2 - V_1)}_{Squeeze\ \text{action}} \qquad (2)$$

1.1.3 Pressure distribution in bearing

The classical Reynolds equation for hydrodynamic lubrication is used to find the pressure distribution in a bearing with an incompressible lubricant, for the geometry of the figure (2).Integrating equation (2) Twice with respect to x given

$$p = \frac{12\eta\ (U_1 - U_1)}{2}.\frac{x}{h^2} + \frac{12\eta\ (V_2 - V_1)}{h^3}.\frac{x^2}{2} + \frac{12\eta A}{h^3}x + 12\,\eta\,\frac{\mathrm{B}}{h^3} \qquad (3)$$

Where A and B are constants of integration . Since the lower side of the loaded static and straight level ,thus film thickness can be written as a function of x

$$h = h_i - (h_i - h_{\circ})\frac{\mathrm{x}}{\mathrm{L}} \qquad (4)$$

$$\frac{dh}{dx} = -\frac{(h_i - h_0)}{L}$$

Where L length of the loader towards the two movement surfaces, h_i and h_0 represent film thickness inlet and film thickness outlet respectively Boundary conditions for the pressure and film thickness in the synovial hip joint as given :

$$\left.\begin{array}{lll} P = P_i & ,x = 0 & ,h = h_i \\ P = P_{\circ} & ,x = L & ,h = h_{\circ} \end{array}\right\} \qquad (5)$$

Substituting in equation (1.3) and applying the boundary condition it was obtained pressure distribution in the film region .

$$P = \frac{12\eta\ (U_1 - U_2)}{2}.\frac{1}{(h_i - (h_i - h_0)\frac{L}{L})^2}L + \frac{12\eta\ (V_2 - V_1)}{(h_i - (h_i - h_0)\frac{L}{L})^3}.\frac{L^2}{2} + \frac{12\eta A}{(h_i - (h_i - h_0)\frac{L}{L})^3}.x$$
$$+12\eta\frac{B}{(h_i - (h_i - h_0)\frac{L}{L})^3} \qquad (6)$$

Where

$$A = \frac{-h_i h_0}{h_i + h_0}(U_1 - U_2) - \frac{{h_i}^2 {h_0}^2}{6\eta L(h_i + h_0)}(P_i - P_0) - \frac{L h_i}{(h_i + h_0)}(V_2 - V_1) \qquad (7)$$

and $$B = \frac{-L}{2(h_i^2 - h_0^2)}(U_1 - U_2) + \frac{P_i h_i^2 - P_{0_i} h_i^2}{12\eta(h_i^2 - h_0^2)} + \frac{L^2(V_2 - V_1)}{(h_i^2 - h_0^2)(h_i + h_0)}$$

It was easy to see by figure (2) that two section open this fact lead to values pressure inlet (P_i) and pressure outlet ($P_\circ$) is equal zero.

1.1.4 Load carrying capacity in bearing:

In general ,The load carrying capacity is obtained by integration of the pressure distribution (6) over the plane

$$W = \int_0^L P \ \ \mathrm{dx} \qquad (8)$$

$$W = \frac{6\eta\,(U_1 - U_2)L^2}{h_\circ^2} H_{W1} - \frac{12\eta\,(V_2 - V_1)L^2}{h_\circ^3} H_{W2} + L(H_{W3} P_i + H_{W4} P_\circ) \qquad (9)$$

Where

$$H_{w1} = \left\{ \frac{Ln\,\mathrm{n}}{(n-1)^2} - \frac{2}{(n^2 - 1)} \right\} \qquad (9a)$$

$$\mathrm{H}_{w2} = \left\{ \frac{Ln\,\mathrm{n}}{(\mathrm{n}-1)^3} - \frac{2}{(n^2 - 1)(n+1)} \right\} \qquad (9b)$$

$$H_{W3} = \left\{ \frac{n}{n+1} \right\} \qquad (9c)$$

$$H_{W4} = \left\{ \frac{1}{n+1} \right\} \qquad (9d)$$

and n represent $h_i / h_\circ$ through our knowledge of the values of film thickness inlet and film thickness outlet seen table (1.2) it was found that n = 2. Most studies agree that the maximum extent of load is when n = 2.2. The values in table (5.2) represent normal human hip joint

