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Effects of Couple Stress and Porous Medium on Transient Magneto Peristaltic Flow

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Abstract :

In this investigation we have analyzed the influence of a transverse magnetic field in the transient peristaltic transport through porous medium. This model is designed for computing the intestinal pressures during the movement of food bolus in the digestive system under magneto-hydro dynamic effects. we consider in the present article the transient magneto-fluid flow through a finite length channel by peristaltic pumping. Long wavelength($a \ll \lambda \ll \infty$) and low Reynolds number approximations($Re \to 0$) have been employed to reduce the governing equations from nonlinear to linear form and that enough to neglect inertial effects . Analytical approximate solution for axial velocity, transverse velocity , pressure gradient , local wall shear stress and volumetric flow rate are obtained for the non-dimensional equations subject to appropriate boundary conditions. This study is done through drawing many graphs by using the MATHEMATICA package.

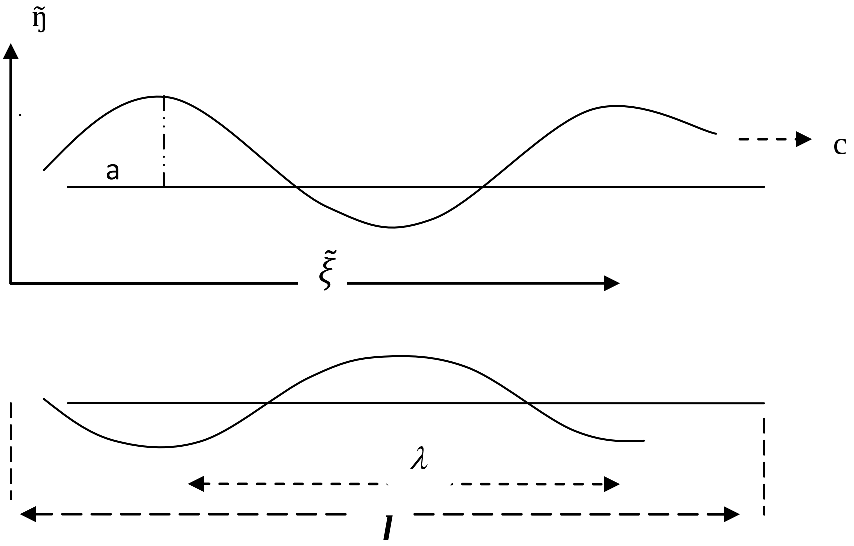
Keywords : Transient peristaltic flow , Pressure gradient , Local wall shear stress , Couple stress , Magnetic field , porous medium.

1.Introduction

Peristaltic motion in a channel / tube is now known as an important type of flow occurring in several engineering and physiological processes .A variety of complex rheological fluid can easily be transport from one to another place with a special type of pumping known as Peristaltic pumping The digestive system is essentially a long ,twisting tube that runs from mouth to the anus .After chewing and swallowing, the food enters the esophagus . The esophagus is long tube that runs from the mouth to the stomach . Further an interesting fact is that in esophagus, the movement of food is due to peristalsis .(M. Li, J.G. Brasseur,1993) studied peristaltic flow of Newtonian viscous fluid through a finite length tube , this study differing from the earlier analysis of(A.H Shapiro et al.1969).They(M. Li, J.G. Brasseur,1993) also compare their results with manometric observation of intra bolus oesophageal pressure. (Misra and Pandey, 2001) have extended this model for power law fluid behavior index on flow patterns, with the objective of simulating rheological bio fluids more accurately. (Tripathi ,Pandy,and . A. Běg 2012) analyzed the unsteady transient peristaltic flow of variable viscosity fluid in a finite length cylindrical tube. The non - Newtonian behavior of many fluids in transport has also been established for some time. The micro-continuum theory of couple stress fluid proposed by (V.K.Stokes ,1966) defines the rotational field in terms of the velocity field for setting up the constitutive relationship between the stress and the strain rate . In this fluid model , the couple stress effects are considered as a consequence of the action of a deforming body on its neighborhood . Some investigations(Mekheimer 2008&2004, Kothondponi ,Srinivas ,2008) on peristaltic flow of the couple stress fluids through uniform and non uniform tubes have been presented . these studies deal with the effect of couple stress parameter and magnetic field on pressure and friction force distributions, where steady flows discussed. Flows with slip would be useful for problems in chemical engineering , for example flows through pipe in chemical reactions occure at the walls(Hummady and Abdulhadi,2014) discussed influence of MHD on peristaltic flow of couple stress fluid through a porous medium with slip effect .This transient couple stress peristaltic flow of non - Newtonian fluid through a finite length channel wall through porous medium..Intra-bolus pressure distribution, local wall shear stress and velocity profiles computed for the effects of the key hydrodynamic parameters .The influence of various pertinent parameters on the flow characteristics , this study are discussed through graphs.

