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Application of Smooth Transition autoregressive (STAR) models for Exchange Rate

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Abstract

This study evaluates the suitability of the Smooth transition autoregressive (STAR) models specification for real exchange rate Modeling. Our paper investigates the stationarity of real exchange rates which assume linearity in it; we also apply the tests to check stationarity that assume nonlinearity in a particular time series. The focus of this study is to explain the simple matter of time series stationarity or non-stationarity regarding modeling; its principle aim is application of logistic Smooth transition autoregressive (LSTAR) and exponential Smooth transition autoregressive (ESTAR) modeling to Exchange rate series to find the model which better explain its deviation from mean. We found ESTAR adjustment for our data series.

1. Introduction

Smooth transition autoregressive (STAR) was initially proposed in its univariate form by Chan and Tong (1986),Terasvirta (1988) and others. The STAR model can be seen as a continuous mixture of two AR (k) models, with the weighting function defining the degree of non-linearity. We focus on two subclasses of STAR model, the logistic STAR model or LSTAR where the weighting function has the form of a logistic function and ESTAR where the weighting function has the form of an exponential function.

The exercise of ESTAR and LSTAR models for real exchange rates analysis has become massive and is still growing rapidly. Non-Linear class of models and the alternative available of different models that have been developed since the introduction of ESTAR and LSTAR models are remarkable. There has been a move away from single equation models to multivariate models, to vector error correction ESTAR models, and to fractionally integrated ones (see, for example, Rothman, van Dijk and Franses, 2001; Milas and Legrenzi, 2006; Smallwood, 2005, 2006, and many others).

Non stationarity or unit root process of the real bilateral exchange rates of many countries has not been a new concept now. Which demonstrate the movement of Excahnge rate meanders away and do not tend to revert to a long run mean. A number of tests have been theoretically developed and empirically employed to detect the nonstationarity of real exchange rates around the world. These tests include both parametric ones (such as the augmented Dickey–Fuller (ADF) [Dickey and Fuller,1981] and its variants like the ADF–GLS [Elliott *et al.* 1996] tests) and nonparametric tests (such as the Phillips and Perron (PP) [Phillips and Perron, 1988] test). A different type of unit root test is the KPSS test (Kwiatkowski et al., 1992) that assumes the null of stationarity unlike other tests that assume non stationarity under the null.

The tests that have been traditionally applied assume that the real exchange rates exhibit linear movements. But, researchers now increasingly believe that real exchange rates exhibit nonlinearity rather than linearity. Nonlinearity in real exchange rates may arise from the perceived shipping time following a profitable deviation (Coleman, 1995). Cerrato and Sarantis (2006) cite other causes of nonlinearity in real exchange rates, namely, diversity in agents' beliefs and heterogeneity in investors' objectives. Altogether, one can expect that the greater is the deviation in real exchange rates from its long rum equilibrium level, the tendency of mean–reversion in it will be more evident. There have been many studies that looked into the time series properties of real exchange rates assuming nonlinearity in the data generating process. Michael *et al.* (1997), Sarantis (1999), Baum *et al.* (2001), Taylor *et al.* (2001) are some examples. In all of them, the authors model the presence of nonlinearity in real exchange rates.

Among the models capturing the non-stationarity in real exchange rate ESTAR is considered to be the best one for exchange rate issues. However, there exist several issues when utilizing ESTAR models in empirical statistical analysis that are not addressed by this literature.Daniel(2007) adresees many issues in this model and explains ; A feature of empirical real exchange rate data is that threshold effects generally occur some distance away from the centre of the density of the data, requiring threshold bands to be quite wide. At the same time, adjustment is fairly quick. An exponential transition function, nevertheless, has difficulties in delivering such an effect, as it can only generate a quick transition between regimes in the proximity of its location parameter. What then happens is that the parameter governing the shape of the transition function shrinks towards zero to be able to push the threshold bands further away from the centre. There are a number of consequences to this. As the transition function parameter becomes smaller, it becomes more difficult to classify inner and outer regimes appropriately, as the function weighting the regimes remains close to zero over the entire range of values that are generally observed empirically for the threshold variable. When this occurs, the whole notion of regime modelling within this framework becomes redundant. An additional problem that occurs when the transition function parameter becomes small is that it becomes more difficult to uniquely identify the model, as two sets of parameters effectively enter as a product in the model's conditional mean. How weakly identified the model then becomes depends partly on the size of the threshold parameter and partly on the range of the data. Ample evidence of the symptoms hinting to these issues can be found in the empirical literature fitting ESTAR models to real exchange rate data. Overall, study ended, outlining,that an exponential transition function is not suitable for real exchange rate modelling.

