Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.2, No.9, 2012

Note on $\hat{\Omega}$ -closed sets in topological spaces

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Abstract.

In this paper we introduce a new class of sets known as $\hat{\Omega}$ - closed sets in topological spaces and we study some of its basic properties. It turns out that this class lies between the class of δ -open sets and the class of δg (resp. ω)-closed sets. Unique feature is, this new class of sets forms a topology and it is independent of open sets.

Key words and Phrases: semi open sets, δ -open sets, δ -closure, skerl, $\hat{\Omega}$ -closed sets.

Mathematics Subject Classification 2010: 57N05

1.Introduction.

Levine [11] initiated the study of generalized closed sets (briefly g -closed) in general topology. The concept of g -closed set has been studied further by weaker forms of open sets such as α-open, semi open, pre open, and semi-pre open sets. By using δ -closure operator, Donham and Ganster [8] introduced and studied the concept of δg -closed set, strong than g -closed set. We introduce and study a new class of sets known as $\hat{\Omega}$ -closed set, slightly stronger than the class of δg (resp. ω)-closed sets. Also it properly lies between δ -closed ness and δg (resp. ω)- closed ness.

2. Preliminaries.

Throughout this paper (X, τ) (briefly X) represent topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of (X, τ) , we denote the closure of A, the interior of A and the complement of A as cl(A), int(A) and A^c respectively.

Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. A subset A of a topological space (X, τ) is called a

(i) α-open set [1] if A $\subseteq$ int(cl(int(A)).

(ii) semi-open set [10] if A $\subseteq$ cl(int(A)).

(iii) pre-open set [13] if A $\subseteq$ int(cl(A)).

(iv) β -open (or semi pre open) set[1] if A $\subseteq$ cl(int(cl(A)).

(v) regular open set [14] if A = int(cl(A)).

(vi) b -open set [5] if A $\subseteq$ cl(int(A)) $\cup$ int(cl(A)).

The complement of the above sets are called α-closed, semi-closed, pre-closed, β -closed regular closed and b -closed sets respectively. The α-closure (resp.semi-closure,pre-closure, β -closure) of a subset A of (X, τ) is the intersection of all α-closed (resp.semi-closed ,pre-closed, β -closed,) sets containing A and is denoted by αcl(A) (resp. scl(A), pcl(A), βcl(A)). The intersection of all semi open subsets of (X, τ) containing A is called the semi kernel of A and is denoted by sker(A). The set of all open sets in X is denoted by O(X) and O(X,x) = {U $\in$ X : x $\in$ U $\in$ O(X)}.

Definition 2.2. [17] A subset **A** of X is called δ **-**closed set in a topological space (X, τ) if A = δcl(A) , where δcl(A) = {x $\in$ X : int(cl(U))∩A $\neq \Phi$,U $\in$ O(X,x)}**.**The complement of δ –closed set in (X, τ) is called δ -open set in (X, τ). The set of all δ -closed sets in X is denoted by δ C(X).From [9], lemma 3, δcl(A) = ∩{F $\in$ δC(X) : A $\subseteq$ F} and from corollary 4, δcl(A) is a δ -closed for a subset A in a topological space (X, τ).

Definition 2.3. [17] A subset A of X is called θ -closed in a topological space (X, τ) if A = θcl (A) , where **θcl(A) = {x** $\in$

X : cl(U) ∩ A $\neq \Phi$ **,** U ∈ O(X,x)}.The complement of θ -open set in (X, τ) is called θ -closed set in (X, τ).

Definition 2.4. A subset A of a topological space (X, τ) is called

(i) a generalized closed (briefly g -closed) set [11] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(ii) a generalized α- closed (briefly gα-closed) set [12] if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in (X, τ).

(iii) a α- generalized closed (briefly αg -closed) set[12] if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(iv) a generalized semi-closed (briefly gs -closed) set [2] if scl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(v) a generalized semi-closed (briefly sg -closed) set [3] if scl(A) ⊆ U whenever A ⊆ U and U is semi open in (X, τ).

(vi) a generalized semi-pre closed (briefly gsp -closed) set [7] if spcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(vii) a δ generalized closed (briefly δg -closed) set [8] if δcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

(viii) $\hat{g}$ (or) ω -closed set [15] if cl(A) ⊆ U whenever A ⊆ U and U is semi open in (X, τ).

The complement of g -closed (resp. gα-closed, αg -closed, gs -closed, sg -closed, gsp -closed, δg -Closed, ω -closed) set is called g -open (resp. gα-open, αg -open, gs -open, sg -open, gα-open, gsp -open, δg –open, ω -open).

3. $\hat{\Omega}$ -Closed Sets.

In this section we introduce a basic definition of new class of sets known as $\hat{\Omega}$ -closed sets in topological spaces.

Definition 3.1. A subset A of a space (X, τ) is called $\hat{\Omega}$ -closed if δcl(A) ⊆ U whenever A ⊆ U and U is semi open set in (X, τ). The complement of $\hat{\Omega}$ -closed set in (X, τ) is called $\hat{\Omega}$ - open set in (X, τ).

Theorem 3.2. Every δ -closed set is $\hat{\Omega}$ -closed in (X, τ).

Proof. Let A be any δ -closed and U be any semi open set in (X, τ) such that A ⊆ U. Since A is δ -closed set in (X, τ) , δcl(A) ⊆ U. Thus A is $\hat{\Omega}$ -closed set in (X, τ).

Remark 3.3. The reversible implication is not always possible from the following example.

