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Solvability of Time-Varying Descriptor Systems Using Non-Classical Variational Approach

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Abstract

This paper focuses on studying the solvability of time varying linear descriptor system. A non-classical variational approach have been developed for finding an approximate solution for time varying linear descriptor system in the presence of system uncertainty and the existence of a class of consistence initial conditions.

Keywords: Consistent Initial Condition, Non-classical Variational Approach, Regular System, time-varying linear Descriptor system.

1. Introduction

Descriptor systems are those the dynamics of which are governed by a mixture of algebraic and differential equations .During the past decades descriptor systems have attracted much attention due the comprehensive applications in chemical engineering, control theory [3], electrical [1] and mechanical models [6] atc.

A constructive method to give a variational formulation to every linear equation or a system of linear equations by changing the associated bilinear forms was given in [4], [5]. this method has a more freedom of choice a bilinear form that makes a suitable problem has a variational formulation. The solution then may be obtained by using the inverse problem of calculus of variation .To study this problem and its freedom of choosing such a bilinear form and make it easy to be solved numerically or approximately, we have mixed this approach with some kinds of basis, for example Ritz basis of completely continuous functions in a suitable spaces, so that the solution is transform from indirect approach to direct one. That since the linear operator is then not necessary to be symmetric, this approach is named as a non-classical variational approach.

2. Description of the Problem

For the time varying linear descriptor system

$$E(t)X'(t) = A(t)X(t) + Bu(t) \quad \ldots(1)$$

Where

$E(t), A(t)$ are $n \times n$ matrices , $x(0) = x_0$

$$u(t) \in A\Omega\,(x_0\,, J) = c^k(J) \cap \left\{x_0 + T(t_0)\sum_{i=0}^{k} N^k(t)\,(S(t)f(t))^{(k)}\right\}$$

and $u(t)$ is k-time continuously differentiable control .

3. Some Basic Concept

Some basic definitions, lemmas and theorems as well as necessary requirements for the solvability of time varying uncertain linear descriptor system have been presented and as follows:

3.1 Definition [7]

Consider the time – varying linear descriptor system

$$E(t)X'(t) = A(t)X(t) \quad \ldots(2)$$

Where $(E(t), A(t)) \in C(I, \mathbb{R}^{n\times n})^2$, $n \in \mathbb{N}$ then a function $X: \mathcal{J} \to \mathbb{R}^n$ is called solution of **(2)** if and only if X is a continuously differentiable function on open interval $\mathcal{J} \leq \mathbb{R} \leq I$ and solve (2) for $t \in \mathcal{J}$;it called global solution if $\mathcal{J} = I \leq \mathbb{R}$ for $t \in \mathcal{J}$.

3.2 *Remark* [7]

If $(S(t), T(t)) \in C(I; GL_n(\mathbb{R}) \times C^1 (I; GL_n (\mathbb{R})), then$
$X: \mathcal{J} \to \mathbb{R}^n$Solves (2) if and only if $w(t) = T^{-1}(t)X(t)$Solves

$$S(t)E(t)\dot{w} = \left[S(t)A(t)T(t) - S(t)E(t)\dot{T}(t)\right]w$$

$$\tilde{E}(t)\dot{w} = \tilde{A}(t)w \qquad \ldots(3)$$

Where

$$\tilde{E}(t) = S(t)E(t)T(t), \tilde{A}(t) = S(t)A(t)T(t) - S(t)E(t)\dot{T}(t)$$

This new system is equivalent to the system (2) where

$$\frac{d}{dt}T^{-1}(t) = -T^{-1}(t)\dot{T}(t)T^{-1}(t)$$

3.3 Definition [7]

The descriptor system $(E(t), A(t)) \in C(I, \mathbb{R}^{n\times n})^2$ is called transferable into standard Canonical form (SCF), if and only if there exist

$$(\mathrm{S}(t), \mathrm{T}(t)) \in C\left(I; GL_n(\mathbb{R})\right) \times C^1\left(I; GL_n(\mathbb{R})\right)$$

such that

$$E(t) = diag\left(I_{n_1}, N(t)\right), A(t) = diag\left(J(t), I_{n_2}\right) \qquad \ldots(4)$$

Where $N(t): I \to \mathbb{R}^{n_2\times n_2}$ is a pointwise strictly lower triangular and $J(t): I \to \mathbb{R}^{n_1\times n_1}$.