Table 2 .Values parameters in equation (.9)

H_{W1}	H_{W2}	H_{W3}	H_{W4}
0.0257	0.2935	0.5875	0.3125

After substitute values in table (1) in the equation (9) then equation loads carrying capacity (W) become

$$W = 0.1602\frac{\eta\ (U_1 - U_2)L^2}{h_\circ^2} - 3.522\frac{\eta\ (V_2 - V_1)L^2}{h_\circ^3} + L\,(0.6875P_i + 0.3125P_\circ) \qquad (10)$$

1.1.5 Center of pressure

The location of the center of pressure x_{cp} indicates the position at which the resulting force is acting .The expression for calculating the location is

$$Wx_{cp} = \int_0^L P\ x\ dx \qquad (11)$$

$$Wx_{cp} = \frac{6\eta(U_1 - U_2)L^3}{h_\circ^2} Hw_{cp1} - \frac{12\eta(V_2 - V_1)L^4}{h_\circ^3} Hw_{cp2} - L^2 Hw_{cp3} P_\circ \qquad (12)$$

Where

$$Hw_{cp1} = \left\{ \frac{0.5(n-1)(5n+1) - n\,(n+2)Ln\,n}{(n-1)^3\,(n+1)} \right\} \qquad (12a)$$

$$Hw_{cp2} = \left\{ \frac{0.5(\mathrm{n}-1)(\mathrm{n}-1)^2 - n\,(n+2)Ln\,\mathrm{n}}{(\mathrm{n}-1)^3\,(n+1)} \right\} \qquad (12b)$$

$$Hw_{cp3} = \left\{ \frac{n^2\ Ln\,\mathrm{n} - \mathrm{n}^2(\mathrm{n}-1) + 0.5(\mathrm{n}-1)^2}{(\mathrm{n}-1)^3\,(n+1)} \right\} \qquad (12c)$$

Table following clear that values $Hw_{cp1}, Hw_{cp2}, Hw_{cp3}$

Table(3) .Values parameters in equation (12)

N	Hw_{cp1}	Hw_{cp2}	Hw_{cp3}
2	-0.04	-2.52	-0.35

Now substitute values (12a), (12b) and (12c) in equation(1.12), it was obtained formula following :

$$Wx_{cp} = -\frac{0.24\ \eta\,(U_1 - U_2)L^3}{h_\circ^2} + \frac{30.2\,\eta\,(V_2 - V_1)L^4}{h_\circ^3} + 0.36\,L^2 P_\circ \qquad (13)$$

After that divide the load carrying capacity represent in equation (11) on result of integration of the equation (13) .Therefore, the center of pressure can be written as

$$x_{cp} = (\frac{-0.24\ \eta\ (U_1 - U_2)}{h_\circ^2} L^3 - \frac{30.2\ \eta\ (V_2 - V_2)}{h_\circ^3} L^4 + 0.36L^2 P_\circ)/(\ 0.1602\frac{\eta\ (U_1 - U_2)}{h_\circ^2} L^2$$

$$-3.522\frac{\eta\ (V_2 - V_2)}{h_\circ^3} L^2 + L(0.6875P_i - 0.3125P_\circ) \qquad (14)$$

It is easy calculate value center of pressure through compensation for values sliding motion , squeeze action and length mentioned in the table (1) in the equation (14) then we will find that the value of center of pressure in the x direction is x_{cp} =30 m

1.1.6 Friction force

The friction force can be obtained to integrate the shear stress on the surfaces along the entire lubricating region. Assume that Newtonian fluid

$$\tau = \eta \,(\frac{\mathrm{du}}{\mathrm{dy}}) \qquad (15)$$

Where the term $(\frac{du}{dy})$ at both values of y is obtained from the velocity distribution

$$u = \frac{1}{2\eta}\ \frac{dP}{dx}\, y(y-h) + \frac{h-y}{h} U_1 + \frac{y}{h} U_2 \qquad (16)$$

Now we calculate the first derivative of the function (16) with respect to y:

$$\frac{\partial u}{\partial y} = \frac{1}{\eta}\frac{dp}{dx}(y-h) + \frac{(U_2 - U_1)}{h} \qquad (17)$$

