

## Modeling Volatility in Financial Time Series: Evidence From Nigerian Inflation Rates

\* AWOGBEMI, C.A. \*\*AKOMOLAFE, ABAYOMI \*\*\* BERNARD, E.BASSEY

\*National Mathematical Centre, Sheda, Kwali, Abuja, Nigeria.

E-mail of Corresponding Author: awogbemiadeyeye@yahoo.com

\*\*Dept. of Mathematical Sciences, Osun State University of Science & Technology,

P.M.B.353, Okitipupa, Ondo State, Nigeria.

\*\*\*Dept. of Banking and Finance, Ebonyi State University, Abakaliki, Nigeria.

### ABSTRACT

This research work tends to describe volatility in the consumer prices of some selected commodities in Nigerian market. This is achieved by examining the presence or otherwise of the volatility in their prices using ARCH and GARCH models with the monthly Consumer Price Index (CPI) of five selected commodities over a period of time (1997 – 2007) collected from National Bureau of Statistics, Headquarters, Abuja. The data obtained were analyzed using MS-Excel and E-view software packages. Akaike information Criteria (AIC) and Bayesian Information Criteria were used to test the adequacy of the models. Langragean Multiplier test was also used to test for the presence of ARCH effects and the data for the prices showed varying degree of ARCH effects.

**Keywords:** Volatility, Inflation Rates, ARCH, GARCH, Volatility Clustering

### INTRODUCTION

Inflation and its volatility entail large real costs to the economy (Antonio Morenio, 2004). Among the harmful effect of inflation, the negative consequences of inflation volatility are of particular concern (Philip, 2004). These include higher risk premia for long term arrangement, unforeseen redistribution of wealth and higher costs for hedging against inflation risks. Thus inflation volatility can impede growth even if inflation on average remains restrained.

Volatile periods are hectic periods with price fluctuations. Intuitively, such periods reflect investors' uncertainty about the fundamentals in the economy. The inflation rate as measured by Consumer Price Index in Nigeria varied widely over time, and that high variability has been associated with periods of high inflation. Friedman (1997) opined that higher inflation leads to greater volatility and higher inflation volatility is detrimental to economic growth. A lack of price stability exerts harmful effects on the economy not only through changes in the

price level but also through increased price level uncertainty. High volatility of inflation over time raises such price level uncertainty.

According to Emmanuel (2008), the major component of instability in inflation rates is exhibited by the volatility of the inflation rates. Hence, to be able to establish and maintain a viable economy that will contribute its quota to the welfare and development of its citizens, there must be an in depth and comprehensive understanding of inflation volatility.

Gujarati (2004) is of the opinion that the knowledge of volatility is of crucial importance in many areas. For example, considerable macro economic work has been done in studying the volatility of inflation rate over time. For decision makers, inflation in itself may not be bad, but its high variation makes the forecasting of economic planning difficult and the decision making processes may be negatively affected.

Brooks (2004) opined that financial data exhibit a number of features such as volatility clustering, leptokurtosis, among others. He also raised the question of how financial time series for instance could be modeled because of their volatility. In the light of this, the researcher investigated varying variances of monthly inflation rates in Nigeria by modeling volatility of some selected prices of items using ARCH and GARCH models.

## **RESEARCH PROBLEM**

The underlying characteristic of most financial time series is that in their level form they are random walks, that is, they are non-stationary (Gujarati, 2004). The question is how do we model such financial data which is known to exhibit volatility? Since volatility has to do with variability of financial data, the volatility characterized by the data can be brought out by modeling the variability or variance by establishing a variance model of the data (Joutz, 2006).

The special problem involved in forecasting financial time series such as inflation rates is that of capturing the pattern of volatility that characterizes their movement. That is, they are characterized by a phenomenon known as volatility clustering which typifies periods in which they exhibit wide swings for an extended period followed by a period of comparative tranquility. The need for stability on the price levels is inevitable for general performance of Nigerian economy.

