

EFFECT OF NON-LINEAR DENSITY VARIATION ON CONVECTIVE HEAT AND MASS TRANSFER WITH THIRD ORDER BOUNDARY CONDITIONS

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ABSTRACT

In this paper, we analyze the combined influence of thermal radiation, Soret, Dufour effects, heat sources on convective heat and mass transfer flow of a viscous, electrically conducting, chemically reacting fluid past a vertical plate with a convective surface boundary conduction. The governing equations are transformed by employing similarity transformations and the resultant non-dimensional equations are solved numerically using Runge-Kutta method along with Shooting technique. The effects of various parameters velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are exhibited in figures, tables and analyze in detail.

Keywords : Thermal Radiation, Radiation Absorption, Soret Effect and Dufour effects, Dissipation

NOMENCLATURE:

$$M \text{ Magnetic parameter} = \frac{\sigma \mu_e^2 H_o^2 x}{\rho U_\infty}, \quad Gr \text{ Thermal Grashof number} = \frac{\beta g (T_w - T_\infty) x}{U_\infty^2}$$

$$N \text{ Buoyancy ratio} = \frac{\beta^* (C_w - C_\infty)}{\beta (T_w - T_\infty)}, \quad Sr \text{ Soret number} = \frac{D_m k_T (T_w - T_\infty)}{\nu T_m (C_w - C_\infty)}$$

$$Du \text{ Dufour parameter} = \frac{D_m k_T (C_w - C_\infty)}{C_s C_p (T_w - T_\infty)}, \quad Ec \text{ Eckert Number} = \frac{U_\infty^2}{C_p (T_w - T_\infty)}$$

$$Pr \text{ Prandtl number} = \frac{\mu C_p}{k_f}, \quad Bi \text{ Convective heat transfer parameter} = \left(\frac{h}{k} \right) \sqrt{\frac{\nu x}{U_\infty}}$$

$$Sc \text{ Schmidt number} = \frac{\nu}{D_m}, \quad Kr \text{ Chemical reaction parameter} = \frac{k_r' x}{U_\infty}$$

Re Reynolds number = $\frac{U_{\infty}x}{\nu}$,

Q Heat source parameter = $\frac{Q_0x}{U_{\infty}Cp}$

Q₁ Radiation absorption parameter = $\frac{k'_r x}{U_{\infty}}$

N₁ Thermal radiation parameter = $\frac{3\beta_R k_f}{4\sigma^* T_{\infty}^2}$

u, v velocity components along x and y-axes respectively

T fluid temperature ,

C fluid concentration

K_f thermal conductivity,

D_m mass diffusivity

g acceleration due to gravity,

C_∞ free stream concentration

T_∞ free stream temperature,

K permeability of the porous medium

f dimensional stream function ,

C_p specific heat at constant pressure

Greek symbols

γ Chemical reaction parameter = $\frac{k'_r x}{U_{\infty}}$,

ν Kinematic viscosity

σ electrical conductivity ,

μ_e magnetic permeability

β, β* thermal and concentration expansions

θ dimensionless temperature $\frac{T - T_{\infty}}{T_w - T_{\infty}}$,

φ dimensionless concentration $\frac{C - C_{\infty}}{C_w - C_{\infty}}$

η Similarity variable, ψ Stream function ,

λ plate surface concentration exponent

1.Introduction:

Hydro magnetic boundary layers with heat and mass transfer over a flat surface are found in a many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, cooling of nuclear reactors. Many processes in engineering areas occur at high temperature and a knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Bestman [5] examined the natural convection boundary layer with suction and mass transfer in a porous medium. His results confirmed the hypothesis that suction stabilizes the boundary layer and affords the most efficient method in boundary layer control yet known. Abdul Sattar and Hamid Kalim [1] investigated the unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate.