Friction force on down surface coordinate (y = 0) is given by

$$F \ = \int_0^{\mathrm{L}} \tau_{y=0}\ dx \qquad (18)$$

Substitute equation (15) and (1.17) in formula (18) and after integration for $F_{y=0}$ was obtained the friction forces

$$F = \frac{-\eta\ (U_1 - U_2)L}{h_\circ}\left\{\frac{4}{n-1} Ln\,\mathrm{n} - \frac{6}{\mathrm{n}+1}\right\} - \frac{-6\eta\ (V_2 - V_1)L^2}{h_\circ}\left\{\frac{2}{n^2-1} - \frac{1}{(\mathrm{n}-1)^2} Ln\,\mathrm{n}\right\} + \frac{nh_\circ}{n+1}(P_i - P_\circ) \qquad (19)$$

Friction force on up surface coordinate y = h is given by

$$F \ = \int_0^{\mathrm{L}} \tau_{y=h}\ dx \qquad (20)$$

Substitute equation (15) and (17) in formula (20) and after integration for $F_{y=h}$ was obtained the friction forces

$$F = \frac{\eta\ (U_1 - U_2)L}{h_\circ}\left\{\frac{2}{n-1}Ln\,\mathrm{n} - \frac{6}{\mathrm{n}+1}\right\} - \frac{-6\eta\ (V_2 - V_1)L^2}{h_\circ}\left\{\frac{2}{n^2-1} - \frac{1}{(\mathrm{n}-1)^2}Ln\,\mathrm{n}\right\} - \frac{nh_\circ}{n+1}(P_i - P_\circ) \qquad (21)$$

It was calculate the coefficient of friction substituting friction force (F) from equation (21) and load carrying capacity (W) from equation (10) into formula $C_f = F/W$.

$$C_f = \frac{\dfrac{\eta\ (U_1 - U_2)L}{h_\circ}\left\{\dfrac{2}{n-1}Ln\,\mathrm{n} - \dfrac{6}{\mathrm{n}+1}\right\} - \dfrac{-6\eta\ (V_2 - V_1)L^2}{h_\circ}\left\{\dfrac{2}{n^2-1} - \dfrac{1}{(\mathrm{n}-1)^2}Ln\,\mathrm{n}\right\} - \dfrac{nh_\circ}{n+1}(P_i - P_\circ)}{\left(0.1602\dfrac{\eta\ (U_1 - U_2)L^2}{\mathrm{h}_\circ^2} - 3.522\ \dfrac{\eta\ (V_2 - V_1)L^2}{h_\circ^3} + \mathrm{L}(0.6875\,\mathrm{P_i} + 0.3125\mathrm{P}_\circ)\right)} \qquad (22)$$

1.1.7 Flow rate in thin gap between two surface articular

The lubricant flow through gap between the two articular cartilage in a figure (5.1) is given by

$$Q_x = \int_0^h u\ \mathrm{dy} \qquad (23)$$

Substituting for you from equation (16) and integrating with the assumption that the flow will be the same at all values of x if the side leakage is neglected givens .

$$Q_x = h_\circ\frac{(U_1 - U_2)}{2}n - h_\circ\frac{(U_1 - U_2)}{2}n\frac{n-1}{n+1} + \frac{1}{6\eta L}\frac{n^2h_\circ^3}{n+1}(P_i - P_\circ) + \frac{Ln\,\mathrm{n}}{n+1}(V_2 - V_1) \qquad (24)$$

1.1.8 Resultes

The variation of the pressure distribution (P) generated in film regime as a function of length of hyaluronic acid (L) for different values of squeeze action parameters (V) are as shown in figure (3) with using equation (10) and value (η= 2×10-2 Ns/m2) .It is observed that the effect of the squeeze action on increase film pressure and length ,especially in the vicinity of the position (L = 20 at V= 15×10^{-3} m/s). The percentage rate of increase in pressure distribution was approximately 75 % at (L= 20) this result appears important hyaluronic acid in synovial fluid for about increase pressure distribution .The effect of length parameter (L) on the variation of pressure (P) with(film thickness) is shown in figure (4) with the parametric (U= 75×10^{-3} m/s). It is observed that the curves (P) increases with increasing values of (L).