3.4 Proposition [2]

The system (2) is transferable into standard canonical form if and only if $(E(t), A(t))$ is regular.

3.5 Definition [8]

The set of all pairs of consistent initial values of (1) is denoted by

$$V_{E,A} = \{(t_0, x_0) \in \mathbb{R}\times\mathbb{R}^n | \, \exists \, (local) solution \; x(t) of \; (1): t_0 \in dom \, x(t), x(t_0) = x_0\}$$

and the linear subspace of initial values which are consistent at time $t_0 \in I$ is denoted by

$V_{E,A}(t_0) = \left\{x_0 \in \mathbb{R}^n | (t_0, x_0) \in V_{E,A}\right\}$ and since $X: \mathcal{J} \to \mathbb{R}^n$ is a solution of (1), then $x(t) \in V_{E,A}(t)$ for all $t \in \mathcal{J}$.

3 Proposition [8]

suppose that the Descriptor system (2) is transferable into (SCF) as in (4), then

1)$(t_0, x_0) \in V_{E,A} \leftrightarrow x_0 \in im\, T(t_0)\begin{bmatrix} I_{n1} \\ O \end{bmatrix}$

2) Any solution of the initial values problem (1), $X(t_0) = x_0$, where $(t_0, x_0) \in V_{E,A}$, extends uniquely to a global solution $X(t)$, and this solution satisfies

$x(t) = U(t, t_0)x_0$, $U(t, t_0) = T(t)\begin{bmatrix} \phi_J(t, t_0) & 0 \\ 0 & 0 \end{bmatrix}T^{-1}$, where $t \in I, \phi_J(t, t_0)$ denotes the transition matrix of $\dot{w} = J(t)w$

4. Solvability of non-homogenous time- varying Descriptor system

One can deal in this subsection with a variation of constants formula for inhomogeneous time- varying linear differential – algebraic initial value problems

$$E(t)X'(t) = A(t)X(t) + f(t), X(t_0) = x_0 \quad \ldots(5)$$

Where $(t_0, x_0) \in \mathbb{R} \times \mathbb{R}^n and\ f \in C(I; \mathbb{R}^n)$.

4.1 Theorem [8]

Suppose that the descriptor system (5) is transferable into standard canonical form by some $(S(t), T(t)) \in C^n(I; GL_n(\mathbb{R}))^2$, then the following statements hold for $f \in C^{n2}(I; \mathbb{R}^n)$

1) The initial value problem (5) has a solution if and only if

$$\left\{x_0 + T(t_0)\begin{bmatrix} 0 \\ I_{n2} \end{bmatrix} \sum_{K=0}^{n2-1} \left(N(t)\right)^k \left([0, I_{n2}]S(t)f(t)\right)^{(k)}|_{t=t_0}\right\} \in imT(t_0)\begin{bmatrix} I_{n1} \\ 0 \end{bmatrix} \quad (6)$$

2) Any solution of (5) such that (6) can be uniquely extended into a global solution $x(t)$, and this solution satisfies, for the generalized transition matrix $U(t, t_0)$ of $(E(t), A(t))$for $t \in I$

$$x(t) = U(t, t_0)x_0 + \int_{t_0}^{t} U(t, s)T(s)S(s)f(s)ds - T(t)\begin{bmatrix} 0 \\ I_{n2} \end{bmatrix} \sum_{K=0}^{n2-1} \left(N(t)\right)^k \left([0, I_{n2}]S(t)f(t)\right)^{(k)}$$

5 . Solvability of time Varying Singular Linear system using non classical Variational Method.

The main theoretical requirements (theorem) of the solvability and then finding the solution / approximate solution are the main theme of this section.

