Effect the Magnetic field over an inclined stretching sheet of three dimensional Maxwell fluid in obscene of Mixed convection

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Abstract
Three dimensional flow of Maxwell fluid with boundary condition is investigated. transformations are victimized to reduce the partial differential equations into ordinary differential equations. Effect of two parameters ,Magnetic field parameter and Deborah number parameter on non dimensional velocity are discussed .homotopy analysis method(HAM) is used to solve the velocity equations.

Keywords: steady flow , Maxwell fluid , Magnetic field, HAM solution.

1. Introduction
The problem of non –Newtonian fluids are very important in many applications such as metallurgical process, wire drawing, polymer extrusion, food processing industry, and many others. Lost all the fluids occurring in industry and biomedicine are non – Newtonian.

In the recent years the flow of Maxwell fluid with magnetic field have been studied by some researchers, In [7] M.Qasim and S. Noreen studied the falkner –skan flow of Maxwell fluid with heat transfer and magnetic field, he used the homotopy method to solve the flow and heat equations . In another paper Vigendra Singh ,Shwet agranal in[14] discussed MHD flow and heat transfer for Maxwell fluid over exponentially stretching sheet, the implicit finite difference scheme is used to solve the problem .The flow of Maxwell fluid due to constantly moving flat radiative surface with convective condition under the influence of non uniform transverse magnetic field are studied by M. Mustafa [6],Mixed convection radiative flow of three dimensional Maxwell fluid over an inclined stretching sheet in presence of thermophoresis and convective condition investigation by[4]

In this paper we studied the effect the magnetic field of Maxwell fluid in three dimensional in the obscene the mixed convection radiative, homotopy method is used to obtain the analysis solutions. This method is general and its power technique for the non linear differential equations.

2. Homotopy analysis method (HAM) [1], [2],[10]
In order to show the basic idea of HAM, consider the following differential equation

\[ N[u(\tau)] = 0 \] .... (1)

where \( N \) is a nonlinear operator, \( \tau \) denote the independent variables and \( u \) is an unknown function. For simplicity, we ignore all boundary or initial conditions,

By means of the HAM, we construct the zeroth-order deformation equation

\[ (1-q)L[\Phi(\tau; q)-u_0(\tau)]=qhH(\tau)N[\Phi(\tau; q)] \] ....(2)

where \( q \in [0; 1] \) is the embedding parameter, \( H( \tau \neq 0) \) is an auxiliary parameter, \( L \) is an auxiliary linear operator, \( u_0(\tau) \) is an initial guess . It is obvious that when the embedding parameter \( q = 0 \) and \( q = 1 \), it holds

\[ \Phi(\tau; 0) = u_0(\tau); \Phi(\tau; 1) = u(\tau); \] ....(3)

Thus as \( q \) increases from 0 to 1, the solution varies from the initial guess \( u_0(\tau) \) to the solution \( u(\tau) \). Expanding \( \Phi(\tau; q) \) in Taylor series with respect to \( q \), one has

\[ \Phi(\tau; q) = u_0(\tau) + \sum_{m=1}^{\infty} u_m(\tau) \cdot q^m \] ....(4)
If the auxiliary linear operator, the initial guess, the auxiliary h, and the auxiliary function are properly chosen, the series (4) converges as \( q=1 \), then we have

\[
\sum_{m=1}^{\infty} u_m(t) = u_0(t) + \sum_{m=1}^{\infty} u_m(t)
\]

define the vector

\[
(\tau) = [u_0(\tau), u_1(\tau), \ldots, u_n(\tau)]^T
\]

Differentiating equation (2) m times with respect to the embedding parameter q and then setting q=0 and finally dividing them by m!, we obtain the mth-order deformation equation

\[
\sum_{m=1}^{\infty} \frac{\partial^{m-1} \Phi(q)}{\partial q^{m-1}} u_m = \sum_{m=1}^{\infty} \frac{\partial^{m-1} \Phi(q)}{\partial q^{m-1}} x_m
\]

..(8)

Applying \( L^{-1} \) on both sides of equation (7), we get

\[
u_m(\tau) = \sum_{m=0}^{M} u_m(\tau)
\]

..(10)

when \( M \to \infty \), we get an accurate approximation of the original equation (1).