2. Mathematical Formulation:

Consider a transient peristaltic flow of non – Newtonian couple stress fluid through a finite length channel , in two dimensional conduit with constant speed c along the channel walls.



Fig(1) : Geometry of the problem

The geometry of wall surface (see Fig.1) is described as :

$$\tilde{h}\,(\tilde{\xi},\tilde{t}) = a - \tilde{\varphi}\cos^2\frac{\pi}{\lambda}(\tilde{\xi} - c\tilde{t}) \tag{1}$$

Where $\tilde{h}$, $\tilde{\xi}$, $\tilde{t}$, a , $\tilde{\varphi}$, λ and c represent the transverse vibration of the wall , the axial coordinate , time , the half width of the channel , the amplitude of the wave , the wavelength and the wave velocity respectively .

3. Basic equations:

The basic equations governing the non-Newtonian fluid flow under the effect of a transverse magnetic field are given by :

The continuity equation :

$$\frac{\partial\tilde{u}}{\partial\tilde{\xi}} + \frac{\partial\tilde{v}}{\partial\tilde{\eta}} = 0 \tag{2}$$

The momentum equations are:

$$\rho\left(\frac{\partial\tilde{u}}{\partial\tilde{t}} + \tilde{u}\frac{\partial\tilde{u}}{\partial\tilde{\xi}} + \tilde{v}\frac{\partial\tilde{u}}{\partial\tilde{\eta}}\right) = -\frac{\partial\tilde{p}}{\partial\tilde{\xi}} + \mu\nabla^2\tilde{u} - \mu_1\nabla^4\tilde{u} - \sigma B_0^2\tilde{u} - \frac{\mu}{\tilde{k}}\tilde{u} \tag{3}$$

$$\rho\left(\frac{\partial\tilde{v}}{\partial\tilde{t}} + \tilde{u}\frac{\partial\tilde{v}}{\partial\tilde{\xi}} + \tilde{v}\frac{\partial\tilde{v}}{\partial\tilde{\eta}}\right) = -\frac{\partial\tilde{p}}{\partial\tilde{\eta}} + \mu\nabla^2\tilde{v} - \mu_1\nabla^4\tilde{v} - \sigma B_0^2\tilde{v} - \frac{\mu}{\tilde{k}}\tilde{v} \tag{4}$$

Where ρ is the fluid density, $\tilde{u}$ axial velocity , $\tilde{v}$ transverse velocity , $\tilde{\eta}$ transverse coordinate , $\tilde{p}$ pressure , μ viscosity , μ_1 constant associated with couple stress, σ electrical conductivity, B_0 transverse magnetic field and $\tilde{k}$ permeability parameter respectively .

$$\nabla^2 = \frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2} \quad , \quad \nabla^4 = \frac{\partial^4}{\partial\xi^4} + \frac{\partial^4}{\partial\eta^4} + 2\,\frac{\partial^4}{\partial\xi^2\,\partial\eta^2}$$

In order to simplify the governing equations of the motion ,We introducing the following dimensionless parameters:

$$\left.\begin{array}{l}\xi=\frac{\tilde{\xi}}{\lambda} \quad ,\eta=\frac{\tilde{\eta}}{a} \quad ,u=\frac{\tilde{u}}{c} \quad ,v=\frac{\tilde{v}}{c\delta} \quad ,\varphi=\frac{\tilde{\varphi}}{a} \quad ,h=\frac{\tilde{h}}{a} \quad ,p=\frac{\tilde{p}a^2}{\mu c\lambda}, \\ Re=\frac{\rho ca}{\mu},\delta=\frac{a}{\lambda} \quad ,\alpha=a\sqrt{\frac{\mu}{\mu_1}} \quad ,M=\sqrt{\frac{\sigma}{\mu}}aB_0 \quad ,K=\frac{\tilde{k}}{a^2} \quad ,t=\frac{c\tilde{t}}{\lambda}.\end{array}\right\}$$

Where δ is the wave number , Re Reynolds number , α is the couple stress parameter , M is the Hartmann number , and K is the thermal conductivity.