Example 3.4. Let X = {a, b, c} and τ = { Φ ,{a}, {b, c},X}. Here {b} is $\hat{\Omega}$ -closed set in (X, τ) but not , δ -closed in (X, τ).

Theorem 3.5. In a topological space (X, τ) ,every $\hat{\Omega}$ -closed set is

(i) $\hat{g}$ (or ω) -closed set in (X, τ).

(ii) g (resp. gα,αg , sg , gs)-closed set in (X, τ).

(iii) δg -closed set in (X, τ).

Proof. (i) Suppose that A is a $\hat{\Omega}$ -closed and U be any semi open set in (X, τ) such that A ⊆ U. By hypothesis, δcl(A) ⊆ U. Then, cl(A) ⊆ U and hence A is ˆg -closed set in (X, τ).

(ii) By [16], every $\hat{g}$ -closed set is g (resp. gα,αg , sg , gs)-closed set in (X, τ). Therefore, it holds.

(iii) Suppose that A is a $\hat{\Omega}$ -closed and U be any open sets in (X, τ) such that A ⊆ U. Since every open set is semi open in (X, τ) and by hypothesis, δcl(A) ⊆ U. Hence A is δg -closed set in (X, τ).

Remark 3.6. The following example reveals that the reversible implications are not true in general .

Example 3.7. Let X = {a, b, c, d} and τ = { Φ , {a}, {a, b},X}. Then the set {b, c} is g -closed,gα-closed, sg -closed, δg -closed but not $\hat{\Omega}$ -closed in (X, τ). Also {c, d} is $\hat{g}$ -closed but not $\hat{\Omega}$ -closed in (X, τ).

Remark 3.8. The following examples show that $\hat{\Omega}$ -closed set is independent of closed,α-closed, semi closed, and δ -semi-closed sets.

Example 3.9. Let X = {a, b, c, d} and τ = { Φ , {a}, {a, b},X}. Then the set {c, d} is closed,semi closed and α-closed but not $\hat{\Omega}$ -closed set in (X, τ).

Example 3.10. Let X = {a, b, c} and τ = { Φ , {a, b},X}. Then the set {a, c} is $\hat{\Omega}$ -closed,but not closed or semi closed or α-closed in (X, τ).

Example 3.11. Let X = {a, b, c, d} and τ = { Φ , {a}, {b}, {a, b},X}. Then the set {c} is δ -semi-closed but not $\hat{\Omega}$ -closed set in (X, τ).

Example 3.12. Let X = {a, b, c, d} and τ = { Φ , {c}, {a, d}, {a, c, d},X}. Then the set {a, b, c} is $\hat{\Omega}$ -closed but not δ -semi-closed in (X, τ).

Remark 3.13. The pictorial representation of the above discussions and existing results is shown in Figure-1.Also in Figure-1, any reversible implication is not possible in general.

4. Characterizations.

In this section we characterize $\hat{\Omega}$ -closed sets by giving three necessary and sufficient conditions.

Theorem 4.1. If A is $\hat{\Omega}$ -closed subset in (X, τ) ,then δcl(A) \ A does not contain any nonempty closed set in (X, τ).

Proof. Let F be any closed set in (X, τ) such that F ⊆ δcl(A) \ A. Then A ⊆ X \ F and X \ F is open in (X, τ). Since A is $\hat{\Omega}$-closed and X \ F is semi open, δcl(A) ⊆ X \ F. Hence F ⊆ X \ δcl(A). Thus F ⊆ (δcl(A) \ A) ∩ (X \ δcl(A)) = Φ .

Remark 4.2. The converse is not possible in general from the following example.

Example 4.3. Let X = {a, b, c} and τ = { Φ , {a},X}. Let A = {b}. Then δcl(A) \ A = X \ {b} = {a, c} does not contain any non-empty closed set and A is not a $\hat{\Omega}$ -closed subset of (X, τ).

Theorem 4.4. If A is $\hat{\Omega}$ -closed subset in (X, τ) if and only if δcl(A)\A does not contain any non-empty semi closed set in (X, τ).

Proof. Necessity- Let F be any semi closed such that F ⊆ δcl(A) \ A. Then A ⊆ X \ F and X \ F is semi open in (X, τ). Since A is $\hat{\Omega}$ -closed set in (X, τ) , δcl(A) ⊆ X \ F, F ⊆ X \ δcl(A). Thus, F ⊆ (δcl(A) \ A) ∩ (X \ δcl(A)) = Φ .

Sufficiency- Suppose that A ⊆ U and U is any semi open set in (X, τ). If A is not $\hat{\Omega}$-closed set,then δcl(A) U and hence δcl(A) ∩ (X \ U) ≠ Φ .We have a nonempty semi closed set δcl(A)∩(X \ U) such that δcl(A)∩(X \ U) ⊆ δcl(A)∩(X \ A) = δcl(A) \A, which contradicts the hypothesis.

Theorem 4.5. Let A be any $\hat{\Omega}$ -closed set in (X, τ). Then A is δ -closed in (X, τ) if and only if δcl(A)\A is semi closed set in (X, τ).

Proof. Necessity- Since A is δ -closed set in (X, τ) , δcl(A) = A. Then δcl(A) \ A = Φ is semi closed set in (X, τ).

Sufficiency- Since A is $\hat{\Omega}$ -closed set (X, τ) ,by theorem 4.4, δcl(A) \ A does not contain any non-empty semi closed set. Therefore, δcl(A) \ A = Φ .Hence δcl(A) = A. Thus, A is δ -closed in (X, τ).