When relative prices vary because of inflation, such movements may decrease economic welfare for society as a whole. The efficiency of resource allocation also decreases because decision makers have less useful information on prices to guide their decision. The main

thrust therefore is to undertake a careful study of volatility in prices of some selected items that accompanies inflation

## MATERIAL AND METHOD

In this study, the data used were collected from price unit of National Bureau of Statistics, Abuja. The data also consists of Consumer Price Index of five randomly selected commodity items: In this study, the monthly consumer prices of Food, Clothing & Footwear, Housing, Health, and Transport from 1997 – 2007 inclusive are examined. The data are published on monthly basis by National Bureau of Statistics (NBS) and readily available on national dailies. Therefore, the data are of public interest.

The plotting of graphs and analysis were carried out using E views Software .

### AUTO REGRESSIVE CONDITIONAL HETEROSCEDASTICITY (ARCH) MODELS

ARCH models represent the changes of variance along time (heteroscedasticity). This implies that changes in variability are related to or predicted by recent past values of the observed series. The models were introduced by Robert Engle (1982) to model and forecast conditional variance of error terms and later generalized by Bollerslev(1986)as GARCH models.

ARCH models considered the variance of the current error term to be a function of the variances of the previous time period's error terms. ARCH relates the error variance to the square of a previous period's error. It is commonly employed in modeling financial time series that exhibit time varying volatility clustering i.e., periods of swings followed by periods of relative calm.

ARCH ( $q$ ) model is expressed as:

$$\begin{aligned}\delta_t^2 &= \alpha_0 + \alpha_1 \mu_{t-1}^2 + \dots + \alpha_q \mu_{t-q}^2 \\ &= \alpha_0 + \sum_{i=1}^q \alpha_i \mu_{t-i}^2\end{aligned}$$

where  $\alpha_0 > 0$  and  $\alpha_i \geq 0, i > 0$ .

The conditional disturbance variance is the variance of  $\mu_t$ , conditional on information available at time  $t - 1, \dots, t - p$ . It may be expressed as:

$$\delta_t^2 = \text{var}(\mu_t / \mu_{t-1}, \dots, \mu_{t-p}) = E(\mu_t^2 / \mu_{t-1}, \dots, \mu_{t-p})$$

### 1.7.6 GENERALIZED ARCH (GARCH) MODELS

If an Autoregressive Moving Average Model (ARMA) is assumed for the error variance, the model is a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. In that case, the GARCH ( $p, q$ ) model is given by:

$$\begin{aligned}\delta_t^2 &= \alpha_0 + \alpha_1 \mu_{t-1}^2 + \dots + \alpha_p \mu_{t-p}^2 + \beta_1 \delta_{t-1}^2 + \dots + \beta_q \delta_{t-q}^2 \\ &= \alpha_0 + \sum_{i=1}^p \alpha_i \mu_{t-i}^2 + \sum_{i=1}^q \beta_i \delta_{t-i}^2\end{aligned}$$

where  $p$  is the order of the ARCH terms:  $\mu_{t-i}^2$ ,  $q$  is the order of GARCH terms:

$\delta_{t-i}^2$ ,  $\delta_0$  is constant,  $\alpha_i \geq 0$ ,  $\beta_i \leq 1$ , and  $(\alpha_i + \beta_i) < 1$ . GARCH ( $p, q$ ) models express the conditional variance as a linear function of  $p$  lagged squared disturbances and  $q$  lagged conditional variances.

### TIME SERIES PLOT OF THE DATA

In this study, time series plots were carried out to illustrate the pattern of price movement of various commodity prices for a period of 11 years on monthly basis. The time plots of  $x_t$  are presented in figures 1a, 2a, 3a, 4a, and 5a for Food, Clothing & Footwear, Housing, Health and Transport respectively. See attached figures.