Substituting (5) into equations (1)-(4), we obtain the following non-dimensional equations and boundary conditions :

$$h(\xi,t)=1-\varphi\cos^2\pi(\xi-t). \tag{6}$$

$$\frac{\partial u}{\partial \xi}+\frac{\partial v}{\partial \eta}=0\ . \tag{7}$$

$$Re\,\delta\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial \xi}+v\frac{\partial u}{\partial \eta}\right)=-\frac{\partial p}{\partial \xi}+\delta^2\frac{\partial^2 u}{\partial \xi^2}+\frac{\partial^2 u}{\partial \eta^2}-\frac{1}{\alpha^2}\delta^4\frac{\partial^4 u}{\partial \xi^4}-\frac{1}{\alpha^2}\frac{\partial^4 u}{\partial \eta^4}-2\frac{\delta^2}{a^2}\frac{\partial^4 u}{\partial \xi^2\,\partial \eta^2}$$
$$-M^2u-\frac{1}{k}u\ . \tag{8}$$

$$Re\,\delta^3\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial \xi}+v\frac{\partial v}{\partial \eta}\right)=-\frac{\partial p}{\partial \xi}+\delta^4 v\,c\,\frac{\partial^4 u}{\partial \eta^4}+\delta^2 v\,c\frac{\partial^2 u}{\partial \eta^2}-\delta^4\frac{\mu_1}{\mu\lambda^2}v\,c\,\frac{\partial^4 u}{\partial \xi^4}-$$
$$\delta\frac{\mu_1}{\mu}v\,c\,\frac{\partial^4 u}{\partial \eta^4}-2\delta\frac{\mu_1}{\mu}\frac{a}{\lambda^3}v\,\frac{\partial^4 u}{\partial \xi^2\partial \eta^2}-\delta\frac{\mu}{ka^2}v\,c\ . \tag{9}$$

The above problem, will be solve subject to the following boundary conditions :

$$\left.\begin{array}{ll}
\text{(no slip condition)} & u = 0 \ \text{ at } \ \eta = h\,, \\
\text{(the regularity condition)} & \dfrac{\partial u}{\partial \eta} = 0 \ \text{ at } \ \eta = 0\,, \\
\text{(the vanishing of couple stresses)} & \dfrac{\partial^2 u}{\partial \eta^2} = 0 \text{ at } \ \eta = h\,,\ \dfrac{\partial^3 u}{\partial \eta^3} = 0 \text{ at } \ \eta = 0 \\
\text{(the transverse velocity at the wall)} & v = \dfrac{\partial h}{\partial t} \quad \text{ at } \eta = h\,,\ v = 0 \text{ at } \eta = 0 \\
\text{(finite length conditions) :} & p(\xi\,,t) = 0 \ \text{ at } \ \xi = 0\,,\ p(\xi\,,t) = 0 \ \text{ at } \ \xi = l
\end{array}\right\} \qquad (10)$$

4. Solution of the problem :

The general solution of the governing equations (6) -(9) in the general case seems to be impossible ; therefore , we shall confine the analysis under the assumption of small dimensionless wave number. It follows that $\delta \ll 1$. In other words , we considered the long - wavelength approximation. Along to this assumption, equations (6)-(9) become:

$$h(\xi\,, t) = 1 - \varphi \cos^2 \pi (\xi - t). \qquad (11)$$

$$\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} = 0\,. \qquad (12)$$

$$-\frac{\partial p}{\partial \xi} + \frac{\partial^2 u}{\partial \eta^2} - \frac{1}{\alpha^2}\frac{\partial^4 u}{\partial \eta^4} - N^2 u = 0\,. \qquad (13)$$

$$-\frac{\partial p}{\partial \eta} = 0\,. \qquad (14)$$

Let $N^2 = M^2 + \frac{1}{\kappa}$

Equation (14) shows that p dependents on ξ only. The analytical solution of Eq.(13) , subject to boundary conditions (10) , yields an expression for the axial velocity ..