Notations 4.6. In a topological space (X, τ) ,Xs = {x ∈ X : {x} is semi closed in (X, τ) } and $X_{\hat{\Omega}}$ = {x ∈ X : {x} is $\hat{\Omega}$ - open in (X, τ) }.

Proposition 4.7. In a topological space (X, τ) ,for each x ∈ X, either {x} is semi closed or {x}c is $\hat{\Omega}$ -closed set in (X, τ). That is, X = Xs ∪ $X_{\hat{\Omega}}$

Proof. Suppose that {x} is not a semi closed set in (X, τ). Then $\{x\}^c$ is not a semi open set and the only semi open set containing $\{x\}^c$ is X. Therefore, δcl($\{x\}^c$) ⊆ X and hence $\{x\}^c$ is $\hat{\Omega}$-closed set in (X, τ).

Theorem 4.8. Let A be any $\hat{\Omega}$ -closed set in (X, τ). If A ⊆ B ⊆ δcl(A) , then B is also a $\hat{\Omega}$-closed set in (X, τ).

Proof. Let B ⊆ U where U is any semi open set in (X, τ). Then A ⊆ U. Since A is $\hat{\Omega}$-closed set, δcl(A) ⊆ U. Since δcl(B) ⊆ δcl(δcl(A)) = δcl(A) ⊆ U, B is a $\hat{\Omega}$-closed set in (X, τ).

Definition 4.9. The intersection of all $\hat{\Omega}$ -open subsets of (X, τ) containing A is called the $\hat{\Omega}$-kernel of A and is denoted by $\hat{\Omega}$ker(A).

Theorem 4.10. A subset A of a topological space (X, τ) is $\hat{\Omega}$-closed in (X, τ) if and only if δcl(A) ⊆ sker(A).

Proof. Necessity. Suppose that A is $\hat{\Omega}$-closed set in (X, τ) and x ∈ δcl(A) and x ∉ sker(A). Then there exists a semi open set U in (X, τ) such that A ⊆ U and x ∉ U. Since A is $\hat{\Omega}$-closed set in (X, τ) , δcl(A) ⊆ U which is a contradiction to x ∈ δcl(A) and x ∉ U.

Sufficiency. Suppose that δcl(A) ⊆ sker(A) and U is any semi open set in (X, τ) such that A ⊆ U. Then sker(A) ⊆ U and hence δcl(A) ⊆ U. Thus, A is $\hat{\Omega}$ -closed set in (X, τ).

Justification 4.11. By the following results, we justify that the original axioms for the topology are preserved by the class of

$\hat{\Omega}$ -open sets in a topological space (X, τ). It is denoted by $\tau_{\hat{\Omega}}$ which is weaker than τ_δ ,the class of δ open sets and stronger than the topology formed by the class of ω -open sets.

Theorem 4.12. If A and B are $\hat{\Omega}$ -closed sets in a topological space (X, τ) ,then A ∪ B is $\hat{\Omega}$-closed set in (X, τ).

Proof. Suppose that A ∪B ⊆U where U is any semi open in (X, τ). Then A ⊆U and B ⊆U. Since A and B are $\hat{\Omega}$-closed sets in (X, τ) , δcl(A) ⊆ U and δcl(B) ⊆U. Always δcl(A ∪ B) = δcl(A) ∪ δcl(B). Therefore, δcl(A ∪ B) ⊆ U.Thus,A ∪ B is a $\hat{\Omega}$ -closed set in (X, τ).

Lemma 4.13. [6] Let x be any point in a topological space (X, τ). Then {x} is either nowhere dense or pre-open in (X, τ). Also,X = X_1 ∪ X_2 ,where X1 = {x ∈ X : {x} is nowhere dense in (X, τ) } and X_2 = {x ∈ X : {x} is pre-open in (X, τ) } is known as Jankovic-Reilly decomposition.

Theorem 4.14. In a topological space (X, τ) , X2 ∩ δcl(A) ⊆sker(A) for any subset A of (X, τ).

Proof. Suppose that x ∈ X_2 ∩ δcl(A) and x ∉ sker(A) .Since x ∈ X_2 , scl({x}) = int(cl({x})).

Moreover,x ∉ X_1 implies that scl({x}) ≠Φ. Since x ∈ δcl(A), A∩int(cl(U)) ≠Φ for any U∈O(X,x).Choose U = int(cl({x})). Then A ∩ int(cl({x}))≠Φ. Choose y ∈ A ∩ int(cl({x})). Since x ∉ sker(A) ,there exists a semi open set V in (X, τ) such that A ⊆ V and x ∉ V . If F = X \ V , then F is a semi closed such that x ∈ F ⊆ X \ A. Also int(cl({x}))⊆ int(cl(F)) ⊆ F and hence y ∈ A ∩ F, a contradiction. Thus, x ∈ sker(A) .

Theorem 4.15. A subset A is $\hat{\Omega}$-closed set in a topological space in (X, τ) if and only if X_1 ∩ δcl(A) ⊆ A.

Proof. Necessity- Suppose that A is $\hat{\Omega}$ -closed set in (X, τ) and x ∈ X_1 ∩ δcl(A) but not in A. Therefore, {x} is semi closed set in (X, τ) and hence X \ {x} is semi open set in (X, τ).Since X\{x} is the semi open set in (X, τ) containing A and by hypothesis, δcl(A) ⊆ X \ {x},a contradiction to x ∈δcl(A). Therefore, X_1 ∩ δcl(A) ⊆ A.