As shown in figure (5) the values of the film thickness in fluid film lubrication has a profound effect on the load carry capacity (W) that are developed. It is observed load carry capacity increase with decreasing values of film thickness and this increasing (W) is more accentuated for smaller values of film thickness. The variation of the load carry capacity (W) as function of length of hyaluronic acid chain (L) for different values of viscosity parameters are as shown in figure (6) .It is observed load carry capacity decrease with decrease values of viscosity and this decrease (W) is more accentuated for osteoarthritis joint .

Figures (7-8) demonstrate that there is a non- linear relation between friction force and length hyaluronic acid chain (L) for different values of sliding motion (U) and film thickness parameters (h_i) with using equation (22) with fixed values (V = 0.035 m/s and h_0 = 3-1.5 μm). In figure (7) shown the percentage rate of increase in friction force approximately 65 % at (L = 40 (mm). It is observed in figures (7-8) that friction decrease with increasing values of (h) and (U) . It is worth mentioning that an increase in friction force result to decreasing values of film thickness the percentage rate of increase in friction force approximately 70 % at (L= 40 mm) ,

The variation coefficient of friction (COF) as a function of film thickness (h) for different values of viscosity fluid are as shown in figure (9) with using equation (23)and with fixed values (V = 0.035 m/s and U = 0.075m/s) .It observed that increase in coefficient of friction with increase viscosity in normal synovial fluid comparison with disease where decrease viscosity lead to decrease in coefficient of friction.

Figure (10-11) demonstrate that there is a linear relation between flow rate and (film thickness – velocity) after applying equation (24) in the computer program with the parametric values (V = 0.035 m/s). while in figure (11) it is observed that flux decrease with decrease velocityIt is worth mentioning that an increase in the rate flow with increase film thickness and thus the maximum flow rate is achieved at film thickness 2 μm . flow rate it same of all regimes since $Q_a = Q_d$

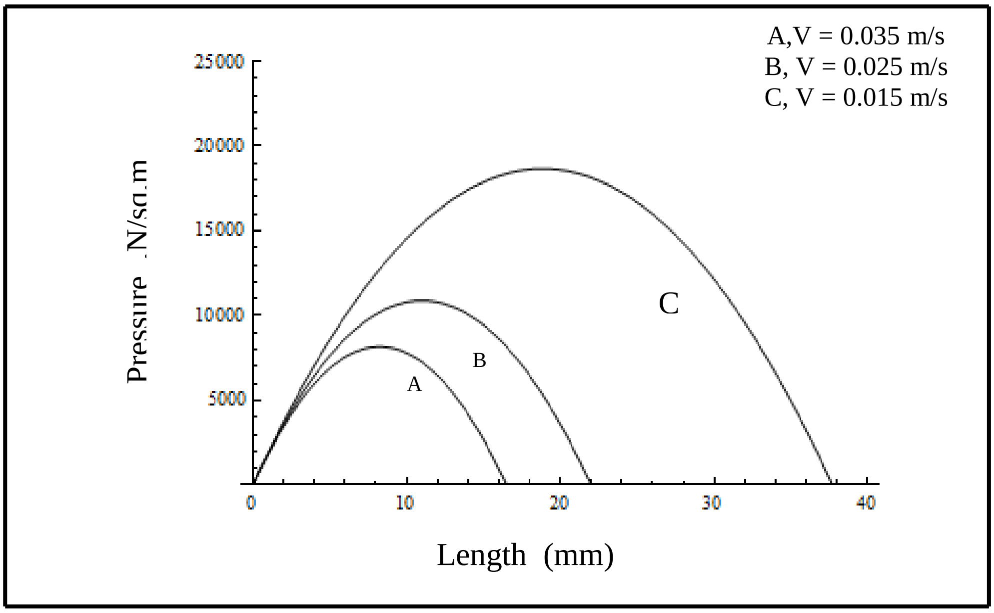

Figure (3) Hydrodynamic pressure distribution (P) with length (L) for different squeeze action (V).