$$u = \frac{\partial p/_{\partial \xi}}{N^2}\left[\frac{1}{(m_2^2 - m_1^2)}\left\{\frac{m_2^2 \cosh(m_1 \eta)}{\cosh(m_1 h)} - \frac{m_1^2 \cosh(m_2 \eta)}{\cosh(m_2 h)}\right\} - 1\right] \tag{15}$$

Where

$$m_1 = \sqrt{\frac{\alpha^2 + \alpha\sqrt{\alpha^2 - 4N^2}}{2}} \quad , \quad m_2 = \sqrt{\frac{\alpha^2 - \alpha\sqrt{\alpha^2 - 4N^2}}{2}}$$

Then , by using Eq.(15) and , v = 0 at ŋ = 0, the transverse is obtained as:

$$\mathrm{v} = \frac{1}{\mathrm{N}^2}\left[\frac{\partial \mathrm{p}}{\partial \xi}\frac{\partial \mathrm{h}}{\partial \xi}\left(\frac{1}{\mathrm{m}_2^2 - \mathrm{m}_1^2}\right)\{\mathrm{m}_2^2 \sinh(\mathrm{m}_1 \eta)\tanh(\mathrm{m}_1 \mathrm{h})\,\mathrm{sech}(\mathrm{m}_1 \mathrm{h}) - \mathrm{m}_1^2 \sinh(\mathrm{m}_2 \eta)\tanh(\mathrm{m}_2 \mathrm{h})\mathrm{sech}(\mathrm{m}_2 \mathrm{h})\} - \frac{\partial^2 \mathrm{p}}{\partial \xi^2}\left[\frac{1}{\mathrm{m}_2^2 - \mathrm{m}_1^2}\left\{\frac{\mathrm{m}_2^2 \sinh(\mathrm{m}_1 \eta)}{(\mathrm{m}_1 \cosh(\mathrm{m}_1 \mathrm{h})} - \frac{\mathrm{m}_1^2 \sinh(\mathrm{m}_2 \eta)}{(\mathrm{m}_2 \cosh(\mathrm{m}_2 \mathrm{h})}\right\} - \eta\right]\right]. \tag{16}$$

At the boundary of the wall , transverse velocity is obtained by substituting the conditions , $v = \frac{\partial \mathrm{h}}{\partial \mathrm{t}}$ at ŋ = h in Eq. (16) . It is given by

$$\frac{\partial \mathrm{h}}{\partial \mathrm{t}} = \left[\frac{1}{\mathrm{N}^2}\left[\frac{\partial \mathrm{p}}{\partial \xi}\frac{\partial \mathrm{h}}{\partial \xi}\left(\frac{1}{\mathrm{m}_2^2 - \mathrm{m}_1^2}\right)\{\mathrm{m}_2^2 \tanh^2(\mathrm{m}_1 \mathrm{h}) - \mathrm{m}_1^2 \tanh^2(\mathrm{m}_2 \mathrm{h})\} - \frac{\partial^2 \mathrm{p}}{\partial \xi^2}\left[\frac{1}{(\mathrm{m}_2^2 - \mathrm{m}_1^2)}\left\{\frac{\mathrm{m}_2^2}{\mathrm{m}_1}\tanh(\mathrm{m}_1 \mathrm{h}) - \frac{\mathrm{m}_1^2}{\mathrm{m}_2}\tanh(\mathrm{m}_2 \mathrm{h})\right\} - \mathrm{h}\right]\right]\right]. \tag{17}$$

When integrating Eq.(17) with respect to ξ , we get the pressure gradient across the length the channel as follows :

$$\frac{\partial \mathrm{p}}{\partial \xi} = \frac{\mathrm{N}^2(\,G(t) + \int_0^{\xi}\frac{\partial h}{\partial t}\,d\xi}{F(\xi, t)} \tag{18}$$