Sufficiency- Suppose that X_1∩δcl(A) ⊆ A. Since A⊆ sker(A), X_1∩δcl(A)⊆ sker(A).By theorem 4.14,X_2 ∩δcl(A)⊆ sker(A) . Therefore, δcl(A) = (X_1∪ X_2) ∩ δcl(A) = (X_1 ∩ δcl(A)) ∪ (X_2 ∩ δcl(A)) ⊆ sker(A) .By theorem 4.10,A is $\hat{\Omega}$-closed set in X.

Theorem 4.16. Arbitrary intersection of $\hat{\Omega}$ -closed sets in a topological space (X, τ) is $\hat{\Omega}$-closed set

in (X, τ).

Proof. Let {Ai : i∈ I} be any family of $\hat{\Omega}$ -closed sets in (X, τ) and A = $\bigcap_{i\in I} A_i$. Therefore,X_1∩δcl(Ai) ⊆Ai for each i∈I and hence X_1∩δcl(A)⊆ X_1∩δcl(Ai)⊆Ai for each i ∈ I .Then X_1∩δcl(A)⊆ $\bigcap_{i\in I} A_i$= A. By theorem 4.15,A is $\hat{\Omega}$ -closed set in (X, τ). Thus, arbitrary intersection of $\hat{\Omega}$-closed sets in a topological space (X, τ) is $\hat{\Omega}$ -closed set in (X, τ).

Notations 4.17. In a topological space (X, τ) ,the set of all semi (resp. pre, $\hat{\Omega}$) open sets are denoted by SO(X) (resp. PO(X), $\hat{\Omega}$O(X)). The set of all δ -closed (resp. $\hat{\Omega}$-closed) sets are denoted by δC(X) (resp. $\hat{\Omega}$C(X)).

Lemma 4.18. If A is $\hat{\Omega}$ -closed and B is δ -closed sets in (X, τ) then A∩B is $\hat{\Omega}$ -closed in (X, τ) because of arbitrary intersection of $\hat{\Omega}$ -closed sets is a $\hat{\Omega}$ -closed set.

Let us characterize partition space via $\hat{\Omega}$ -closed sets.

Remark 4.19. [8] A partition space is a topological space (X, τ) where every open set is closed. Also a topological space is partition space if and only if every subset is pre open.

Theorem 4.20. In a topological space (X, τ) ,

(i) SO(X) ⊆ δC(X) if and only if $\hat{\Omega}$O(X) = P(X) .

(ii) (X, τ) is a partition space if and only if $\hat{\Omega}$O(X) = P(X) .

Proof. (i) Necessity-Let A be arbitrary subset of (X, τ) such that A ⊆ U where U∈SO(X) .By hypothesis, δcl(A) ⊆ δcl(U) = U. Therefore, A is $\hat{\Omega}$ -closed set in (X, τ).

Sufficiency- Let U be any semi open set in (X, τ). By hypothesis, U is $\hat{\Omega}$ -closed set in (X, τ). Since every $\hat{\Omega}$ -closed set is pre closed set, U is a pre closed set in (X, τ). It is clear that if U is both semi open and pre closed, then U is a regular closed set and hence it is a δ -closed set in (X, τ).

(ii) Necessity- Let A be arbitrary subset of (X, τ) and suppose that x ∈ X_1 ∩ δcl(A), x ∉ A. We have {x} is a semi closed set and hence it is a closed set in (X, τ). Therefore, X\{x} is an open set in (X, τ) and by hypothesis, it is a closed set in (X, τ). Now X\{x} is a clopen set in (X, τ) and then δ -closed set in (X, τ). Therefore, δcl(A)⊆δcl(X \{x}) = X \{x},a contradiction to $x \in \delta cl(A)$. Thus, $X_1 \cap \delta cl(A) \subseteq A$. By theorem 4.15,A is $\hat{\Omega}$ -closed set in (X, τ).

Sufficiency- Let U be any open and hence semi open set in (X, τ). By hypothesis, $\hat{\Omega}$-closed set in (X, τ). Since every $\hat{\Omega}$ -closed set is pre closed set, U is a pre closed set in (X, τ). It is clear that if U is both semi open and pre closed, then U is a regular closed and hence it is a δ -closed in (X, τ). Therefore, U is a closed set in (X, τ). Thus, every open set is closed in (X, τ).

Remark 4.21. From the above discussions, a topological space is partition space if and only if $\hat{\Omega}O(X) = PO(X) = P(X)$.

5. $\hat{\Omega}$ -closure.

In this section we define the closure of $\hat{\Omega}$ -closed sets and prove that it is a **"Kuratowski closure operator."**

Definition 5.1. Let A be a subset of a topological space (X, τ). Then the $\hat{\Omega}$ -closure of A is defined to be the intersection of all $\hat{\Omega}$ -closed sets containing A and it is denoted by $\hat{\Omega}cl(A)$. That is $\hat{\Omega}cl(A) = \bigcap\{F : A \subseteq F \text{ and } F \in \hat{\Omega}C(X)\}$. Always, $A \subseteq \hat{\Omega}cl(A)$.

Remark 5.2. From the definition and 4.16, $\hat{\Omega}cl(A)$ is the smallest $\hat{\Omega}$ -closed set containing A.

Theorem 5.3. Let A and B be subsets of a topological space (X, τ). Then,

(i) $\hat{\Omega}cl(\Phi) = \Phi$ and $\hat{\Omega}cl(X) = X$.