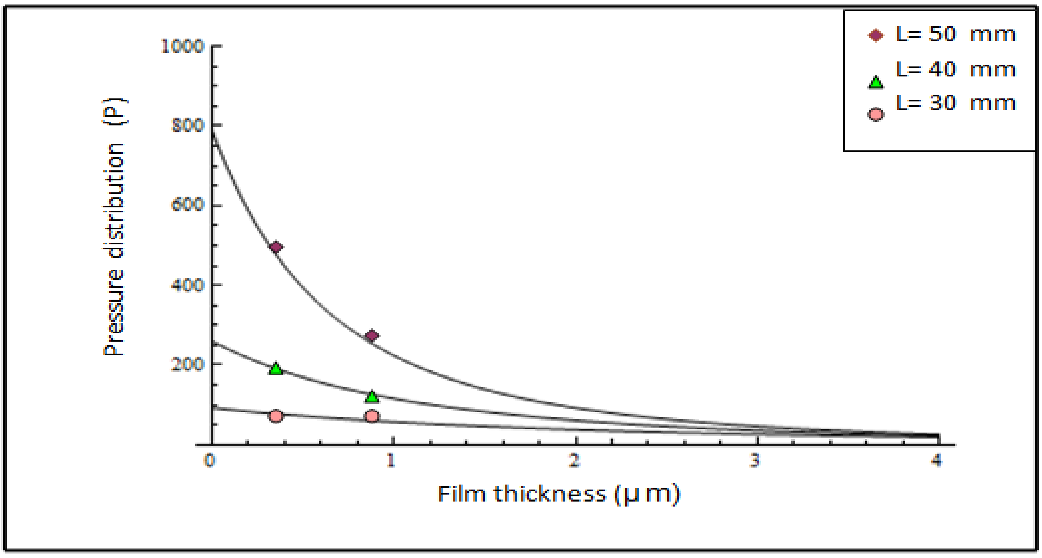

Figure (4) show that variation pressure distribution (P) with film thickness (h) for different length hyaluronic acid chain (L).

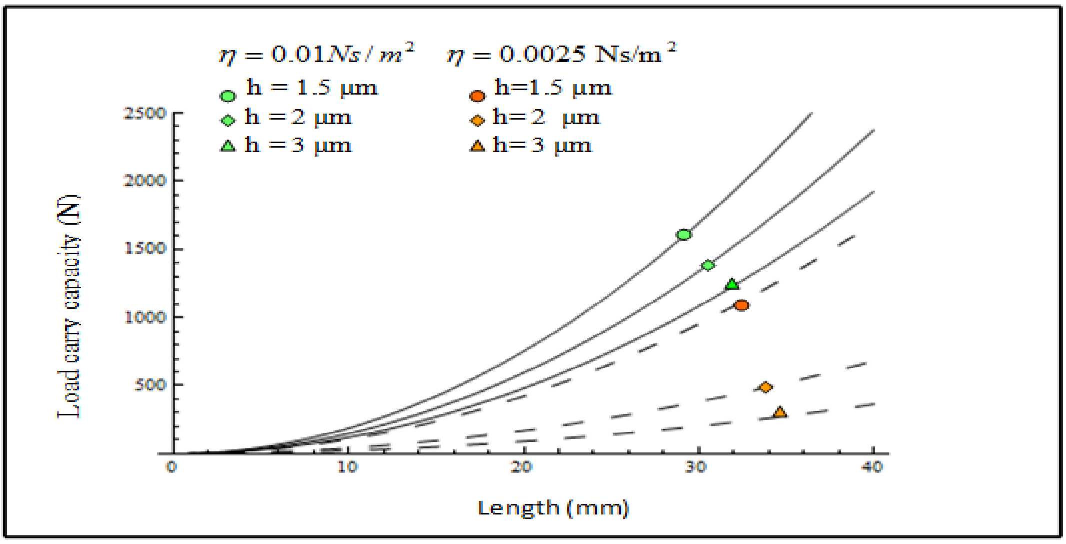

Figure (5) load carry capacity W versus length (L) for different values film thickness h_0 and viscosity (η).