Makinde [15,16,17] examined the transient free convection interaction with thermal radiation of an absorbing-emitting fluid along moving vertical permeable plate. Raptis [20] analyzed the thermal radiation and free convection flow through a porous medium by using perturbation technique. With regard to thermal radiation heat transfer flows in porous media, Chamkha [6] studied the solar radiation effects on porous media supported by a vertical plate. Impulsive flows with thermal radiation effects and in porous media are important in chemical engineering systems, aerodynamic blowing processes and geophysical energy modeling. Such flows are transient and therefore temporal velocity and temperature gradients have to be included in the analysis. Raptis and Singh [19] studied numerically the natural convection boundary layer flow past an impulsively started vertical plate in a Darcian porous medium. The thermal radiation effects on heat transfer in magneto – aerodynamic boundary layers has also received some attention, owing to astronomical re-entry, plasma flows in astrophysics, the planetary magneto-boundary layer and MHD propulsion systems. Abd-El-Naby et al [2] have presented a finite difference solution of radiation effects on MHD unsteady free convection flow over a vertical porous plate. Shateyi et al [21] have analyzed the Thermal Radiation and Buoyancy Effects on Heat and Mass Transfer over a Semi-Infinite stretching Surface with Suction and Blowing. Dulal Pal et al [7] have analyzed unsteady magneto hydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Rajesh et al [18] have considered the radiation effects on MHD flow through a porous medium with variable temperature or variable mass diffusion.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic chemical reactions. The volumetric heat generation has been assumed to be constant or a function of space variable. For example, a hypothetical core – disruptive accident in a liquid metal fast breeder reactor (LMFBR) could result in the setting of fragmented fuel debris on horizontal surfaces below the core. The porous debris could be saturated sodium coolant and heat generation will result from the radioactive decay of the fuel particulate. Vajravelu and Hadjinicolaou [22] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Hossain et al [12] studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the

presence of heat generation or absorption. Alam et al [3] studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation. Hady et al [11] studied the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect.

In all these studies Soret/Dufour effects are assumed to be negligible. Such effects are significant when the density differences exist in the flow regime. For example, when species are introduced at a surface in fluid domain, with different(lower) density than surrounding fluid, both Soret and Dufour effects can be significant. Also, when heat and mass transfer occur simultaneously in a moving fluid, the relation between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is called Dufour or diffusion –thermo effect. On the other hand mass fluxes can also be generated by temperature gradients and this is called Soret or thermo-diffusion effect. The thermal diffusion(Soret effect),for instance it has been utilized for isotope separation, and in mixture between gases with very like molecular weight(H_2, H_e) and of medium weight(N_2, air),the diffusion thermo (Dufour)effect was to be of a considerable magnitude such that it can not be ignored. In view of the importance of these above mentioned effects, Dursunkaya and Worek [8] studied diffusion thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface. Kafoussias and Williams [13] presented the same effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Angel et al [4] investigated the Dufour and Soret effects on free convection boundary layer flow over a vertical surface embedded in a porous medium. Gnaneswar Reddy et al [10] reported a Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Recently Gangadhar [9] and Madhusudhan Rao [14] have investigated soret and dufour effect on convective heat and mass transfer past a vertical plate with convective surface.

In this paper we analyze the combined influence of thermal radiation, Soret, Dufour effects on convective heat and mass transfer flow of viscous, electrically conducting, chemically reacting fluid past a vertical plate with a convective surface boundary conduction.

Similarity solution of hydro-magnetic heat and mass transfer over a vertical plate. The governing equations are transformed by employing similarity transformations and the resultant non-dimensional equations are solved numerically using Runge-Kutta method along with Shooting technique. The effects of various parameters velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are exhibited in figures and tables and analyze in detail.

2. Mathematical Analysis:

Let us consider a steady, laminar, hydro magnetic convective heat and mass transfer flow of a viscous electrically conducting chemically reacting fluid past a vertical plate. A uniform magnetic field of strength H_0 is applied normal to the plate. Since the magnetic Reynolds is very small for most of the fluid used in industrial applications, we neglect the induced magnetic field in comparison to the applied magnetic field. The density variation and the effects of buoyancy are taken into account in the momentum equation (Boussinesq's approximation). In addition, there is no applied electric field and all of the Hall effects and Joule heating are neglected. We choose a rectangular coordinate system $O(x,y,z)$ with x -axis lying along the plate and the y -axis normal to the wall. The plate at $y=0$ is under a convective surface boundary condition while the concentration on the wall is constant C_0 . The equations governing the flow, heat and mass transfer under boundary layer approximations are