Before applying the estimation procedures, it is necessary to study the data to see what behavior they can produce. Non stationarities were observed by the plots of the various series. To induce stationarity in the series, the differencing transformation of the data was carried using Box and Jenkins procedures. The logarithm transformations are carried out on various series. See figures 1b, 2b, 3b, 4b, 5b and differencing of logged series were performed. See figure 1c, 2c, 3c, 4c and 5c

An examination of the plots in figures 1c, 2c, ... 5c reveals that some element of stationarity has been induced into the data. The Box and Jenkins procedure was then applied to the variance series  $Z_t^2$  obtained from differenced – logged series according to the procedure described in chapter 3. In this case, the model building commences with the evaluation of the sample (ACF) and sample (PACF) by using correlogram plots. See figures below:

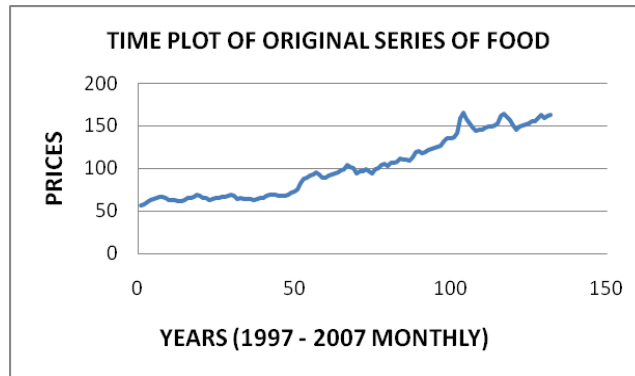


Figure 1a

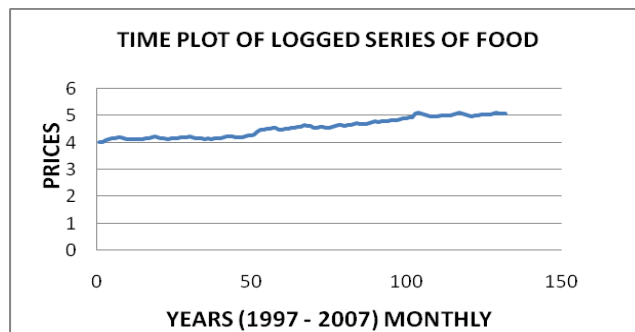


Figure1b

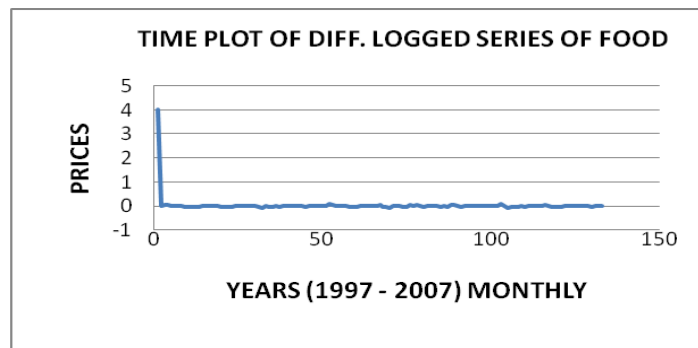


Figure 1c

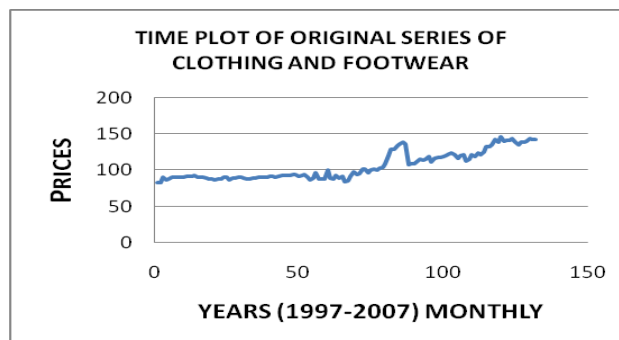


Figure 2a