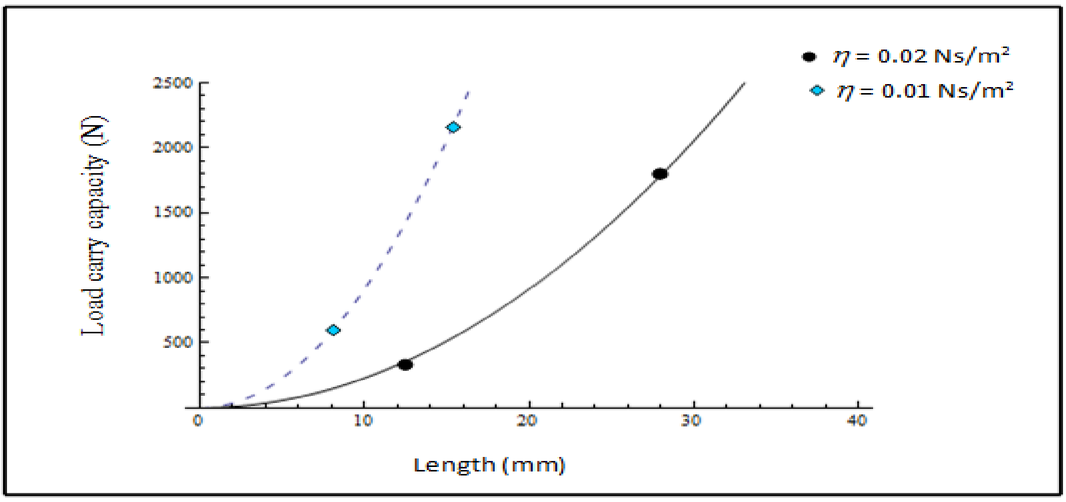

Figure (6) load carry capacity W versus length (L) for different values viscosity in healthy and osteoarthritis joint

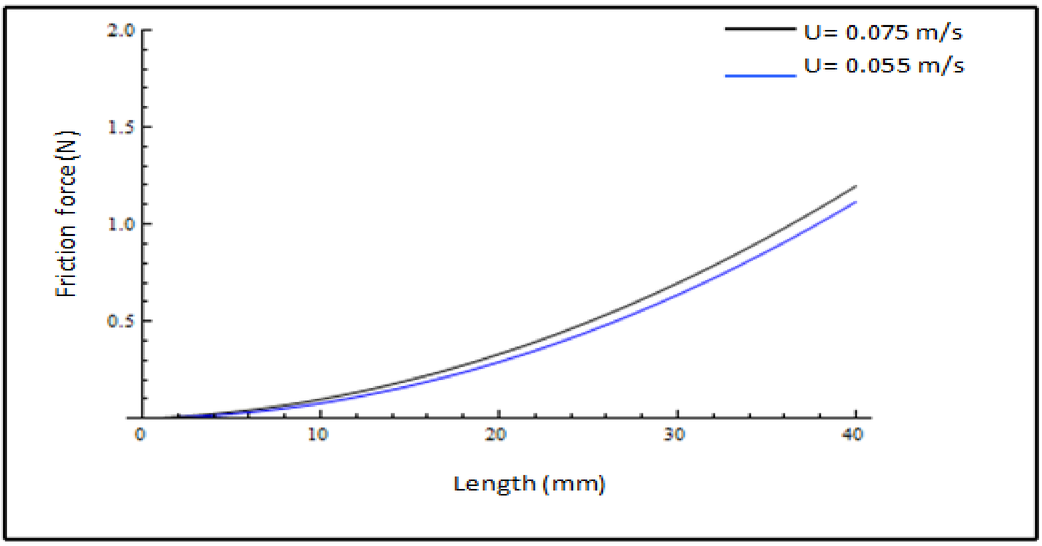

Figure (8) friction force F versus length (L) for different values sliding motion