Equation of Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{k} \right) (u - U_\infty) + \beta g (T - T_\infty) + \beta_1 g (T - T_\infty)^2 + \beta^* g (C - C_\infty) \quad (2)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = k_f \frac{\partial^2 T}{\partial y^2} + Q_o (T_\infty - T) + \frac{D_m k_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{\partial (q_R)}{\partial y} + Q_1 (C - C_\infty) \quad (3)$$

Diffusion equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} - k_r (C - C_\infty) \quad (4)$$

Where u, v are the velocity components along the x -and y -axis respectively. T and C are the fluid temperature and concentration

The boundary conditions at the plate surface and far away from the plate are

$$u(x,0) = v(x,0) = 0, -k_f \frac{\partial T}{\partial y} = h(T - T_w(x,0))$$

$$C_w(x,0) = Ax^\lambda + C_\infty$$

$$u(x, \infty) = U_\infty, T(x, \infty) = T_\infty, C(x, \infty) = C_\infty \quad (5)$$

In view of the continuity equation (2.1) we define the Stream function ψ as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (6)$$

A similarity solution of equations (1)-(6) is obtained by defining an independent variable η and a dependent variable 'f' in terms of the stream function ψ are

$$\eta = y \sqrt{\frac{U_\infty}{\nu x}}, \quad \psi = \sqrt{\nu x U_\infty} f(\eta) \quad (7)$$

We introduce the non-dimensional temperature and concentration as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (8)$$

Where T_∞ is the temperature of the hot fluid at the left surface of the plate. Substituting the equations (6)-(8) in the equations (1)-(5), we obtain

$$f''' + \frac{1}{2} f f'' - (M^2 + D^{-1})(f' - 1) + G(\theta + \gamma \theta^2 + N\phi) = 0 \quad (9)$$

$$\left(1 + \frac{4}{3N_1}\right) \theta'' + \frac{1}{2} \text{Pr} f \theta' + \frac{1}{2} \text{Pr} \text{Du} \phi'' + \frac{1}{2} \text{Pr} (\text{Ec}(f'')^2 + D^{-1}(f')^2) = 0 \quad (10)$$

$$\phi'' + \frac{1}{2} \text{Sc} f \phi' + \frac{1}{2} \text{Sc} \text{Sr} \theta'' - \text{kr} \text{Sc} \phi = 0 \quad (11)$$

$$f(0) = 0, f'(0) = 0, \theta'(0) = \text{Bi}(\theta(0) - 1), \phi(0) = 1, \quad (12)$$

$$f'(\infty) = 1, \theta(\infty) = \phi(\infty) = 0 \quad (13)$$

where prime denotes differentiation w.r.to η . It is noteworthy that the local parameters Bi , M , G and N in equations (9)-(13) are function of x . However, in order to have a similarity solution all the parameters Bi , M , F , N , Sr , Du , Ec must be constant and we therefore assume

$$h = cx^{-0.5}, \sigma = ax^{-1}, \beta_T = bx^{-1}, \beta_c = dx^{-1} \quad (14)$$

Where a, b, c and d are constants.

The physical quantities of interest in this problem are skin friction parameter

$C_f = 2(\text{Re})^{-0.5} f''(0)$, the plate surface temperature $\theta(0)$, Nusselt Number

$Nu = -(\text{Re})^{0.5} \theta'(0)$ and the Sherwood Number $Sh = -(\text{Re})^{0.5} \phi'(0)$

can be calculated. For local similarity case, integration over the entire plate is necessary to obtain the total skin friction, total heat and mass transfer rate.

3. Solution of the problem:

We investigate the effect of non-linear density temperature variation on convective heat & mass transfer flow of a viscous electrically conducting chemically reacting fluid past a vertical plate in the presence of dissipation, radiation absorption. The set of coupled non-linear governing boundary layer equations $x-x$ together with the boundary conditions $x-x$ are solved numerically by employing the Runge-Kutta fourth order method along with shooting technique. First of all, higher order non-linear differential equations $x-x$ are conventional in to simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique. The resultant initial value problem is solved by using Runge-Kutta fourth order technique. The step size $\Delta_n = 0.05$ is stressed to obtain the numerical solution with decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction Coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0)-\Theta^1(0)$ are also sorted out and their numerical values of presented in a tabular form.