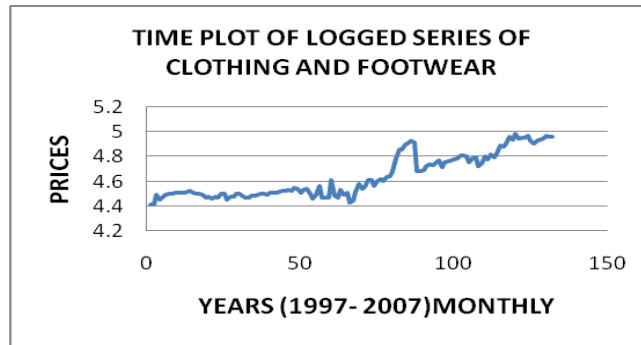


Figure 2b

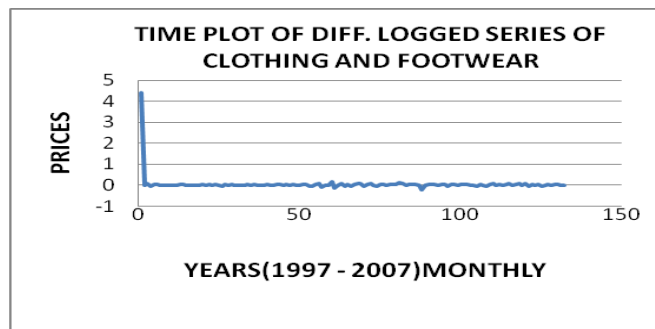


Figure 2c

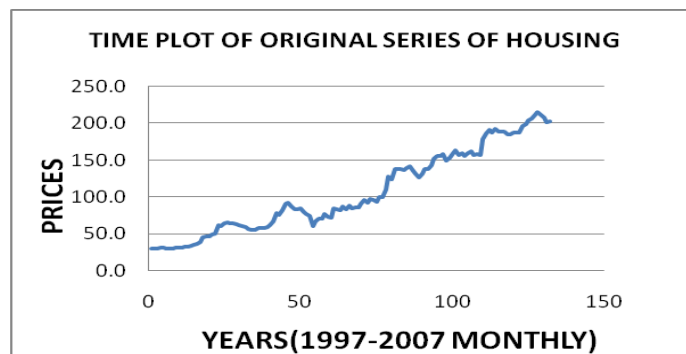


Figure 3a

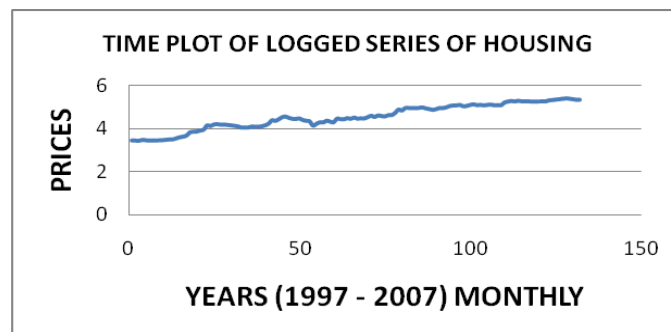


Figure 3b

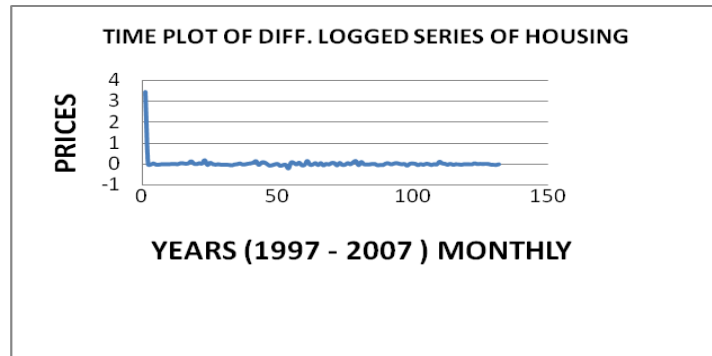


Figure 3c

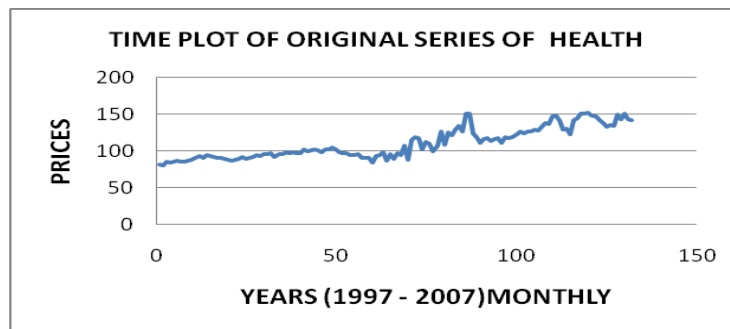


Figure 4a

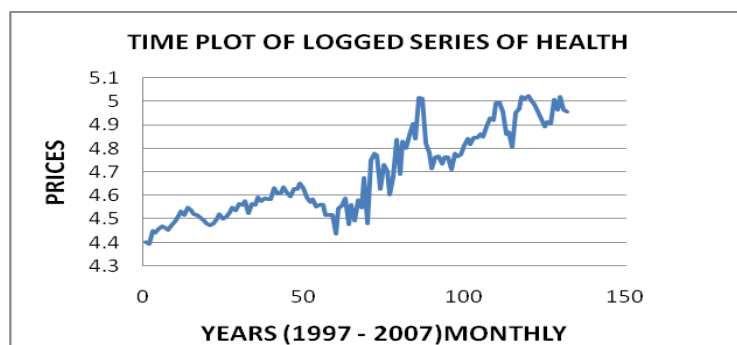


Figure 4b

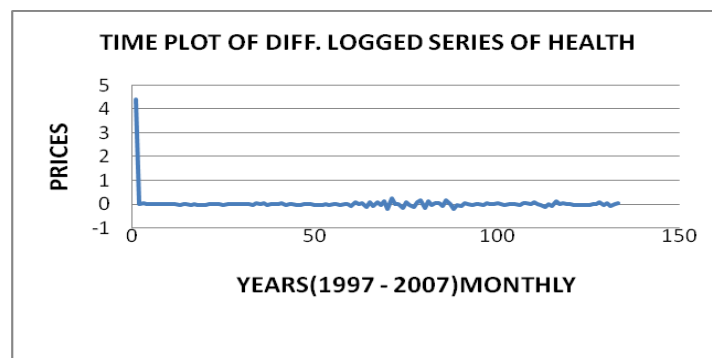


Figure 4c

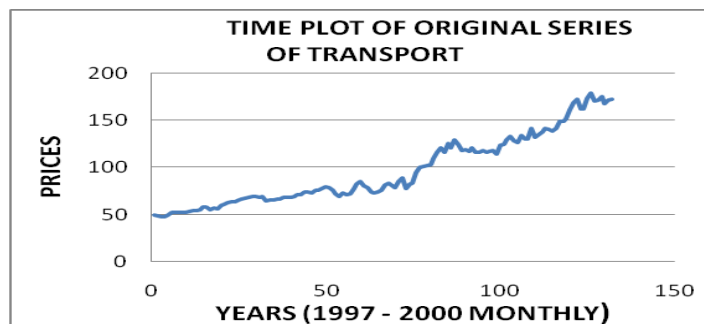


Figure 5a

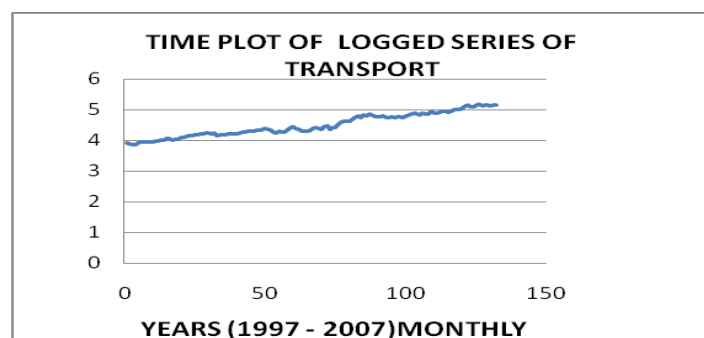


Figure 5b

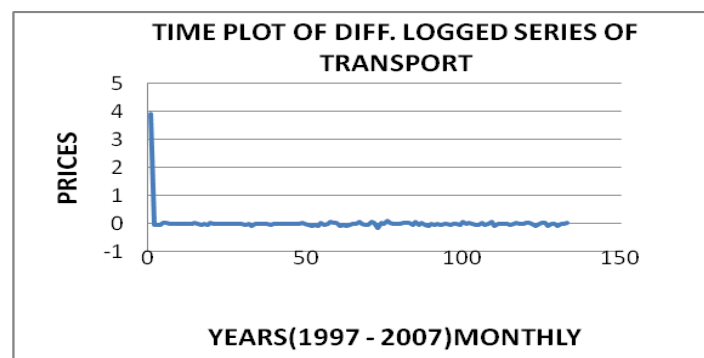


Figure 5c

### ESTIMATION OF THE PARAMETERS AND TEST FOR ARCH EFFECTS

The fixed values of the parameters associated with the ARIMA models are estimated while ARCH disturbances are also tested using Lagrange Multiplier Test by considering the hypothesis of no ARCH effect:  $H_0: \alpha_1 = \alpha_2 = \alpha_3 \dots \alpha_p = 0$  versus the alternative hypothesis that the conditional variance is given by ARCH(p) process.

### DESCRIPTIVE STATISTICS

The various  $Z^2_t$  were collated and analyzed to produce various features or statistics that describe the distribution of various commodity prices over time. The summary of the result for the five selected commodity items are shown in the table below:



**Table 1: DESCRIPTIVE MEASURE**

STATISTICS	FOOD	CLOTHING & FOOTWEAR	HOUSING	HEALTH	TRANSPORT
MEAN	0.123558	0.149311	0.090126	0.147996	0.114513
STANDARD DEVIATION	1.4110682	1.696749	1.003789	1.657608	1.298082
SKEWNESS	11.35813	11.35799	11.35748	11.35763	11.35808
KURTOSIS	130.0073	130.0052	129.9974	129.997	130.0066

From the table above, and considering the mean (which is a more volatility in the case) of CLOTHING & FOOTWEAR has the highest volatility followed by HEALTH. Next is Food and TRANSPORT while HOUSING has the least volatility.

We measure the skewness of any distribution based on how far the peak is from the centre of the distribution.

Food has the highest skewness followed by TRANSPORT, CLOTHING & FOOTWEAR, HEALTH; while the least is observed in HOUSING. They are all positively skewed; meaning that the mean is greater than the mode of their respective distributions. The kurtosis values for the different commodity items show that the FOOD has the highest value, closely followed by HEALTH and with HOUSING as the least. From the observation above, it means that FOOD item has the highest peakedness at the centre of the distribution i.e. there is long tail.

**ESTIMATED MODEL FOR FOOD PRICES**

**Table 2: SUMMARY STATISTICS**

MODEL ESTIMATED	PARAMETER	AIC	BIC	R <sup>2</sup>	SIG. PROB.
ARIMA(0,1,3)	$\theta_1 = 0.152660$ $\theta_2 = 0.103078$ $\theta_3 = 0.922582$	2.232879	2.298398	0.735786	0.0000 0.0000 0.0000

$$Z^2_t = 0.1527\varepsilon_{2t-1} + 0.1031 \varepsilon^2_{t-2} + 0.9226 \varepsilon^2_{t-3} + \mu_t \dots\dots\dots$$

This is the familiar ARMA (0, 3) without a drift.

Performing Lagrangean Multiplier test on FOOD prices series  $Z^2_t$  leads to rejection of  $H_0$  : that there is no ARCH effect or disturbance at  $\alpha = 0.05$  ( i.e.  $nR^2 > X^2_p$ ). Hence, the model estimated above using Box and Jenkins approach with Least Squares estimation is not adequate. Thus, we try ARCH/GARCH models as postulated by Engle (1982) and Bollerslev (1986) respectively.

$$\text{ARCH (1)/GARCH (0, 1) Model: } Z^2_t = 3.33E - 06 + 0.4329 \varepsilon^2_{t-1} + \mu_t \dots\dots\dots$$

This gives smaller values of both AIC = -8.303221 and BIC = -8.172185 than when the popular Box Jenkins approach was used.