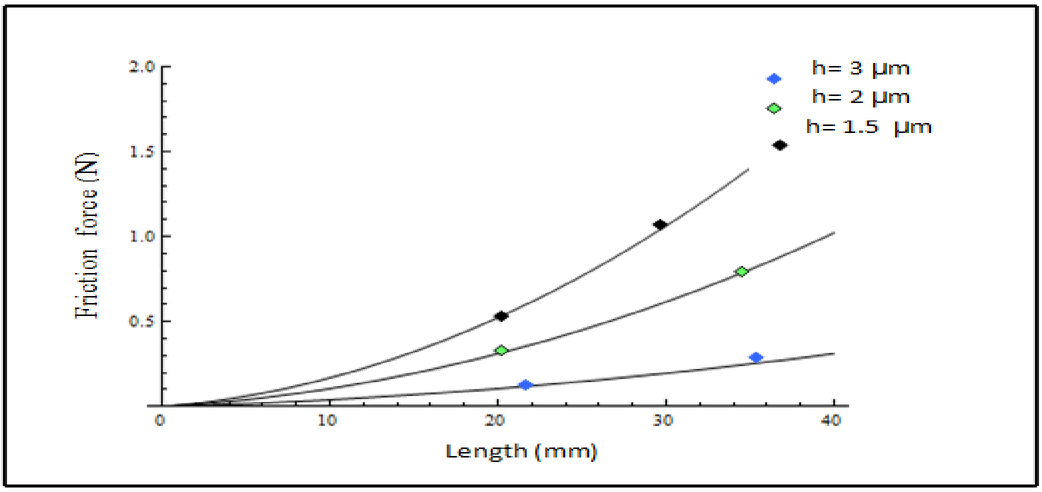

Figure (7) friction force F versus length (L) for different values film thickness

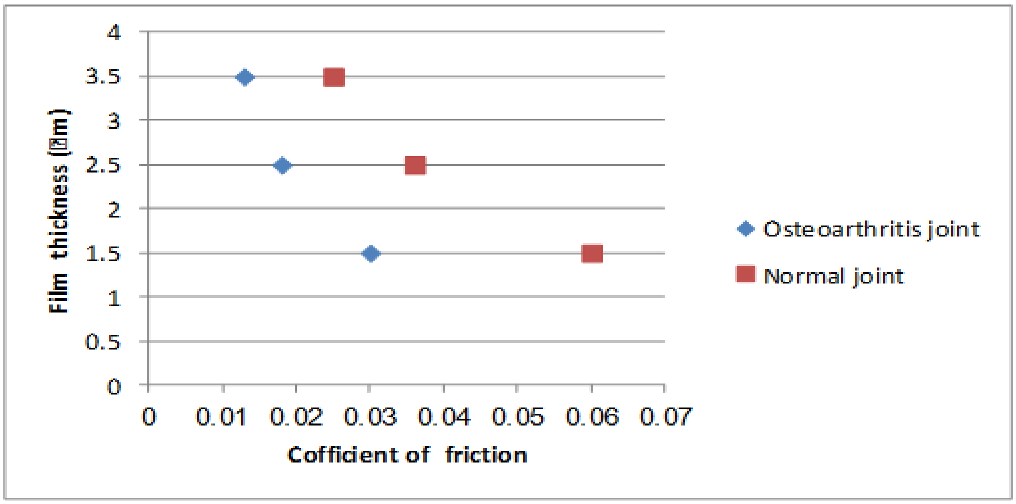

Figure (9) show that variation coefficient of friction (C_f) with film thickness (h) for different velocity in healthy and osteoarthritis.

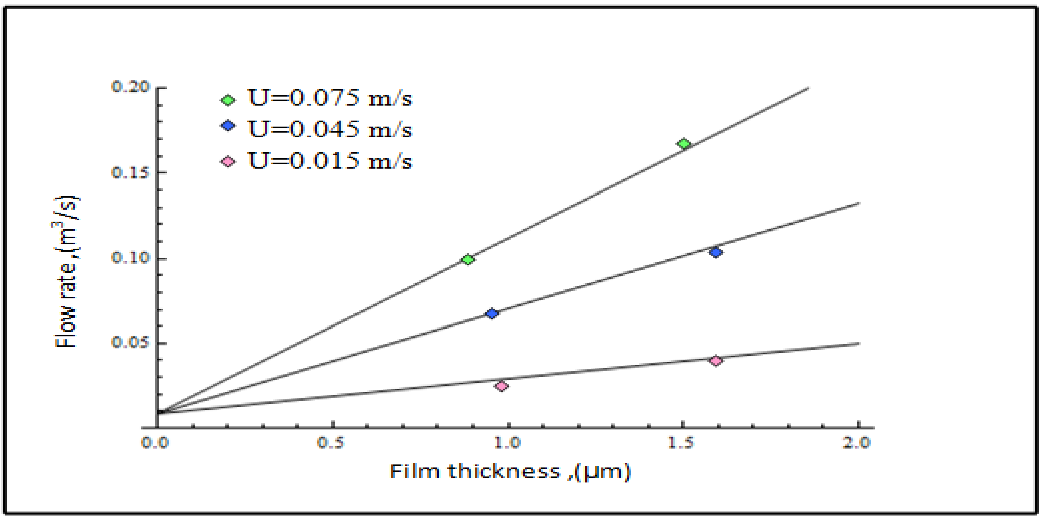

Figure (10)show that different flow rate with different film thickness for different sliding motion.