4. Results and discussion:

The governing equations(9)-(11) subject to the boundary conditions(12)-(13) are integrated as described in section.3.Numerical results are reported in tables.1.The Prandtl number Pr was taken to be $Pr=0.72$ which corresponds to air, the value of Schmidt number(Sc) were chosen to $Sc=0.24,0.6,1.3,2.01$,representing diffusing chemical species of most common interest in air like, H_2,H_2O,NH_3 and Propyl Benzene respectively. Attention is focused on positive value of buoyancy parameters that is, Grashof number $G>0$ (which corresponds to the cooling problem) . The buoyancy ratio $N>0$ when the buoyancy forces are in the same direction and $N<0$, when they are in opposite directions. In order to benchmark our numerical results, we have compared the plate surface temperature $\theta(0)$ and the local heat transfer rate at the plate surface $\theta'(0)$ in the absence of both magnetic field and buoyancy

forces for various values of Bi with those of Gangadhar [9] and found them in good agreement as shown in table.1.

The velocity component f^1 is shown for different values of k, Q_1, Ec, Sr, Du . f^1 reduces in both the degenerating and generating chemical reaction cases (fig.1). An increase in the radiation absorption parameter Q_1 leads to a depreciation in f^1 (fig.4). Higher the dissipative heat larger f^1 (fig.7). Increase Sr (or decreasing Du) results in a depreciation in f^1 at $\eta=0$ (fig. 10). The non-dimensional temperature (Θ) is shown for different parametric values. We follow the convention that the non-dimensional temperature is positive/negative according as the actual temperature is greater/lesser than T_∞ . From figs.2,5 &8 we find that the actual temperature reduces with increase in chemical reaction parameter $|k|$ and radiation absorption parameter Q_1 and enhances dissipative heat (Ec). Increasing the Soret parameter Sr (or decreasing Du) leads to a depreciation in the actual temperature (fig.11). The concentration dissipation (C) is shown for different parametric values. The concentration is positive for all parametric values. From figs.3 & 6 we notice that the actual concentration reduces with increase in $|k|$ and reduces with Q_1 . Higher the dissipative heat (Ec) lesser the actual concentration (fig.9). Increasing Soret parameter Sr (or decreasing Du) results in a depreciation in C (fig.12).

With reference to the chemical reaction parameter k , We find that $|\tau|$ & $|Sh|$ enhances and $|Nu|$ reduces in both degenerating and generating chemical reaction cases. An increase in Ec enhances $|Nu|$ & $|Sh|$ and reduces $|\tau|$ at $\eta=0$. An increase in the radiation absorption parameter Q_1 enhances $|\tau|$, $|Nu|$ & $|Sh|$ at $\eta=0$. Increasing soret parameter Sr (or decreasing Dufour parameter Du) results in an enhancement in $|\tau|$, $|Nu|$, $|Sh|$. (table.1).

6. Comparison:

Comparison with Gangadhar(2013) for skin friction C_f , Nusselt Number Nu and Sherwood Number for $G=2, N=1, Sc=0.6, Du=0.3, Ec=0.001, Sr=2, Pr=0.71, Bi=0.5$

Gangadhar (2013)

Present Work ($N_1=0, Q=0, D^{-1}=0$)

M	C_f	Nu	Sh	C_f	Nu	Sh
0.1	0.56966	0.07553	0.38155	0.57012	0.07558	0.38159
0.4	0.81362	0.0			0.169	0.39119
0.6	0.93045	0.07643	0.39554	0.93048	0.07649	0.39559