5.1 Theorem

For the time varying linear descriptor system (1) define a linear operator $L(t)$ by $L(t) = \left(E(t)\frac{d.}{dt} - A(t).\right)$ with domain $D(L(t)) \in imT(t_0)\begin{bmatrix} I_{n1} \\ 0 \end{bmatrix}$ and rang $\mathcal{R}(L(t))$ in $\mathbb{R}$

then the linear equation equivalent to the system (1) be as:

$$L(t)x = Bu(t), \quad (7)$$

if the system regular for a given $u(t) \in A\Omega\ (x_0, J)$ and L is symmetric with respect to a certain bilinear then the solution of equation (7) are critical points of the functional

$$J[x] = 0.5 < L(t)x, x > - < Bu(t), x >$$

Moreover, if the chosen bilinear form $< x, y >$ is non-degenerate on $D(L(t))$ and $\mathcal{R}(L(t))$ it furthermore valid that the critical points of the functional $J[x]$ are solution to the given equation (7).

Proof:

The solution exist since the system is regular and construct a linear equation form our system as follows:

Let $L(t)x = Bu(t)$, where $L(t) = \left(E(t)\frac{d.}{dt} - A(t).\right)$ be linear time varying operator with domain $D(L(t))$

Where $D(L(t)) = \left\{\{x \in \mathbb{R} | x_0 + T(t_0)\begin{bmatrix} 0 \\ I_{n2} \end{bmatrix} \sum_{K=O}^{n2-1} \left(N(t)\right)^k \quad [0, I_{n2}]\left(S(t)f(t)\right)^{(k)}|_{t=t_0}\right\}$

$D(L(t)) \in imT(t_0)\begin{bmatrix} I_{n1} \\ 0 \end{bmatrix}$ and rang $\mathcal{R}(L(t))$ in $\mathbb{R}$.

The existences of time varying, leads to select a bilinear form

$(x, y) = \int_0^{\tau} x(t)y(t)dt, \quad x, y : C[0, \tau] \to \mathbb{R}$.

In general, since $\frac{d}{dt}$ is appeared in $L(t)$, then $L(t)$ is not symmetric with respect to (x, y) . Define a new bilinear form <x, y> = $(x, L(t)y)$

Since $< L(t)x, y > = (L(t)x, L(t)y)$

$$= \left(\left(E(t)\frac{d}{dt}x - A(t)x\right), \left(E(t)\frac{d}{dt}y - A(t)y\right)\right)$$
$$= \left(\left(E(t)\frac{d}{dt}y - A(t)y\right), \left(E(t)\frac{d}{dt}x - A(t)x\right)\right)$$
$$= (L(t)y\,, L(t)x)$$
$$= < L(t)y, x >$$

then $< L(t)x, y > = < x, L(t)y >$ which is symmetric with respect to (x, y)

Since we are concerning for a given u , we can assume that the critical point $x_u = x$, for a given $u(t)$

Define the functional $J[x]$ in the term of the new bilinear form and all for a given $u(t)$

as $\delta J[x] = J[x + \delta x] - J[x]$

$$= 0.5 < L(t)\delta x, x > +0.5 < L(t)x, \delta x > - < Bu(t)\,, \delta x >$$
$$= 0.5(L(t)\delta x, L(t)x) + 0.5(L(t)x, L(t)\delta x) - (Bu(t)\,, L(t)\delta x)$$
$$= 0.5(L(t)x, L(t)\delta x) + 0.5(L(t)x, L(t)\delta x) - (Bu(t)\,, L(t)\delta x)$$
$$= 0.5 < L(t)x, \delta x > +0.5 < L(t)x, \delta x > - < Bu(t)\,, \delta x >$$
$$= < L(t)x - Bu(t), \delta x >$$

Where the symbol δ is the customary symbol of variation of a function used in calculus of variation .

If the x^* is a solution of (7)

$L(t)x^* - Bu(t) = 0$ for a given $u(t) \in A\Omega\,(x_0\,, J)$

And then $\delta J[x^*] = 0$.

If the chosen bilinear form $< x\,, y >$ is non- degenerate on $D(L(t))$ and $\mathcal{R}(L(t))$ let $\bar{x}$ is critical point of $J[X]$ for every $\delta\bar{x} \in D(L(t))$ i.e.

$\delta J[\bar{x}\,] = 0$

$J[\bar{x} + \delta\bar{x}] - J[\bar{x}] = 0$.