## ESTIMATED MODEL FOR CLOTHING & FOOTWEAR

**Table 3: SUMMARY STATISTICS**

MODEL ESTIMATED	PARAMETER	AIC	BIC	R <sup>2</sup>	SIG. PROB.
ARIMA(0,1,2)	$\theta_1 = 0.001173$ $\theta_1 = 0.979979$	0.622330	0.6666009	0.962957	0.0000 0.0000

$$Z^2_t = 0.0012\varepsilon^2_{t-1} + 0.9800 \varepsilon^2_{t-2} + \mu_t \dots\dots\dots$$

This is GARCH (0, 2) model.

Testing ARCH disturbance effect in the CLOTHING & FOOTWEAR series shows that there is an ARCH effect at  $\alpha = 0.05$ .

The conditional variance equation is thus given as :

$$Z^2_t = 2.99E-05 + 0.0030 \varepsilon^2_{t-1} + \mu_t \dots\dots\dots$$

This is GARCH (0, 1) model which gives lower values of AIC = -7.068391 and BIC = -6.959194 than the ARIMA approach.

## ESTIMATED MODEL FOR HOUSING PRICES

**Table 4: SUMMARY STATISTICS**

MODEL ESTIMATED	PARAMETER	AIC	BIC	R <sup>2</sup>	SIG. PROB.
ARIMA(0,1,2)	$\theta_1 = 0.002567$ $\theta_2 = 0.979992$	0.594031	0.637710	0.962270	0.0000 0.0000

$$Z^2_t = 0.0132\varepsilon^2_{t-1} + 0.9800 \varepsilon^2_{t-2} + \mu_t \dots\dots\dots$$

This is also ARMA (0, 2) model.

The Langrangean Multiplier test indicates the presence of ARCH disturbance at  $\alpha = 0.05$  in the series  $Z^2_t$  of the HOUSING prices.

The conditional variance equation:

$$Z^2_t = 4.57E-05 - 0.0163 \varepsilon^2_{t-1} + \mu_t \dots\dots\dots$$

This is also GARCH (0, 1) model which gives lower values of AIC = -6.6866 and BIC = -6.5774 than the ARIMA approach.

## ESTIMATED MODEL FOR HEALTH PRICES

**Table 5: SUMMARY STATISTICS**

MODEL ESTIMATED	PARAMETER	AIC	BIC	R <sup>2</sup>	SIG. PROG.
ARIMA(0,1,2)	$\theta_1 = 0.002567$ $\theta_2 = 0.979992$	0.594031	0.637710	0.962270	0.0000 0.0000

$$Z^2_t = 0.0026\varepsilon^2_{t-1} + 0.9800 \varepsilon^2_{t-2} + \mu_t \dots\dots\dots$$

This is ARMA (0, 2) model without a drift. Lagrangean Multiplier test indicates the presence of ARCH effect at  $\alpha = 0.05$ .

Conditional variance equation:

$$Z^2_t = 3.87E -06 + 0.031869 \varepsilon^2_{t-1} + \mu_t \dots\dots\dots$$

This is GARCH (0, 1) model which gives smaller values of AIC = -7.243096 and BIC = -7.133899 than the ARIMA approach.

## ESTIMATED MODEL FOR TRANSPORT PRICES

**Table 6: SUMMARY STATISTICS**

MODEL ESTIMATED	PARAMETER	AIC	BIC	R <sup>2</sup>	SIG. PROB.
ARIMA(0,1,2)	$\theta_1 = 0.016191$ $\theta_2 = 0.979940$	0.231913	0.275592	0.362465	0.0000 0.0000

$$Z^2_t = 0.0162\varepsilon^2_{t-1} + 0.9799 \varepsilon^2_{t-2} + \mu_t \dots\dots\dots$$

Testing for ARCH effect shows the presence of ARCH disturbance at  $\alpha = 0.05$ .

The conditional variance equation is thus given as:

$$Z^2_t = 4.56E - 06 + 0.0698 \varepsilon^2_{t-1} + \mu_t \dots\dots\dots (5.5b)$$

This is GARCH (0, 1) model which gives lower values of AIC = -8.047575 and BIC = -7.938378 than the ARIMA approach.