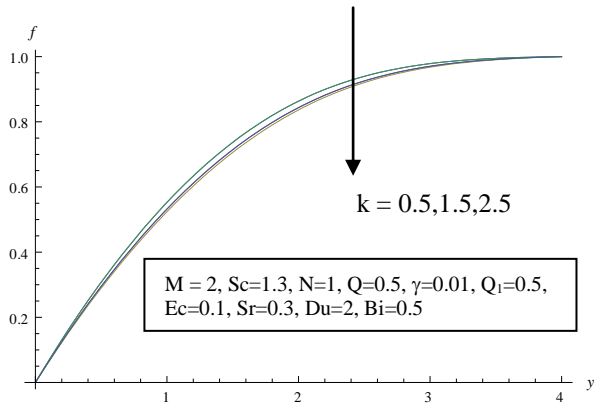


Fig.1 : Variation of f' with k

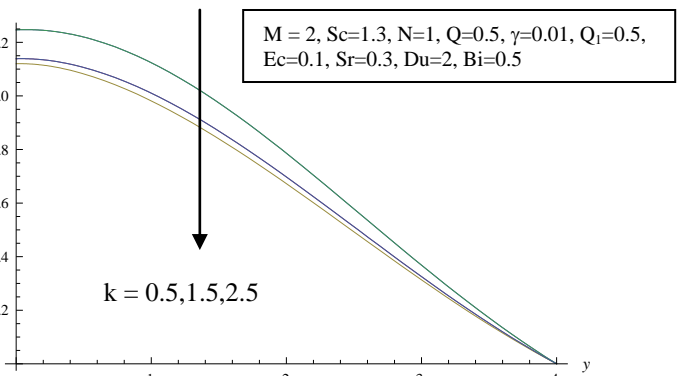


Fig.2 : Variation of Temperature (θ) with k

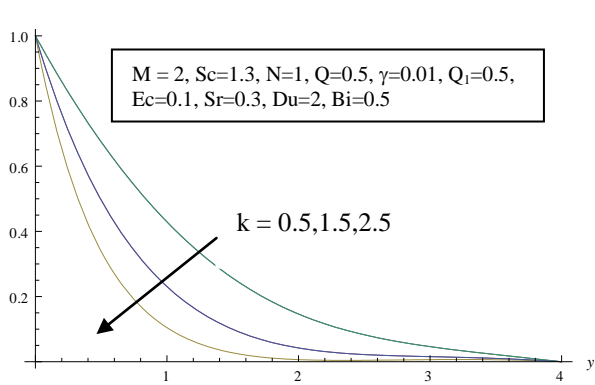


Fig.3 : Variation of Concentration (ϕ) with k

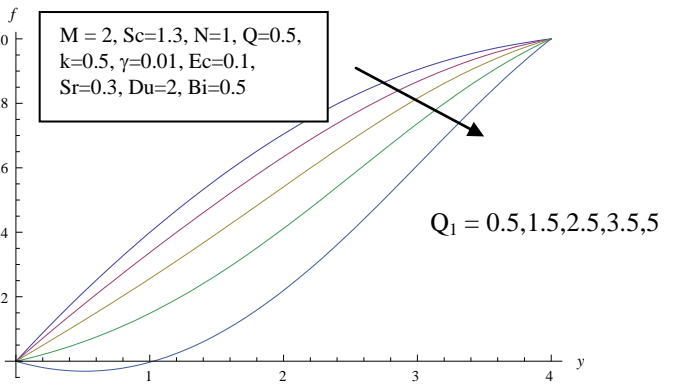


Fig.4 : Variation of f' with Q_1

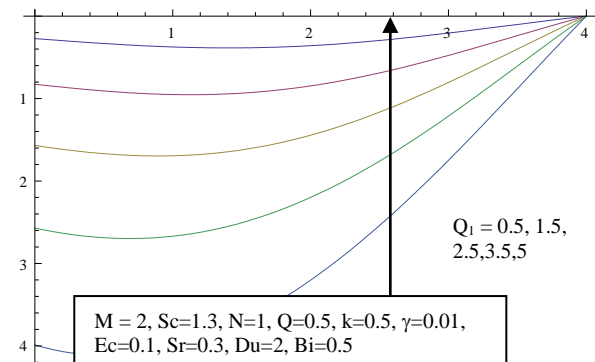


Fig.5 : Variation of Temperature (θ) with Q_1

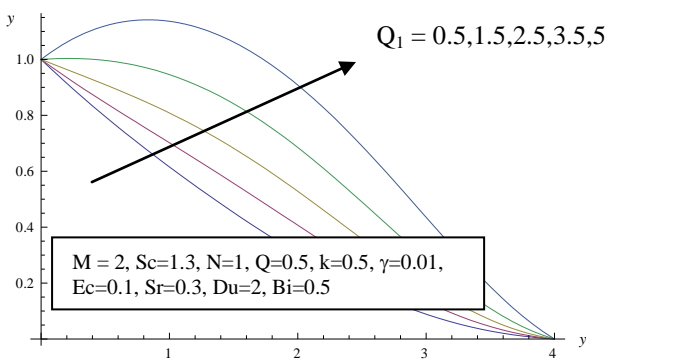


Fig.6 : Variation of Concentration (ϕ) with Q_1

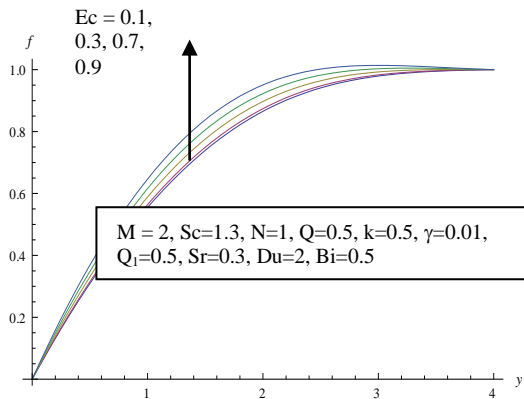


Fig.7 : Variation of f' with Ec

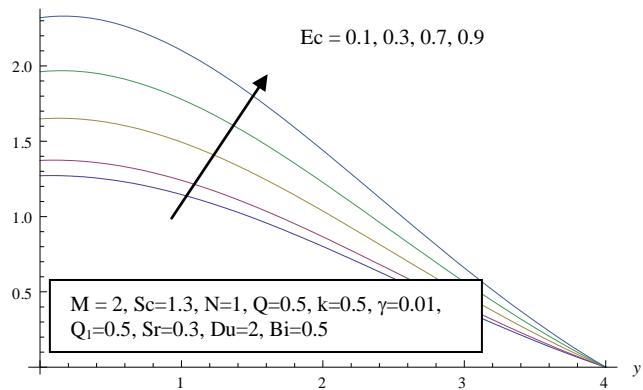


Fig.8 : Variation of Temperature (θ) with Ec

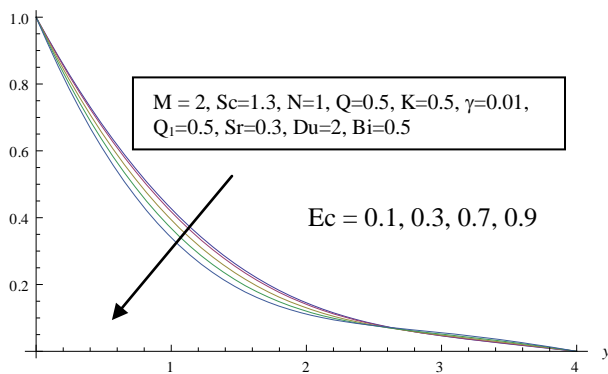


Fig.9 : Variation of Concentration (ϕ) with Ec

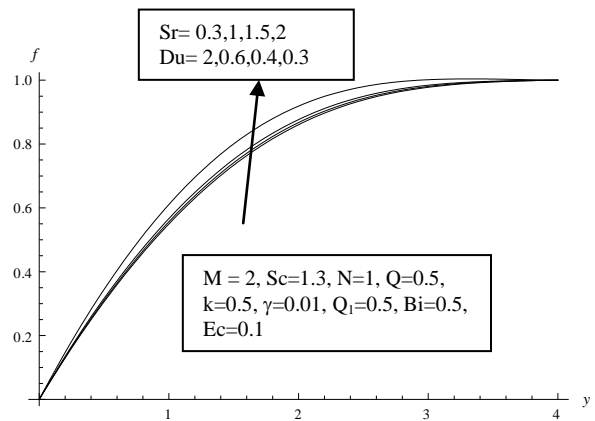


Fig. 10 : Variation of f' with Sr and Du

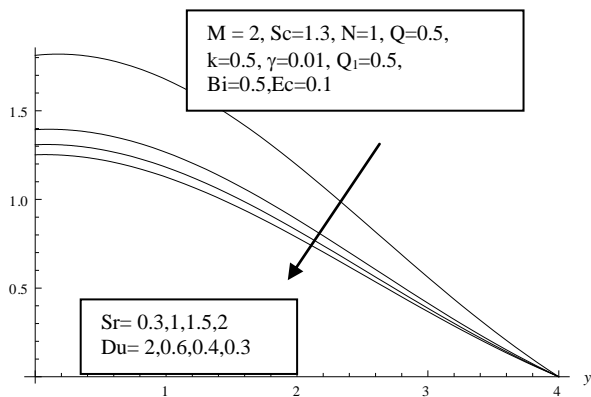


Fig. 11 : Variation of (θ) with Sr and Du