$0.5 < L(t)\bar{x} + \delta\bar{x}, \bar{x} + \delta\bar{x} > - < Bu, \bar{x} + \delta\bar{x} > - < L(t)\bar{x}, \bar{x} > + < Bu, \bar{x} > = 0$

$0.5 < L(t)\bar{x}, \bar{x} > +0.5 < L(t)\bar{x}, \delta\bar{x} > +0.5 < L(t)\delta\bar{x}, \bar{x} > +0.5$
$< L(t)\delta\bar{x}, \delta\bar{x} > - < Bu, \bar{x} > - < Bu, \delta\bar{x} > -0.5 < L(t)\bar{x}, \bar{x} > + < Bu, \bar{x} > = 0$

$0.5 < L(t)\bar{x}, \delta\bar{x} > +0.5 < L(t)\delta\bar{x}, \bar{x} > +0.5 < L(t)\delta\bar{x}, \delta\bar{x} > - < Bu, \delta\bar{x} > = 0$

$0.5(L(t)\bar{x}, L(t)\delta\bar{x}) + 0.5(L(t)\delta\bar{x}, L(t)\bar{x}) + 0.5(L(t)\delta\bar{x}, L(t)\delta\bar{x}) - (Bu, L(t)\delta\bar{x}) = 0$

$0.5(L(t)\bar{x}, L(t)\delta\bar{x}) + 0.5(L(t)\bar{x}, L(t)\delta\bar{x}) + 0.5(L(t)\delta\bar{x}, L\delta\bar{x}) - (Bu, L(t)\delta\bar{x}) = 0$

$(L(t)\bar{x}, L(t)\delta\bar{x}) + 0.5(L(t)\delta\bar{x}, L(t)\delta\bar{x}) - (Bu, L(t)\delta\bar{x}) = 0$

$(L(t)\bar{x}, L(t)\delta\bar{x}) + 0.5(L(t)\delta\bar{x}, L(t)\delta\bar{x}) - (Bu, L(t)\delta\bar{x}) = 0$

Since $(L(t)\delta\bar{x}, L(t)\delta\bar{x})$ is the nonlinear part

$< L(t)\bar{x}, \delta\bar{x} > - < Bu, \delta\bar{x} > = 0$

Therefore $< L(t)\bar{x} - Bu, \delta\bar{x} > = 0$, for every $\delta\bar{x} \in D(L)$.

And then from the non- degeneracy condition we have $L(t)\bar{x} - Bu = 0$

Hence if a given linear operator L is symmetric with respect to a non- degenerate bilinear form $< x\,, y >$ then there is a variational formulation of the given linear equation (7) and the critical point of the variational also solution to the original problem (1).

5.2 Theorem

For the descriptor system (1) with $x(0) = x_0$,
$x_0 \in W_k$ (the class of consistent initial condition) , $u(t) \in A\Omega(X_0, J)$.

If the solution $x(t)$ has been approximated by a linear combination of a suitable basis

i.e. $x(t) = x(0) + \sum_{i=1}^{n} a_i G_i(t)$ satisfies
1- $x_0 \in W_k$
2- $G_i(x_0) = 0$
3- G_i are continuous as required by the variational statement being
4- $\{G_i\}_i$ must be linearly independent
5- Satisfies the homogeneous from the specified condition.
Then the solution for the system $\frac{dJ}{da_j} = 0\,, \forall j = 1,2, \dots, n$ leads to the parameters a_i and implies the approximate solution for the descriptor system (1) for a given $u(t) \in A\Omega\ (x_0\,, J)$.
proof:

Using theorem (5.1)

$$J[x] = 0.5 < L(t)x, Lx > - < Bu(t), L(t)x > \qquad \dots (8)$$

with the classical bilinear form

$$< L(t)x, L(t)x > = \int_0^{\tau} L(t)x(t).\, L(t)x(t)dt,$$

$0 \le t \le \tau$, and the linear operator $L(t)$ as $L(t) = \left(E(t)\frac{d.}{dt} - A(t).\right)$ be linear time varying operator with domain $D(L(t))$ and rang $\mathcal{R}(L(t))$ in $\mathbb{R}$, then

$$J[x] = 0.5 \int_O^{\tau} [E(t)x' - A(t)x]^T [E(t)x' - A(t)x]dt - \int_0^{\tau} B^T u(t)[E(t)x' - A(t)x]\, dt \qquad \dots(9)$$