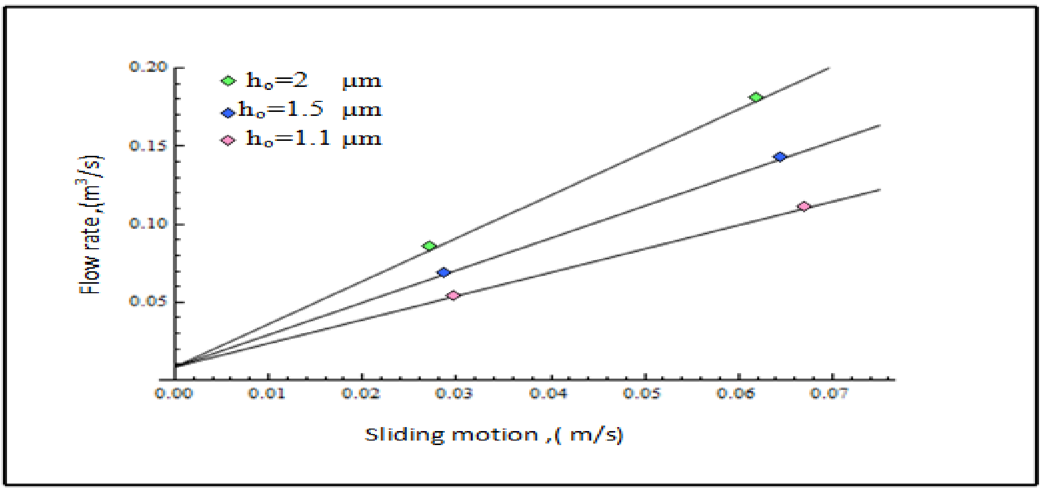

Figure (11)show that variation flow rate with velocity for different film thickness (h) .

1.1.9.Conclusions

The effects of hyaluronic acid on the hydrodynamic lubrication between a two articular cartilage are presented in synovial human hip joint . The modified Reynolds equation, governing the hydrodynamic film pressure is derived using the mathematical model describe long chin polymer exit in synovial fluid .According to the results obtained the following conclusions :

1) The effect of hyaluronic acid on increase pressure film with different squeeze action and length of hyaluronic acid with particle , Comparing between squeeze action and length ,It was found squeeze has influence more than length.

2) The effect of hyaluronic acid on increase load carrying capacity of articular cartilage with different film thickness in (hydrodynamic –squeeze- elastohydrodynmic)and viscosity in healthy and disease.

3) The effect of film thickness and sliding motion on friction force between surface articulars that decreases with the presence of hyaluronic acid in synovial fluid in healthy hip joint .

4) The effect of film thickness and sliding motion on increase flow rate synovial fluid between articulars cartilage . This increase is attributed to the presence hyaluronic acid.

1.1.10 Reference

Albert.A and Ali.A .,(2013)"The hydrodynamic squeeze film lubrication of the ankle joint".International Journal of Mechanical Engineering and Applications .Vol.1,No.2,pp.34-42.

Higginson.G and Norman. R .(1974) " Model Investigation of Squeeze-Film Lubrication in Animal Joints", Physics in Medicine and Biology 19(6), 78.

Ruggiero.A, and Gomez.E.,(2013) "Approximate closed-form solution of the synovial fluid film force in the human ankle joint with non-Newtonian lubricant", Teratology International ,57,pp.156-161.

Lin.J and Liao.W (2005) "The effects of couple stresses in the squeeze film characteristics between a cylinder and a plane surface " Journal of Marine science and technology,Vol.12,No.2,pp.119-123,

Stokes ,V.K.(1966)" Couple stresses in fluids", Physics of Fluids, Vol. 9, 1966,pp. 1709-1715.

Tawer R.I (1966) "lubrication in Synovial Joints", A Theoretical Analysis , J.Bone Jt.Surg.,45A,(6),1241 .