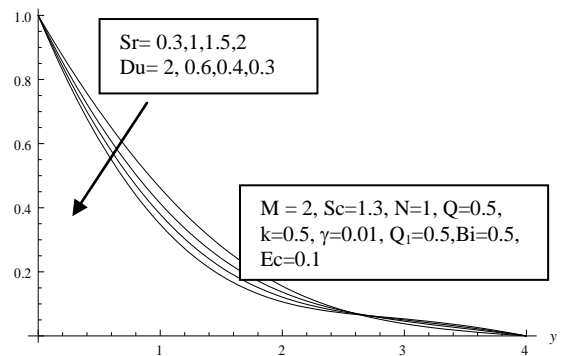


Fig. 12 : Variation of (ϕ) with Sr and Du

		Tow(0)	Nu(0)	Sh(0)
k	0.5	0.084285	0.43464	-0.4029
	1.5	0.17585	0.31177	1.20801
	2.5	0.18104	0.28572	2.40069
	-0.5	0.0428	0.43464	-0.4029
	-1.5	0.175853	0.311778	1.20801
Ec	0.003	0.08422	0.43469	-0.4031
	0.005	0.08416	0.43474	-0.4032
	0.007	0.08409	0.43479	-0.4034
Q ₁	0.5	0.084285	0.434646	-0.40297
	1.5	-1.09257	1.353268	-2.78122
	2.5	1.771958	-0.84952	3.336961
Sr/Du	2/0.3	0.084285	0.434646	-0.40297
	1.5/0.4	0.02834	0.51203	-1.2423
	1/0.6	-0.04135	0.618705	-2.48895
	0.6/1	0.12185	0.38645	0.07837

Table.1

Comparison with Madhusudhan Rao(2012) for skin friction C_f , Nusselt Number Nu and Sherwood Number for $G=0.1$, $N=1$, $Sc=0.6$, $Du=0.3$, $Ec=0.01$, $Sr=2$, $Pr=0.71$, $Bi=0.1$, $D^{-1}=0.5$, $K_r=0.5$

Madhusudhana Rao(2012)

Present Work($N_1=0, Q \neq 0, D^{-1} \neq 0$)

M	C_f	Nu	Sh	C_f	Nu	Sh
0.1	0.63832	0.073329	0.35185	0.63839	0.07333	0.35196
0.4	1.12507	0.074505	0.36504	1.12509	0.074509	0.36509
0.6	1.26275	0.074775	0.36839	1.26282	0.074782	0.368412

7. Conclusions :

- The velocity component f^1 and $|Nu|$ reduces and $|\tau|$ & $|Sh|$ enhances in both the degenerating and generating chemical reaction cases. The actual temperature and the actual concentration reduces with increase in chemical reaction parameter $|k|$.
- An increase in the radiation absorption parameter Q_1 leads to a depreciation in f^1 , the actual temperature and the actual concentration and an enhancement in $|\tau|$, $|Nu|$ & $|Sh|$ at $\eta=0$.
- Higher the dissipative heat larger f^1 , the actual temperature, $|Nu|$ & $|Sh|$ and lesser the actual concentration and $|\tau|$ at $\eta=0$.
- Increase Sr (or decreasing Du) results in a depreciation in f^1 , the actual temperature and the actual concentration and an enhancement in $|\tau|$, $|Nu|$, $|Sh|$.

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