For a given $u(t) \in A\Omega\ (x_0\,, J)$ and approximate the solution x by a linear combination of function where $x_{io} \in W_k$ and select basis $G_i(t)$ satisfied (1) - (5) then

$$x_i(t) = x_{io} + \sum_{i=}^{n} a_i G_i(t)\,, \quad \frac{dx_i}{dt} = \sum_{i=1}^{n} a_i G_i{'}(t) \qquad \dots(10)$$

From (8) and (10) one can get

$$J[X; a] = 0.5 \int_0^{\tau} \left[E(t)\sum_{i=1}^{n} a_i G'(t) - A(t)X_{i0} - A(t)\sum_{i=1}^{n} a_i G_i(t)\right]^T \left[E(t)\sum_{i=1}^{n} a_i G_i{'}(t) - A(t)X_{i0} - A(t)\sum_{i=1}^{n} a_i G_i(t)\right] dt$$

$$- \int_0^{\tau} B^T u(t) \left[E(t)\sum_{i=1}^{n} a_i G_i{'}(t) - A(t)X_{i0} - A(t)\sum_{i=1}^{n} a_i G_i(t)\right] dt$$

In order to find the critical point for the last equation derive $J[X]$, for a_j , $j = 1, \dots, n$ and equate the result to zero i.e. $\frac{dJ}{da_j} = 0, \forall j = 1, \dots, n$, and for a given $(t) \in A\Omega\ (x_0, J)$, one get a system of algebraic equations as follows:

$$\begin{bmatrix} \int_0^\tau [E(t)G_0(t) - A(t)X_{10} - A(t)G_1(t)]^T . [E(t)G_0(t) - A(t)G_1(t)] & & \int_0^\tau [E(t).n.G_{n-1}(t) - A(t)X_{n0} - A(t)G_n(t)]^T . [E(t)G_0(t) - A(t)G_1(t)] \\ \vdots & \ddots & \vdots \\ \int_0^\tau [E(t)G_0(t) - A(t)X_{10} - A(t)G_1(t)]^T . \left[E(t).n.G_{n-1}(t) - A(t)G_n(t)\right] & & \int_0^\tau [E(t).n.G_{n-1}(t) - A(t)X_{n0} - A(t)G_n(t)]^T . \left[E(t).n.G_{n-1}(t) - A(t)G_n(t)\right] \end{bmatrix} . \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \int_0^\tau B^T u(t)\left(E(t)G_0(t) - A(t)G_1(t)\right) \\ \vdots \\ \int_0^\tau B^T u(t)\left(E(t)G_{n-1}(t) - A(t)G_n(t)\right) \end{bmatrix}$$

i.e. $\sum_{i=1}^{n} A(i,j)\ a_i = b_j, \forall j = 1, \dots, n,$ where

$$A(i,j) = \begin{bmatrix} \int_0^\tau [E(t)G_0(t) - A(t)X_{10} - A(t)G_1(t)]^T . [E(t)G_0(t) - A(t)G_1(t)] & & \int_0^\tau [E(t).n.G_{n-1}(t) - A(t)X_{n0} - A(t)G_n(t)]^T . [E(t)G_0(t) - A(t)G_1(t)] \\ \vdots & \ddots & \vdots \\ \int_0^\tau [E(t)G_0(t) - A(t)X_{10} - A(t)G_1(t)]^T . \left[E(t).n.G_{n-1}(t) - A(t)G_n(t)\right] & & \int_0^\tau [E(t).n.G_{n-1}(t) - A(t)X_{n0} - A(t)G_n(t)]^T . \left[E(t).n.G_{n-1}(t) - A(t)G_n(t)\right] \end{bmatrix}$$

$$b_j = \begin{bmatrix} \int_0^\tau B^T u(t)\left(E(t)G_0(t) - A(t)G_1(t)\right) \\ \vdots \\ \int_0^\tau B^T u(t)\left(E(t)G_{n-1}(t) - A(t)G_n(t)\right) \end{bmatrix}$$, and for a given $u(t) \in A\Omega\ (x_0, J)$

Since the consistent initial condition is selected, $x_0 \in W_k$, then obvious that the matrix $A(\mathrm{i,j})$ is nonsingular matrix, then the solution a_i is known and implies the solution x_i is exist for the system (1).