(ii) If $A \subseteq B$, then $\hat{\Omega}cl(A) \subseteq \hat{\Omega}cl(B)$.

(iii) $\hat{\Omega}cl(A \cap B) \subseteq \hat{\Omega}cl(A) \cap \hat{\Omega}cl(B)$.

(iv) $\hat{\Omega}cl(A \cup B) = \hat{\Omega}cl(A) \cup \hat{\Omega}cl(B)$.

(v) A is a $\hat{\Omega}$-closed set in (X, τ) if and only if $A = \hat{\Omega}cl(A)$.

(vi) $\hat{\Omega}cl(\hat{\Omega}cl(A)) = \hat{\Omega}cl(A)$.

(vii) $\hat{\Omega}cl(A) \subseteq \delta cl(A)$.

Proof. (i) Obvious.

(ii) $A \subseteq B \subseteq \hat{\Omega}cl(B)$. But $\hat{\Omega}cl(A)$ is the smallest $\hat{\Omega}$ -closed set containing A. Hence $\hat{\Omega}cl(A) \subseteq \hat{\Omega}cl(B)$.

(iii) $A \cap B \subseteq A$ and $A \cap B \subseteq B$. By (ii), $\hat{\Omega}cl(A \cap B) \subseteq \hat{\Omega}cl(A)$ and $\hat{\Omega}cl(A \cap (B) \subseteq \hat{\Omega}cl(B)$.Hence $\hat{\Omega}cl(A \cap B) \subseteq \hat{\Omega}cl(A) \cap \hat{\Omega}cl(B)$.

(iv) $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By(ii), $\hat{\Omega}cl(A) \subseteq \hat{\Omega}cl(A \cup B)$ and $\hat{\Omega}cl(B) \subseteq \hat{\Omega}cl(A \cup (B)$.Hence $\hat{\Omega}cl(A) \cup \hat{\Omega}cl(B) \subseteq \hat{\Omega}cl(A \cup B)$. On the other hand, $A \subseteq \hat{\Omega}cl(A)$ and $B \subseteq \hat{\Omega}cl(B)$ implies that $A \cup B \subseteq \hat{\Omega}cl(A) \cup \hat{\Omega}cl(B)$ .But $\hat{\Omega}cl(A \cup B)$ is the smallest $\hat{\Omega}$ -closed set containing $A \cup B$. Hence $\hat{\Omega}cl(A \cup B) \subseteq \hat{\Omega}cl(A) \cup \hat{\Omega}cl(B)$.Therefore, $\hat{\Omega}cl(A \cup B) = \hat{\Omega}cl(A) \cup \hat{\Omega}cl(B)$.

(v) Necessity- Suppose that A is $\hat{\Omega}$ -closed in (X, τ). By remark 5.2, $A \subseteq \hat{\Omega}cl(A)$. By the definition of $\hat{\Omega}$ closure and hypothesis, $\hat{\Omega}cl(A) \subseteq A$. Therefore, $A = \hat{\Omega}cl(A)$.

Sufficiency-Suppose that $A = \hat{\Omega}cl(A)$. By the definition of $\hat{\Omega}$ closure, $\hat{\Omega}cl(A)$ is a $\hat{\Omega}$- closed set and hence A is a $\hat{\Omega}$ -closed set in (X, τ).

(vi) Since arbitrary intersection of $\hat{\Omega}$ -closed sets in a topological space (X, τ) is $\hat{\Omega}$ -closed set in (X, τ), $\hat{\Omega}cl(A)$ is a $\hat{\Omega}$-closed set in (X, τ). By v, $\hat{\Omega}cl(\hat{\Omega}cl(A)) = \hat{\Omega}cl(A)$.

(vii) Suppose that $x \notin \delta cl(A)$ .Then there exists a δ -closed set F such that $A \subseteq F$ and $x \notin F$ .Since every δ -closed set is $\hat{\Omega}$ -closed set, $x \notin \hat{\Omega}cl(A)$. Thus, $\hat{\Omega}cl(A) \subseteq \delta cl(A)$.

Remark 5.4. The reversible inclusion of (iii) is not true in general from the following example.

Example 5.5. Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, X\}$. If $A = \{a\}$ and $B = \{b\}$, then $\hat{\Omega}cl(A) = \{a, c, d\}$, $\hat{\Omega}cl(B) = \{b, c, d\}$,$A \cap B = \Phi$, $\hat{\Omega}cl(A \cap B) = \Phi$. But $\hat{\Omega}cl(A) \cap \hat{\Omega}cl(B) = \{c, d\}$.

Remark 5.6. From $\hat{\Omega}cl(\Phi) = \Phi$, $A \subseteq \hat{\Omega}cl(A)$, $\hat{\Omega}cl(A \cup B) = \hat{\Omega}cl(A) \cup \hat{\Omega}cl(B)$,and $\hat{\Omega}cl(\hat{\Omega}cl(A)) = \hat{\Omega}cl(A)$ we can say that $\hat{\Omega}$-closure is the **Kuratowski closure operator on (X, τ).**

Definition 5.7. A point x of a space (X, τ) is called a $\hat{\Omega}$ -limit point of a subset A of (X, τ) if for each $\hat{\Omega}$ -open set U containing x intersects A other than x. That is, $A \cap (U - \{x\}) \neq \Phi$.The set of all limit points of A is denoted by $D_{\hat{\Omega}}$ (A) and is called the $\hat{\Omega}$-derived set of A.