The purpose of our study is not to give a detailed treatment of the uses of ESTAR and LSTAR models in real exchange rate modeling and nor to discover the related issues. It aims a simple application of these models for the understanding of new comer in this field. This is motivated by means of a replication and discussion of the results found in literature.

3. Methodology

Let Yt be the observed value of a time series at time t and Xt=(1,Yt-1,…..,yt-k)$^{/}$,the vector of regressors corresponding to the intercept plus K lagged values ,for t=1,…..,n. Then STAR is defined as follows:

$$yt = xt^{/}\,\epsilon 1 + F(\gamma, c, st)xt^{/}\theta 2 + \varepsilon t \qquad \varepsilon t \sim N(0, \sigma^2) \qquad (1)$$

F playing the role of a smooth transition continuous function bounded between 0 and 1.the parameter $\gamma > 0$ is responsible for the smoothness of F, while c is a location or threshold parameter.st is called the transition variable, with st=yt-d(d a delay parameter).

Since the LM test fails for LSTAR and ESTAR adjustment test, therefore Terasvitra (1994) develops a framework that can detect the presence of non-linear behavior. This method can be used to determine whether a series is best modeled as an LSTAR or ESTAR process. This test is based on a Taylor series expansion of the general STAR model. Thus after forming the products of the regressors with the powers of yt-d we test for the presence of LSTAR or ESTAR behavior by estimating an auxiliary regression.

The test for linearity is identical to testing the joint restriction that all non-linear terms are zero. Then we perform the test using a standard F-test with p number (restrictions) degree of freedom in the numerator. If delay parameter is not known then best way is to run the test using all plausible values of d. the value of d that results in the smallest prob-value is the best estimate of "d".

3.1 Testing procedure follows the following steps

Step1-Estimate the linear portion of the AR (p) model to determine the order p and to obtain the residuals.

Step2- obtain the auxiliary equation (obtain by Taylors series expansion). Test the significance of the entire regression by comparing TR^2 to the critical value of Chi-Square. If TR^2 calculated value>Chi-Square table value, reject the null hypothesis of linearity and accept the alternative hypothesis of a smooth transition model. Alternatively we can perform F-Test.

Step3-if null hypothesis is rejected that is if model is non-linear, test the restriction (on parameters of auxiliary equation) that parameters are zero or not, using an F-test. Then if null is rejected then model have ESTAR or LSTAR form.

3.2 General form of Taylor expansions:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \quad -\infty < x < \infty \qquad (2)$$

In order to test the general form of nonlinearity in the equation against a null hypothesis of linearity we used the the auxiliary model which is transformed by using Taylor Series Expansion:

$$yt = \alpha 1(yt-1) + \alpha 2(yt-2) + \beta 1(yt-1)st + \beta 2(yt-1)st^2 + \beta 3(yt-1)st^3 + \beta 4(yt-1)st^4 + \beta 5(yt-1)st^5 + \beta 6(yt-1)st^6 + \beta 7(yt-1)st^7 + \mu t \qquad (3)$$

We assume st=$\Delta yt-3$

We test the followings hypothesis;

H0= $\beta 1 = \beta 2 = \beta 3 = \beta 4 = \beta 5 = \beta 6 = \beta 7$=0 (model is linear)

H1=at least one βs is notzero(model is nonlinear).

F-Statistics= (SSR0-SSR1/J)/SSR1/T-K

SSR0=Square sum of resid from restricted model estimated under null-hypothesis.

SSR1= Square sum of resid from un- restricted model estimated under alternative-hypothesis

J is number of restrictions and k is number of parameters.

If F-Statistics> F-table value, reject the null that model is linear and accept alternative.

The test used for non-linear models are RESET test, Portmanteau test and Lagrange Multiplier Test.

After testing that model is non-linear next step is specification of STAR model.