3. Formulation of the problem

Consider, the flows are incompressible, steady MHD of Maxwell fluid in three dimensional the flow takes place in the domain \( z > 0 \). The mathematical statements for the Boundary layer problems are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

..(11)

\[
u \frac{\partial^2 u}{\partial y^2} + \nu \frac{\partial^2 v}{\partial x \partial y} + \nu \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 u}{\partial x \partial y} + 2\nu \frac{\partial^2 v}{\partial y \partial z} + 2\nu \frac{\partial^2 w}{\partial x \partial z}
\]

..(12)

\[
u \frac{\partial^2 v}{\partial y^2} + \nu \frac{\partial^2 u}{\partial x \partial y} + \nu \frac{\partial^2 w}{\partial x \partial y} + \nu \frac{\partial^2 v}{\partial x \partial y} + 2\nu \frac{\partial^2 v}{\partial y \partial z} + 2\nu \frac{\partial^2 w}{\partial x \partial z}
\]

..(13)

Where \( u \) and \( v \) and \( w \) are the velocities in the x, y and z directions, respectively, \( \rho \) is the fluid density, \( \nu = \frac{\mu}{\rho} \) is the kinematic viscosity, \( \mu \) is the dynamic viscosity, \( \sigma \) is the electric conductivity, \( \beta_0 \) is the strength of magnetic field, \( \lambda \) is the relaxation time.

The boundary condition are given by

\[
\begin{align*}
u = \eta = \alpha x, \nu = \beta y, \nu = 0 & \text{ at } z = 0 \\
u \to 0, v \to 0, & \text{ as } z \to \infty
\end{align*}
\]

..(14)
\[ u \rightarrow 0, \quad v \rightarrow 0 \text{ as } z \rightarrow \infty \]

In order to simplexes eqs (11-13), we introduce the new quantities:

\[ u = axf(\eta), \quad v = ayg(\eta), \quad w = -\sqrt{a} v \ (f(\eta) + g(\eta)), \quad \eta = \sqrt{a} v, \]

Now, in the above quantities, Eq. (11) satisfies automatically. While Eqs. (12), (13) are reduced as follows:

\[ f''' + (f + g) f'' + 2 \beta_1 [2(f + g)f' - (f + g)^2] - M f = 0, \quad \text{..(15)} \]

\[ g''' + (f + g) g'' + 2 \beta_1 [2(f + g)g' - (f + g)^2 g'] = 0, \quad \text{..(16)} \]

and the boundary conditions (14) reduce to

\[ f = 0, \quad g = 0, \quad f' = 1, \quad g' = \beta, \quad \text{at } \eta = 0, \]

\[ f' \rightarrow 0, \quad g' \rightarrow 0 \text{ as } \eta \rightarrow \infty, \]

Where \( \beta_1 \) is the dimensionless Deborah number, \( \beta \) is ratio of rates parameter, \( \rho \) is the fluid density, \( g \) is the gravitational acceleration, \( M \) is the magnetic field parameter, and prime shows the differentiation with respect to \( \eta \). These are given by

\[ \beta_1 = \frac{\lambda_1 a}{\eta}, \quad \beta = ba, \quad M = \frac{\beta E_3}{\alpha \rho}. \]

**4. Method of solution**

The homotopy analysis method is impetuses to find the solutions of equation (15), (16) which are required the initial approximations and auxiliary linear operators are presented below i.e.

\[ f_0(\eta) = (1 - e^{-\eta}), \quad g_0(\eta) = \beta(1 - e^{-\eta}), \]

\[ L_1 = f''' - f', \quad L_2 = g''' - g', \]

\[ L_1[C_i + C_i e^\eta - C_i e^{-\eta}] = 0, \quad L_2[C_i + C_i e^\eta + C_i e^{-\eta}] = 0, \]

where \( C_i (i = 1 \ldots 10) \) are the arbitrary constants.