Theorem 5.8. Let A and B be any two subsets of a space (X, τ). Then

(i) $D_{\hat{\Omega}}(\Phi) = \Phi$ and $D_{\hat{\Omega}}$ (X) = X.

(ii) If A $\subseteq$ B, then $D_{\hat{\Omega}}$ (A) $\subseteq D_{\hat{\Omega}}$ (B).

(iii) $D_{\hat{\Omega}}$ (A $\cup$ B) = $D_{\hat{\Omega}}$ (A) $\cup$ $D_{\hat{\Omega}}$ (B).

(iv) $D_{\hat{\Omega}}$ (A $\cap$ B) $\subseteq D_{\hat{\Omega}}$ (A) $\cap$ $D_{\hat{\Omega}}$ (B).

(v) A subset A is $\hat{\Omega}$ -closed iff $D_{\hat{\Omega}}$ (A) $\subseteq$ A.

(vi) $\hat{\Omega}$cl(A) = A $\cup D_{\hat{\Omega}}$ (A).

Proof. Follows from the definition and similar to theorem 5.3.

Remark 5.9. The following example shows that the reversible inclusion of (iv) is not true in general.

Example 5.10. Let X = {a, b, c, d} and τ = { Φ, {a}, {b}, {a, b},X} If A = {a} and B = {b}, $D_{\hat{\Omega}}$ (A) = {c, d} and $D_{\hat{\Omega}}$ (B) = {c, d},A $\cap$ B = Φ, $D_{\hat{\Omega}}$ (A $\cap$ B) = Φ. But $D_{\hat{\Omega}}$ (A) $\cap$ $D_{\hat{\Omega}}$ (B) = {c, d}.

Theorem 5.11. In a topological space (X, τ) ,for x $\in$ X, x $\in$ $\hat{\Omega}$cl(A) if and only if U $\cap$A $\neq \Phi$ for every $\hat{\Omega}$ -open set U containing x.

Proof. Necessity- Suppose that x $\in$ $\hat{\Omega}$cl(A) and suppose there exists a $\hat{\Omega}$ -open set U containing x such that U $\cap$A = Φ. Then A $\subseteq U^c$ and U^c is a $\hat{\Omega}$ -closed set. By remark 5.2, $\hat{\Omega}$cl(A) $\subseteq$ U^c .Therefore, x $\notin$ $\hat{\Omega}$cl(A), a contradiction.

Sufficiency- Suppose that x $\notin$ $\hat{\Omega}$cl(A) Then there exists $\hat{\Omega}$ -closed set F containing A such that $\notin$F. Hence F^c is a $\hat{\Omega}$ -open set containing x such that. $F^c \subseteq A^c$ .Therefore, $F^c \cap A = \Phi$ which contradicts the hypothesis.

Definition 5.12. A point x in a topological space (X, τ) is called a $\hat{\Omega}$ -interior point of a subset A of (X, τ) if there exists some $\hat{\Omega}$ -open set U containing x such that U $\subseteq$ A. The set of all $\hat{\Omega}$-interior points of A is called the $\hat{\Omega}$ -interior of A and is denoted by $\hat{\Omega}$int(A).

Remark 5.13. $\hat{\Omega}$int(A) is the union of all $\hat{\Omega}$-open sets contained in A and by theorem 4.16, $\hat{\Omega}$int(A) is the largest $\hat{\Omega}$ -open set contained in A.

Theorem 5.14. A subset A of (X, τ) is $\hat{\Omega}$ -open if and only if F $\subseteq$ δint(A) whenever F is

semi closed set and F $\subseteq$ A.

Proof. obvious.

Theorem 5.15. (i) $\hat{\Omega}$cl(X \ A) = X \ $\hat{\Omega}$int(A).

(ii) $\hat{\Omega}$int(X \ A) = X \ $\hat{\Omega}$cl(A).

Proof. (i) $\hat{\Omega}$int(A) $\subseteq$ A $\subseteq$ $\hat{\Omega}$cl(A). Hence X \ $\hat{\Omega}$cl(A) $\subseteq$X \ A$\subseteq$ X \ $\hat{\Omega}$int(A). Then X \ $\hat{\Omega}$cl(A) is the $\hat{\Omega}$ -open set contained in X \A. But $\hat{\Omega}$int(X $-$A) is the largest $\hat{\Omega}$ -open set contained in X\A. Therefore, X\$\hat{\Omega}$cl(A) $\subseteq$ $\hat{\Omega}$int(X\A). On the other hand, if x $\in$ $\hat{\Omega}$int(X\A),there exists a $\hat{\Omega}$ -open set U containing x such that U $\subseteq$X \ A. Hence U $\cap$ A = Φ .Therefore, x $\notin$ $\hat{\Omega}$cl(A) and hence x $\in$(X \ $\hat{\Omega}$cl(A)) . Thus, $\hat{\Omega}$int(X \ A) $\subseteq$ X \ $\hat{\Omega}$cl(A).

(ii) Similar to the proof of (i).

6. Applications.

Notations 6.1. For any set A $\subseteq$X ,(A, τ |A) represents subspace topological space with respective to τ. Let A and B be any two subsets in a topological space (X, τ) such that B $\subseteq$ A ,then $\delta cl_X(B)$ (resp. $\hat{\Omega}cl_X(B)$) represents δ (resp. $\hat{\Omega}$) closure of B in (X, τ) and $\delta cl_A(B)$ (resp. $\hat{\Omega}cl_A(B)$) represents δ (resp. $\hat{\Omega}$) closure of B in the subspace (A, τ |A) . Also $sker_X$ (B) (resp. $\hat{\Omega}ker_X$ (B)) represents semi (resp. $\hat{\Omega}$) kernel of B in (X, τ) and $sker_A$ (B) (resp. $\hat{\Omega}ker_A$ (B)) represents semi (resp. $\hat{\Omega}$) kernel of B in the

subspace (A, τ |A) .