3.3 Specification of ESTAR MODEL which we used:

$$\Delta yt = \delta yt-1 + \delta yt-1 * \{1-\exp[-\gamma(st-c)^2]\} + \mu t. \qquad (4)$$

where yt is the de-trended series of real exchange rate in our case, and $\gamma > 0$ .The transition functionG(.) is U-shaped and the parameter γ determines the speed of the transition Process between two extreme regimes. When the nonlinear component is zero then our function becomes a linear AR (p) model:

$$\Delta yt = \delta yt-1 + \mu t \qquad (5)$$

Here we Test for the null hypothesis that the series is linear against the alternative hypothesis that the series is nonlinear smooth transition model simplify the testing and consider p =1. That is only AR (1) process.

We Test the null hypothesis of linearity against the alternative hypothesis of nonlinearity. Specifically here, we consider the simple ESTAR nonlinear model Where st is the transition variable. We test;

H0 :γ=0 against H1:γ >0

In order to conduct this test we apply the Taylor series expansion to the non-linear part of the equation and make auxiliary equation. Then apply conditions on auxiliary part and test for non-linearity. We consider only the nonlinear part of the equation.

$$f(\gamma, st) = \delta yt - 1 * \{1 - \exp[-\gamma(st - c)^2\}]$$

To simplify the calculation, we assume that c = 0.We apply the Taylor series expansion to the above function (4) around γ=0.
We get following function after applying expansion;

$$f(\gamma, .) = \alpha 1(yt-1) + \beta 1(yt-1)st^2 + \beta 2(yt-1)st^4 + \beta 3(yt-1)st^6 + \beta 4(yt-1)st^8 + \mu t \qquad (6)$$

We assume st=$\Delta yt - 3$

If γ=0, or alternatively, if Ho=β1= β2= β3= β4=0 then function is linear.if γ>0 or H1=At least one βs is not equal to zero then function is ESTAR non-linear.

Test statistic for decision making is F-Statistics with J and T-K degree of freedom.

After finding that model has non-linear ESTAR specification the next step is to Test the null hypothesis of non-stationary against the ESTAR stationary in the real exchange rate.

If the real exchange rate has the ESTAR non-linearity form, we go to the test for ESTAR stationarity.
To test for ESTAR stationarity we have to follow the two step procedure. In first step we estimate the smoothing parameter γ, and then substitute the estimate of γ in the model we estimateδ. If estimate of δ is negative and numerically large then we can conclude that exchange rate is stationary process. The range of δ can take [-2, 0] for the stationay condition to be satisfied. The lagged dynamics can be included throughout the testing to whiten the noise term.

3.4 Specification LSTAR MODEL which we used:

A first order AR(1) model with LSTAR non-linearity can be specified as:

$$yt = \alpha 1(yt-1) + \delta 1(yt-1) * \{1 + \exp[-\gamma(st-c)]^{-1} + \mu t \qquad (7)$$

Non-linear part is G(st; $\gamma, c) = \{1 + \exp[-\gamma(st-c)]^{-1}$
For simplicity we assume c=0
$f(\gamma, st) = \delta 1(yt-1) * \{1 + \exp[-\gamma(st-c)]^{-1}$

We apply Taylor series approximation to the non-linear form of the function and find the following auxiliary terms as a result;

$$yt = (\alpha 1 + \delta 1/2)(yt-1) + \beta 1(yt-1)st + \beta 2(yt-1)st^3 + \beta 3(yt-1)st^5 + \beta 4(yt-1)st^7 + \mu t$$
(8)

The γ =0 is equivalent to β 1 = β 2 = β 3 = β 4 =0 in (8). Testing the null Hypothesis H0: γ = 0 against H: γ > 0 that is (the eq.(8) is linear against LSTAR nonlinear is equivalent to testing the null hypothesis :
H0= β 1 = β 2 = β 3 = β 4 =0 against H1: At least one βs is not equal to zero

3.5 LSTAR and ESTAR

LSTAR model allows smooth and continuous shift between two extreme regimes.

The expansion for the ESTAR model has the quadratic form, so the auxiliary equation for ESTAR model is nested within that for an LSTAR model. If the ESTAR is appropriate, it should be possible to exclude all of the terms in the cubic expression.

The transition function of LSTAR is of S-shape around c and monotonically increasing in , yielding an asymmetric adjustment process towards the equilibrium, depending on whether or not these deviations are above or below the equilibrium.