5.3 Example

Consider the regular time-varying descriptor system:

$$E(t)x'(t) = A(t)x(t) + Bu(t) \qquad , \quad 0 \le \mathrm{t} \le 1$$

Where $E(t) = \begin{bmatrix} 1 & -\mathrm{t} & -\mathrm{t}^2 \\ 0 & 1 & -\mathrm{t} \\ 0 & 0 & 0 \end{bmatrix}$, $A(t) = \begin{bmatrix} 1 & -\mathrm{t}-1 & \mathrm{t}^2 + 2\mathrm{t} \\ 0 & -1 & \mathrm{t}-1 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $u(t) = \sin \mathrm{t}$

define the bilinear form by $(x, y) = \int_0^{\mathrm{t}} x(t)y(t)\mathrm{dt}$ and construct the linear operator L as follows

$$L\ (t) = \left(E(t)\frac{d.}{dt} + A(t).\right)$$

Then one can get the linear equation $L\,(t)x = Bu(t)$, the solution for this linear equation is the solution for our system.

But since $L(t)$ contain a derivative $\frac{d}{dt}$ then $L(t)$ is not symmetric operator then the new bilinear being as $\langle x, y\rangle = (x, L(t)y)$.

Define the bilinear form as follows:

$$J[x] = \frac{1}{2}(L(t)x, L(t)x) - (Bu(t), L(t)x) \qquad \dots(11)$$

Now substitute L and the new bilinear operator in (11) to get

$$J[x] = 0.5\int_0^{\tau}[E(t)x' + A(t)x]\,[E(t)x' + A(t)\,x]^T dt - \int_0^{\tau} B^T u(t)[E(t)x' + A(t)\,x]dt$$

Now substitute $E(t), A(t), B, u(t),\ \tau = 1$ one gets

$$J[x] = 0.5\int_0^1 \begin{bmatrix} x_1' - tx_2' + t^2x_3' + x_1 - (t+1)x_2 + (t^2+2t)x_3 \\ x_2' - tx_3' - x_2 + (t-1)x_3 \\ x_3 \end{bmatrix} . \begin{bmatrix} x_1' - tx_2' + t^2x_3' + x_1 - (t+1)x_2 + (t^2+2t)x_3 \\ x_2' - tx_3' - x_2 + (t-1)x_3 \\ x_3 \end{bmatrix}^T dt$$
$$- \int_0^1 [0 \quad 0 \quad \sin t] \begin{bmatrix} x_1' - tx_2' + t^2x_3' + x_1 - (t+1)x_2 + (t^2+2t)x_3 \\ x_2' - tx_3' - x_2 + (t-1)x_3 \\ x_3 \end{bmatrix} dt \qquad \dots(12)$$

Approximate the solution x by a linear combination of continuous function as

$x_1(t) = x_{10} + \sum_1^5 a_i G_i \quad , G_i = t^i \quad , i = 1, \dots 5$

$x_2(t) = x_{20} + \sum b_i H_i \quad , H_i = t^i \quad , i = 1, \dots 5$

$x_3(t) = x_{30} + \sum C_i l_i \quad , l_i = t^i \quad , i = 1, \dots 5$

Which satisfies theorem (5.2)

Then $x_1' = x_{10} + \sum_1^5 i a_i G_{i-1}$

$x_2' = x_{20} + \sum_{i=1}^5 i b_i H_{i-1}$

$x_3' = x_{30} + \sum_{i=1}^5 i C_i l_{i-1}$

Where $(x_{10}, x_{20}, x_{30}) \in W_k = \{(x_{10}, x_{20}, x_{30}) : (x_{10}, x_{20}, x_{30}) = (1, s, 0)\}$, take s=1 and substitute $x_1, x_2, x_3, x_1', x_2', x_3'$ in (12) we get