Remark 6.2. [13(a)] Let A be any open set in a topological space (X, τ). Let B $\subseteq$A. Then,

$\delta cl_A(B) = A \cap \delta cl_X(B)$

Remark 6.3. **[13(a)]** Let A be any pre open set in a topological space (X, τ). Let B $\subseteq$ A

Then, $Sker_A(B) = A \cap sker_X(B)$.

Theorem 6.4. If A is both semi open and pre closed set in a topological space (X, τ) , then A is $\hat{\Omega}$ -closed in (X, τ).

Proof. It is clear that if A is both semi open and pre closed, then A is regular closed and hence it is δ -closed in (X, τ). Therefore it is $\hat{\Omega}$ closed in (X, τ).

Theorem 6.5. Let B $\subseteq$ A $\subseteq$ X where A is open in (X, τ). If B is $\hat{\Omega}$ -closed set in (X, τ) ,then B is $\hat{\Omega}$ -closed set in the subspace (A, τ |A).

Proof. Suppose that B is $\hat{\Omega}$-closed set in (X, τ). By theorem 4.10, $\delta cl_X(B) \subseteq sker_X(B)$ and hence $A \cap \delta cl_X(B) \subseteq A \cap sker_X(B)$ .By remarks 6.2 and 6.3, $\delta cl_A(B) \subseteq sker_A(B)$ .Again by theorem 4.10,B is $\hat{\Omega}$-closed set in the subspace (A, τ |A) .

Theorem 6.6. Let B $\subseteq$ A $\subseteq$ X where A is both open and pre closed set in (X, τ). If B is $\hat{\Omega}$-closed set in the subspace (A, τ |A), then B is $\hat{\Omega}$-closed set in (X, τ).

Proof. Suppose that B is $\hat{\Omega}$-closed set in the subspace (A, τ |A). By theorem 4.10, $\delta cl_A(B) \subseteq sker_A(B)$ and hence by remarks 6.2 and 6.3, $A \cap \delta cl_X(B) \subseteq A \cap sker_X(B)$. Since A is δ -closed in (X, τ) , $\delta cl_X(B) = \delta cl_X(A) \cap \delta cl_X(B) = A \cap \delta cl_X(B) \subseteq A \cap sker_X(B) \subseteq sker_X(B)$. Therefore, $\delta cl_X(B) \subseteq sker_X(B)$. By theorem 4.10, B is $\hat{\Omega}$ -closed set in (X, τ).

Theorem 6.7. If F is $\hat{\Omega}$ -closed set in (X, τ) ,then F ∩ A is $\hat{\Omega}$-closed set in the subspace (A, τ |A) provided that A is both open and pre closed set in a topological space (X, τ).

Proof. By theorem 6.4,F ∩ A is $\hat{\Omega}$-closed set in (X, τ).By theorem 4.10, $\delta cl_X(F \cap A) \subseteq sker_X(F \cap A)$ .Then $A \cap \delta cl_X(F \cap A) \subseteq A \cap sker_X(F \cap A)$ and hence by remarks 6.2 and 6.3, $\delta cl_A(F \cap A) \subseteq sker_A(F \cap A)$ Again by theorem 4.10,F ∩ A is $\hat{\Omega}$ -closed set in the subspace (A, τ |A) .

Theorem 6.8. Let U $\subseteq$ A $\subseteq$ X where A is both open and pre closed set in (X, τ). If U is $\hat{\Omega}$-open set in (X, τ) , then U is $\hat{\Omega}$ -open in the subspace (A, τ |A) .

Proof. Suppose that U is $\hat{\Omega}$ -open set in (X, τ). Then X \U is $\hat{\Omega}$ -closed set in (X, τ). By theorem 6.7,(X \U)∩A is $\hat{\Omega}$ -closed set in the subspace (A, τ |A) .That is, A\ (A ∩ U) is $\hat{\Omega}$ -closed set in the subspace (A, τ |A) . Then A \ U is $\hat{\Omega}$ -closed set in the subspace (A, τ |A) .Thus U is $\hat{\Omega}$ -open set in the subspace (A, τ |A).

Theorem 6.9. Let U $\subseteq$ A $\subseteq$ X where A is both δ -open and pre closed set in (X, τ). If U is $\hat{\Omega}$-open set in the subspace (A, τ |A) ,then U is $\hat{\Omega}$ -open in (X, τ).

Proof. Suppose that U is $\hat{\Omega}$ -open set in the subspace (A, τ |A) . Then A\ U is $\hat{\Omega}$-closed set in the subspace (A, τ |A) . By 6.6, A \ U is $\hat{\Omega}$ -closed set in (X, τ). That is A \ U = (X \ U) ∩ A is $\hat{\Omega}$-closed set in (X, τ). By theorem 4.12, U = [X \ ((X \ U) ∩ A)] ∩ A is $\hat{\Omega}$-open set in (X, τ).

Theorem 6.10. Let A be both open and pre closed set in a topological space (X, τ). If U is $\hat{\Omega}$-open set in (X, τ) ,then U ∩ A is $\hat{\Omega}$ -open set in a subspace (A, τ |A) .