The transition function of ESTAR is symmetric and of U-shape around c. The ESTAR model suggests that two regimes have rather similar dynamics, while the transition period can have different dynamics

The main difference between these two STAR models is the discrepancies in the reaction of agents to shocks of the same size with opposite signs. The ESTAR models imply a symmetric U-shaped response about the threshold parameter with respect to positive and negative shocks of the same magnitude. On the other hand the asymmetries of S-shaped are result in the reactions of agents to these shocks in LSTAR.

4. Empirical Findings

For our analysis we have taken the data of exchange rate of Pakistan since 1980-2010.

The series of exchange rate was monthly observation in the log form. First of all we filter the series and get a demeaned and detrended series. Then we determine the stationary of the series by applying ADF-test. If we ignore the intercept, the estimated equation is

$$\Delta yt = -0.027yt - 1 + 0.0031\Delta yt - 1 + \varepsilon t$$

(-4.157) (0.0538)

In absolute terms, the t-statistics (-4.157) is greater than the critical values reported in ADF table (appendix), we reject the null hypothesis of unit root in the real exchange rate. The point estimates of -0.027 implies a fairly slow speed of adjustment, approximately 99.97 percent of the current periods discrepancy is expected to persist into the next year.

The next issue is testing for linearity null hypothesis against a general form of nonlinear alternatives. Nonlinearity alternative hypothesis can include TAR, ESTAR and LSTAR.

We find F-critical= 2.04200850291 and p-value=0.000172981762932

P-value indicates that null is being rejected here that our model has some non-linear type of specification.

When we find that our model is non-linear then in the next step we specify the model for ESTAR non-linearity. We calculate following values after following the whole procedure of testing.

F-critical=2.403320028 p-value= 0.00470738738853

The result show that we reject the null hypothesis, that is, there is clear evidence that the real exchange rate has the ESTAR form with the transition variable st=$\Delta yt-3$. Real exchange rate often has the form of ESTAR non-linearity because it has the symmetric adjustment of positive and negative deviations of the same magnitudes.

After concluding that our model is ESTAR we Test the null hypothesis of non-stationary against the ESTAR stationary in the real exchange rate. In the first step we estimate γ =0.040854

By putting the value of γ =0.040854 in the equation, in the second step we find the value of

δ =-4.202.

As here δ estimate is negative and numerically large we conclude that exchange rate is ESTAR stationary.

When we test for the LSTAR specification we find

F-critical=2.40309970366 P-value= 0.00580738738853

Here again null hypothesis is rejected that our model is nonlinear with LSTAR specification.

The F-critical for ESTAR test has a P_value with enhanced power than that for LSTAR. Though difference between both of them is minute yet it favors the ESTAR specification in real exchange rate.

In order to select a best transition variable among $\Delta yt-1, \Delta yt-2, \Delta yt-3$ and for

$\Delta y^2t-1, \Delta y^2t-2, \Delta y^2t-3$.

We select the general non linear form of model, then define its auxiliary equation by Taylor series expansion and test the model for null=linearity and alternative=non-linearity for all transitional values. We collect for each test F-CRITICAL and P_VALUE.

Here in the table we report the F-critical and p-value. We find $, \Delta yt-3 \text{and}, \Delta y^2t-1$ is best transition variable with enhanced power of rejection

The best transition variable for ESTAR and LSTAR is $, \Delta yt-3$

Transition Variable	F-Critical	P-value	Transition Variable	F-Critical	P-value
$\Delta yt-1$	1.027647	0.30593	$\Delta y^2 t-1$	2.40100	0.0026
$\Delta yt-2$	1.575081	0.13925	$\Delta y^2 t-2$	1.0795	0.0325
$\Delta yt-3$	2.403	0.0047	$\Delta y^2 t-3$	2.05667	0.0039

5. Conclusion

The purpose of this study is neither to explore the issues of exchange rate nor in the modeling of ESTAR and LSTAR. It defines in a simple way how to apply the ESTAR and LSTAR models to a time series which might be non-stationary in nature

We find that real exchange rate series is non-linear and can have ESTAR and LSTAR specification.

The issue to determine the presence of ESTAR and LSTAR adjustment was solved by applying Taylor series approximation. Given the lag length the most plausible value of the delay parameter is d =3.We follows the standard procedure and selects the value of d that results in the best fit of the auxiliary equation. Given that there is presence of threshold adjustment. We find ESTAR adjustment in our real exchange rate series.

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