$J[x] =$

$$0.5\int_0^1 \left[\begin{bmatrix} \sum i a_i t^{i-1} - t\sum i b_i t^{i-1} + t^2 \sum i C_i t^{i-1} + 1 + \sum a_i t^i - (t+1) - (t+1)\sum b_i t^i + (t^2+2t)\sum C_i t^i \\ \sum i b_i t^{i-1} - t\sum i C_i t^{i-1} - 1 - \sum bit^i + (t-1)\sum C_i t^i \\ \sum C_i t^i \end{bmatrix}\right. .$$

$$\begin{bmatrix} \sum ia_i t^{i-1} - t\sum ib_i t^{i-1} + t^2\sum iC_i t^{i-1} + 1 + \sum a_i t^i - (t+1) - (t+1)\sum b_i t^i + (t^2+2t)\sum C_i t^i \\ \sum ib_i t^{i-1} - t\sum iC_i t^{i-1} - 1 - \sum bit^i + (t-1)\sum C_i t^i \\ \sum C_i t^i \end{bmatrix}^T -$$

$$[0 \quad 0 \quad \sin t]\begin{bmatrix} \sum ia_i t^{i-1} - t\sum ib_i t^{i-1} + t^2\sum iC_i t^{i-1} + 1 + \sum a_i t^i - (t+1) - (t+1)\sum b_i t^i + (t^2+2t)\sum C_i t^i \\ \sum ib_i t^{i-1} - t\sum iC_i t^{i-1} - 1 - \sum bit^i + (t-1)\sum C_i t^i \\ \sum C_i t^i \end{bmatrix}\Bigg]dt$$

Now $\frac{dJ}{daj} = \frac{dJ}{dbj} = \frac{dJ}{dcj} = 0$, $\forall j = 1, \dots, 5$ leads to system of algebraic equation

$$\sum_{i=1}^{n} A(i,j)Z_i = Dj \qquad ,\forall j = 1, \dots, 5, \qquad Zi = \begin{pmatrix} a_i \\ b_i \\ c_i \end{pmatrix}$$

we compute $a_i , b_i , c_i, \forall i = 1,\dots,5$, and find the approximate solution

$x_1(t) = 1 + \sum_{i=1}^{5} a_i t^i$

$x_2(t) = 1 + \sum_{i=1}^{5} b_i t^i$

$x_3(t) = \sum_{i=1}^{5} c_i t^i$

The solution by non-classical variational (N.C.V.) and exact solutions are calculated along with absolute error presented in the following table:

Table 1. The numerical results which are compared with given analytical solution

Time	N.C.V. $x_1(t)$	Exact $x_1(t)$	Abso. Error	N.C.V. $x_2(t)$	Exact $x_2(t)$	Abso. Error	N.C.V. $x_3(t)$	Exact $x_3(t)$	Abso. Error
0	1	1	0	1	1	0	0	0	0
0.1	1.015	1.015	0	1.115	1.115	0	0.099	0.099	0
0.2	1.063	1.063	0	1.261	1.261	0	0.198	0.198	0
0.3	1.145	1.145	0	1.438	1.438	0	0.295	0.295	0
0.4	1.267	1.267	0	1.647	1.647	0	0.389	0.389	0
0.5	1.430	1.430	0	1.888	1.888	0	0.479	0.479	0
0.6	1.642	1.642	0	2.160	2.160	0	0.564	0.564	0
0.7	1.906	1.906	0	2.464	2.464	0	0.644	0.644	0
0.8	2.229	2.229	0	2.799	2.799	0	0.717	0.717	0
0.9	2.6202	2.6202	0	3.164	3.164	0	0.783	0.783	0
1	3.086	3.086	0	3.559	3.559	0	0.841	0.841	0

Where $0 \le t \le 1$ and the basis are polynomial of degree 5 .

The exact solution as follows

$x_1(t) = e^{-t} + te^t$

$x_2(t) = e^t + t\sin t$

$x_3(t) = \sin t \qquad \forall \ 0 \le t \le 1$

Conclusion

In this paper a survey was presented of non- classical variational method using bilinear forms and Ritz basis. In this environment the non-classical method is the optimal bridge between exact solution and approximate one.

The above summarized identification algorithm have been tested with success on an example with basis as polynomial of degree 5 as a Ritz basis and we notes that when n=5 gives very powerful technique to solve our system and gives nearly exact solutions.

As to the computational requirements an example presented then we compare our solution with the exact one.

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