Proof. Suppose that U is $\hat{\Omega}$ -open set in (X, τ) ,then X \ U is $\hat{\Omega}$ -closed set in (X, τ). By theorem 6.7,(X \ U) ∩ A is $\hat{\Omega}$ -closed set in a subspace (A, τ |A) . Then A \ (U ∩ A) is $\hat{\Omega}$ -closed set in a subspace (A, τ |A) . Thus U ∩ A is $\hat{\Omega}$ -open set in a subspace (A, τ |A) .

Theorem 6.11. Let A be both open and pre closed set in a topological space (X, τ). If E is any subset of X such that E $\subseteq$ A $\subseteq$ X, then $\hat{\Omega} cl_A(E) \subseteq A \cap \hat{\Omega} cl_X(E)$.

Proof. Suppose that x $\in \hat{\Omega} cl_A(E)$ and F be an arbitrary $\hat{\Omega}$-closed set in (X, τ) such that E $\subseteq$ F By theorem 6.7, F ∩ A is $\hat{\Omega}$ -closed set in a subspace (A, τ |A) such that E $\subseteq$ F ∩ A. Therefore, $\hat{\Omega} cl_A(E) \subseteq F \cap A$ and hence x $\in F \cap A \subseteq F$ By the definition of closure, x $\in \hat{\Omega} cl_X(E)$ and hence x $\in A \cap \hat{\Omega} cl_X(E)$ .Thus $\hat{\Omega} cl_A(E) \subseteq A \cap \hat{\Omega} cl_X(E)$.

Theorem 6.12. Let A be both open and pre closed set in a topological space (X, τ). If E is any subset of X such that E $\subseteq$ A $\subseteq$ X .then $A \cap \hat{\Omega} cl_X(E) \subseteq \hat{\Omega} cl_A(E)$.

Proof. Suppose that x $\in$ A$\cap\hat{\Omega}cl_X(E)$ and F is an arbitrary $\hat{\Omega}$ -closed set in the subspace (A, τ |A) such that E $\subseteq$ F $\subseteq$ A. By theorem 6.6, F is $\hat{\Omega}$ -closed set in (X, τ). Therefore, $\hat{\Omega}cl_X(E) \subseteq \hat{\Omega}cl_X(F)$ = F. Therefore, x $\in$ F. By the definition of $\hat{\Omega}$ -closure in subspace, x $\in$ $\hat{\Omega}cl_A(E)$. Thus A $\cap$ $\hat{\Omega}cl_X(E) \subseteq \hat{\Omega}cl_A(E)$.

Theorem 6.13. Let A be both open and pre closed set in a topological space (X, τ). If E is any subset of X such that E$\subseteq$A $\subseteq$ X, then $\hat{\Omega}ker_A(E) \subseteq A \cap \hat{\Omega}ker_X(E)$.

Proof. Similar to 6.11.

Theorem 6.14. Let A be both δ -open and pre closed set in a topological space (X, τ) and E $\subseteq$A$\subseteq$ X .Then A $\cap$ $\hat{\Omega}ker_X(E) \subseteq \hat{\Omega}ker_A(E)$.

Proof. Similar to 6.12.

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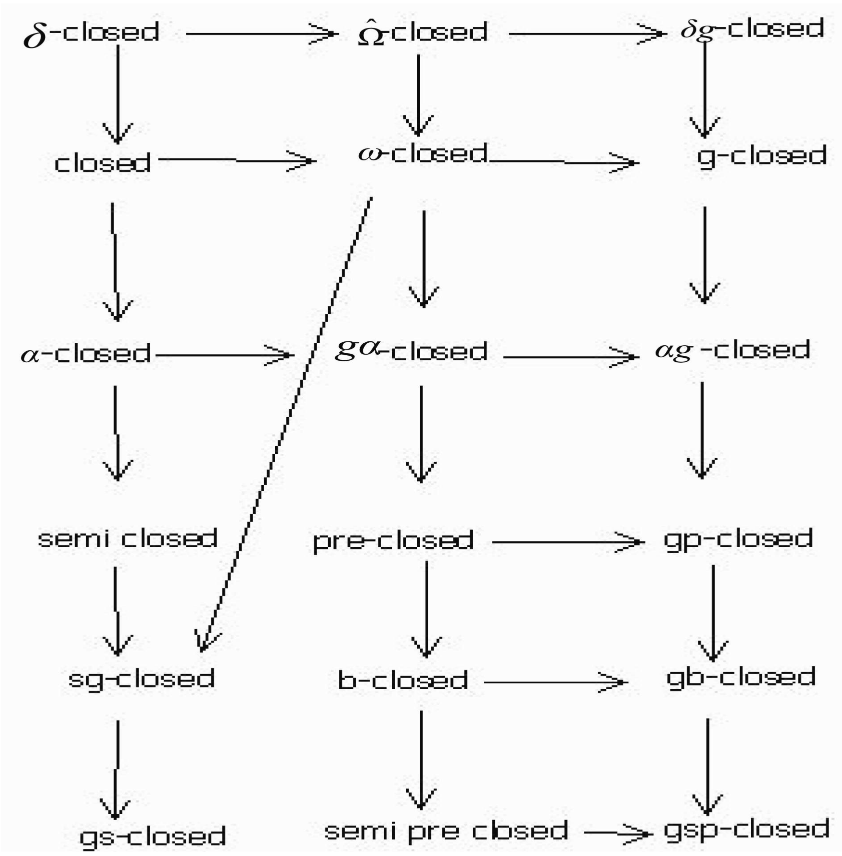

Figure-1