The zeroth order deformation equations are:

\[ (1-q)L_1[f^*(\eta; q) - f_0(\eta)] = qh_1[N_1 f^*(\eta; q), g^*(\eta; q) + q]. \quad \text{..(17)} \]

\[ (1-q)L_2[g^*(\eta; q) - g_0(\eta)] = qh_2[N_2 f^*(\eta; q), g^*(\eta; q) + q]. \quad \text{..(18)} \]

\[ f^*(0; q) = 0, \quad f^*(\infty; q) = 0, \quad g^*(0; q) = 0, \quad g^*(\infty; q) = \beta, \quad g^*(0; q) = 0, \quad g^*(\infty; q) = 0. \]

\[ N_1[f^*(\eta; q), g^*(\eta; q)] = \frac{\partial^2 f^*(\eta; q)}{\partial \eta^2} [\beta^2 f^*(\eta; q)] + \frac{\partial f^*(\eta; q)}{\partial \eta} [\beta f^*(\eta; q)] + g^*(\eta; q) \frac{\partial^2 f^*(\eta; q)}{\partial \eta^2} = 0. \quad \text{..(19)} \]

\[ N_2[f^*(\eta; q), g^*(\eta; q)] = \frac{\partial^2 g^*(\eta; q)}{\partial \eta^2} [\beta^2 g^*(\eta; q)] + \frac{\partial g^*(\eta; q)}{\partial \eta} [\beta g^*(\eta; q)] + g^*(\eta; q) \frac{\partial^2 g^*(\eta; q)}{\partial \eta^2} = 0. \quad \text{..(20)} \]

Where \( q \) is an embedding parameter, \( h_1, h_2 \) are the non-zero auxiliary parameters and \( N_1, N_2 \) the nonlinear operators. When \( q = 0 \) and \( q = 1 \)

\[ f^*(\eta; 0) = f_0(\eta), \quad f^*(\eta; 1) = f(\eta). \]

Clearly when \( q \) is increased from 0 to 1 then \( f(\eta), g(\eta) \) vary from \( f_0(\eta), g_0(\eta) \), to \( f_1(\eta), g_1(\eta) \). By Taylor’s expansion we have

\[ f(\eta, q) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) q_m, \quad f_m(\eta) = \frac{1}{m!} \frac{\partial^m f(\eta, q)}{\partial q^m} \bigg|_{q=0} \]

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\[ g(\eta, q) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta)q^m, \quad g_m(\eta) = \frac{1}{m!} \frac{\partial^m g(\eta, q)}{\partial q^m} \bigg|_{q=0} \quad (21) \]

Where the convergence of above series strongly depends upon \( h_1, h_2 \). Considering that \( h_1, h_2 \) are selected properly so that the series (21) converge at \( q = 1 \) then we can write \( f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \) and \( g(\eta) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta) \).

The resulting problems at nth order deformation can be constructed as follows:

\[ L_n[f_m(\eta) - \chi_m f_m - 1(\eta)] = h_1 R_1^m(\eta) \quad (22) \]

\[ L_2[g_m(\eta) - \chi_m g_m - 1(\eta)] = h_2 R_2^m(\eta) \]

\[ f_m(0) = f_m'(0) = f_m'(\infty) = 0, \quad g_m(0) = g_m'(0) = g_m'(\infty) = 0 \]

\[ R_1^m(\eta) = D[f_{m-1}, \eta, \eta, \eta] - (M + D[f_{m-1}, \eta]) \sum_{k=0}^{m-1} \left( D[f_{m-1-k}, \eta] * D[f_k, \eta] \right) + \sum_{k=0}^{m-1} \left( (f_{m-1-k} * f_{m-k} - f_{m-1-k} * f_k) \right) \]

\[ D[f_{m-1-k}, \eta] \sum_{k=0}^{m-1} \left( D[g_{m-1-k}, \eta] * D[g_k, \eta] \right) + \sum_{k=0}^{m-1} \left( (f_{m-1-k} * g_{m-1-k} * g_{m-k}) \right) \]

\[ f_0(\eta) = 1 - \exp[-\eta] \quad (25) \]

\[ f_2 = \frac{1}{2880} e^{-7\eta} (h_1^2(3 + 2\beta - 22\beta^2 - 32\beta^3 - 25\beta^4) + 9\eta h_1^2 \beta(-1 - 14\beta - 20\beta^2 + 13\beta^3 + 53\beta^4 + 29\beta^5 + 5e^{7h_1^2} \beta \left(96(-1 + \beta^2) + h_1 \left(-26 + 9\beta^5(-15 + 4\eta) + 4\beta^2(-80 + 9\eta) + 8\beta^4(-43 + 9\eta) - 2\beta^2(91 + 36\eta) - \beta(73 + 72\eta) + 2M(-18(1 + \eta) + \beta^2(-7 + 6\eta)) \right) \right) \]