From the correlogram of each of the series, it was noticed that ACF and PACF tail off which suggests a mixed model (ARMA) of  $Z^2_t$ . Generally, volatility in the series seems to be adequately modeled by ARCH AND GARCH.

**Table7: FORECASTING WITH ESTIMATED MODEL**

MODEL	1 <sup>ST</sup>	2 <sup>ND</sup>	3 <sup>RD</sup>	MSE
<b>FOOD</b>				
ARIMA(0,1,3)	-0.1503	0.1083	-0.1769	0.5298
GARCH(0,1)	-0.0675	0.0083	0.083	1.7499
<b>CLOTHING &amp; FOOTWEAR</b>				
ARIMA(0,1,2)	-0.2150	0.0574	0.0000	0.1075
GARCH(0,1)	0.0200	0.0201	0.0201	2.8066
<b>HOUSING</b>				
ARIMA(0,1,2)	0.0007	0.0000	0.0000	0.0433
GARCH(0,1)	0.0303	0.0303	0.0303	0.9893
<b>HEALTH</b>				
ARIMA(0,1,2)	0.0197	-0.0058	0.0000	0.1045
GARCH(0,1)	0.0196	0.0058	0.0000	2.5503
<b>TRANSPORT</b>				
ARIMA(0,1,2)	0.0005	0.0001	0.0000	0.0727
GARCH(0,1)	0.0113	0.0113	0.0113	1.0743

Here, we present forecasts for some time periods based on the estimated models. Particularly, three steps ahead forecast were made and the corresponding Mean Squares Errors obtained. The results are provided in the table below

### SUMMARY OF FINDINGS

Going by the values of kurtosis and skewness obtained for various commodity items as shown in table 4.3.1, various series exhibit high peakedness and are also highly skewed. That is, the distribution is leptokurtic which is one of the features of any financial data. Hence, the present finding is in agreement with the finding of Brook (2002).

Besides, all the commodity items considered show that the error term  $\mu_t$  is time heteroscedastic; which implies that the conventional estimation procedure based on ARMA estimation technique does not produce optimal results. Hence, this result is in line with the finding of Pynnem (2007). That is, despite the popularity of ARMA models in time series, it has limitation(s). One of such limitations is that it assumes a constant conditional variance while most of econometric and financial data exhibit non-constant conditional variance which is self evident from the findings of this research work.

There is no series that shows constant conditional variance; hence, there is no outlier in this regard. Thus this work is in agreement with the findings of Asokan, Shojaeddin and Abbasali (2001).

From our measure of volatility as presented in equations 4.1a – 4.5b, there is quite a bit of persistence in the volatility as the volatility in the current month depends on volatility in the preceding month.

The results of the forecast show that the Box – Jenkins approach has lower values of Mean Squares Error than ARCH/GARCH models for various commodity items considered. The values of the forecast are highly comparable. Also, the Mean Squares Errors are marginally different from one another. Hence, their forecasting ability is comparable.

## CONCLUSION

The study has shown that ARCH and GARCH models are better models because they give lower values of AIC and BIC for data of this type than the conventional Box and Jenkins ARMA models. This is due to the fact that they gave the best model selection criteria for the data analyzed. Since volatility seems to persist in all the commodity items (as can be seen in the results of the analysis); it means the “bullish crowd” will be highly favored in the market of the said commodity items.

## RECOMMENDATION

The Researcher recommends the use of ARCH and GARCH models in the analysis of other Financial time series in Nigeria

Similar research should be carried out using a variety of asymmetric ARCH/GARCH models such as Exponential GARCH (E- GARCH) model of Nelson (1991), Threshold ARCH model attributed to Rabemananjara and Zakoian (1994), Glosten-Jagannathan GARCH (GJR-GARCH) model by Glosten, Jagannathan and Runkle (1993), GARCH – in- Mean (GARCH-M) model, Quadratic GARCH model by Sentana (1995) and Integrated GARCH (I- GARCH) model. Other commodity items' prices at the states level in Nigeria could be investigated to capture their volatility status.

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