\[ f_2 = \frac{1}{14182400} e^{-7\eta} (5h_1^2 \beta^2(-155 - 1408\beta - 3967\beta^2 - 2880\beta^3 + 2695\beta^4 + 4288\beta^5 + 1427\beta^6) - 4e^{7h_1^2} \beta^2(-240 - 4751\beta - 24522\beta^2 - 39788\beta^3 + 36\beta^4 + 67749\beta^5 + 67862\beta^6 + 20054\beta^7) - 7e^{7h_1^2}\beta(840\beta(-3 - 32\beta^2 + 32\beta^3 + 25\beta^4)) \]

\[ f_4 = \frac{1}{17412400} e^{-7h_1^2} \beta^2(-17619 - 213422\beta - 917030\beta^2 - 1745360\beta^3 - 1162248\beta^4 + 85916\beta^2 + 1874270\beta^3 + 1099600\beta^4 + 222627\beta^5 + 1204891\beta^6 + 100352\beta - 7123122\beta^2 - 2207532\beta^3 - 3035742\beta^4 - 581577\beta^5 + 4082514\beta^6 + 5269507\beta^7 + 27280216\beta^8 + 5205080\beta^9) \]

\[ g_0 = \beta (1 - \exp[-\eta]); \]
5. Result and discussion

It's clear that the convergence of homotopic depends upon the parameters $h_1, h_2$. In this section, we show the graphical results of velocity, for this purpose, figures [1,2] explain the effect of the magnetic field on the profile velocity $f$ when the magnetic parameter $M=[1.2, 2.7]$ and ratio of rates parameter $\beta=0.3, 1.5$, its noted that when the magnetic field increasing and $\beta=0.3$ the velocity profile $f$ and momentum boundary layer thickness is decreasing see figure(1). Also, when M is increasing, $\beta=1.3$, the velocity is increasing see figure(2).

Figures [3,4] show the influence of stretching ratio parameter $\beta$ on velocity profile $f$, its clear that at figure 3 the increasing in $\beta$ at $M=1.7$ make the velocity profile and momentum boundary layer thickness is decreasing this is due to the fact that within the increase of ratio of rates parameter $\beta$, relaxation time increases as a result the velocity and boundary layer thickness decreases. Figure 4 explains the effect of the magnetic field $M$ on velocity $g$ it's noted that as the $M$ increase, where $M=[0, 0.5, 1]$, $\beta=0.3$ the velocity $g$ is decreasing. In case take $M=[0, 0.7, 1.3], \beta=1.4$ then the velocity is increase. Figure 6,7 discuss the effect stretching ratio parameter $\beta$ on the velocity $g$, its noted that at $M=0.5$, $1, \beta=[0.2, 0.3, 0.6]$ the velocity is decreasing with increase the magnetic field.
Figure 1: Effect of $M$ on velocity profile $f$, $M = 1.5, 2, 2.7$ and $\beta = 0.3, h = -1.5$

Figure 2: Effect $M$ on velocity profile $f$, $M = 1.5, 2, 2.7$ and $\beta = 1.5, h = -1.5$

Figure 3: Effect $\beta$ on velocity profile $f$, $\beta = 0.0, 0.2, 0.6$ and $M = 1.7, h = -1.5$
Figure 4. Effect $\beta$ on velocity profile $f$, $\beta = 0, 0.2, 0.6$ and $M=3$, $h=-1.5$

Figure 5. Effect $M$ on velocity profile $g$, $M=0, 0.5, 1$ and $\beta=0.3, h=-1.4$
Figure 6. Effect M on velocity profile $g$, $M=0,0.7,1.3$ and $\beta=1.4,h=-1.4$

Figure 7. Effect M on velocity profile $g$, $\beta=0.2,0.3,0.6$ and $M=0.5,h=-1.4$
References


Figure 8. Effect M on velocity profile g, β= 0.2, 0.3, 0.6 and M=1, h=-1.4.


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