Where G(t) is an arbitrary function of t , and

$$F(\xi, t) = h - \frac{1}{(m_2^2 - m_1^2)}\left\{\frac{m_2^2}{m_1}\tanh(m_1 \mathrm{h}) - \frac{m_1^2}{m_2}\tanh(m_2 \mathrm{h})\right\} \tag{19}$$

And by integrate Eq. (18) from 0 to ξ , obtain pressure difference as :

$$p(\xi,t) - p(0,t) = N^2 \int_0^{\xi} \frac{(G(t)+\int_0^{\xi}\frac{\partial h}{\partial t}d\xi}{F(\xi,t)}\, d\xi \tag{20}$$

Using the substitution $\xi = l$, in Eq.(20) gives the pressure difference between the inlet and outlet of the channel as follows :

$$p(l,t) - p(0,t) = N^2 \int_0^{\xi} \frac{(G(t)+\int_0^{\xi}\frac{\partial h}{\partial t}d\xi}{F(\xi,t)}\, d\xi \tag{21}$$

From Eq. (21) , determined G(t) as follows :

$$G(t) = \frac{\frac{p(l,t)-p(0,t)}{N^2} - \int_0^{l}\frac{\int_0^{\xi}\frac{\partial h}{\partial t}dz}{F(\xi,t)}d\xi}{\int_0^{l}\frac{1}{F(\xi,t)}d\xi}. \tag{22}$$

The local wall shear stress[10] defined as $\tau_w(\xi\,,t) = \frac{\partial u}{\partial \eta}\Big|_{\eta=h}$, further by using Eq.(15) ,

$$\tau_w(\xi\,,t) = \frac{1}{N^2}\frac{\partial p}{\partial \xi}\left(\frac{1}{m_2^2-m_1^2}\right)\left\{\frac{m_2^2}{m_1}\tanh(m_1 h) - \frac{m_1^2}{m_2}\tanh(m_2 h)\right\} \tag{23}$$

In view of Eq. (18) , we obtain :

$$\tau_w(\xi\,,t) = \frac{1}{F(\xi,t)}\left(G(t) + \int_0^{\xi}\frac{\partial h}{\partial t}\,dz\right)\left(\frac{1}{m_2^2-m_1^2}\right)\left\{\frac{m_2^2}{m_1}tanh(m_1 h) - \frac{m_1^2}{m_2}tanh(m_2 h)\right\} \tag{24}$$

The volume flow rate [10] is given by :

$$Q(\xi\,,t) = \int_0^{h} u\, d\eta\,. \tag{25}$$

Using Eq. (15) in Eq. (24) , we get :

$$Q(\xi\,,t) = \frac{(\partial p/\partial\xi)}{N^2}\left[\frac{1}{(m_2^2-m_1^2)}\left\{\frac{m_2^2}{m_1}\tanh(m_1 h) - \frac{m_1^2}{m_2}\tanh(m_2 h)\right\} - h\right]. \tag{26}$$

5. Numerical Results and Discussion :

In this section, the numerical and computational results are discussed for the problem non-Newtonian fluid in a tube with porous medium through the graphical illustrations. The transient magneto - peristaltic flow of couple-stress fluids through the porous medium are discussed through Figures (2-19). MATHEMATICA program is used to find out numerical results and illustrations.

A. Based on Eq. 15 , Figs.(2-5) illustrates the effects of the parameters time (t) , Hartmann number(M) , dimensionless wave amplitude(φ) and couple stress parameter(α) on the axial velocity distribution , respectively. It is observed that the axial velocity increase when t and α increase and velocity decrease when the other parameters increase , finally noticed that the velocity take the a trough located and will be minimized at the channel center .

B. Figs.(6-7) illustrate the pressure difference with axial distance for the influence of (α) , M and (φ) at various times whereas $l = 2$

C. Figs. (8-9) shows the evolution of local wall shear stress with axial distance for various couple stress(α= 5,10,20,30) with weak magnetic field, strong wave amplitude and length channel(M=1,φ=0.9,l=2)at different time instant. Shear stress values are maximized with weaker couple stress effects .

D. Finally , Figs. (10-13) illustrate the variation of pressure gradient with volume flow rate at axial distance ($\xi = 1$) for the effect of t , M , α , φ .In all cases a linear relationship is observed between axial, pressure gradient and volume flow rate.

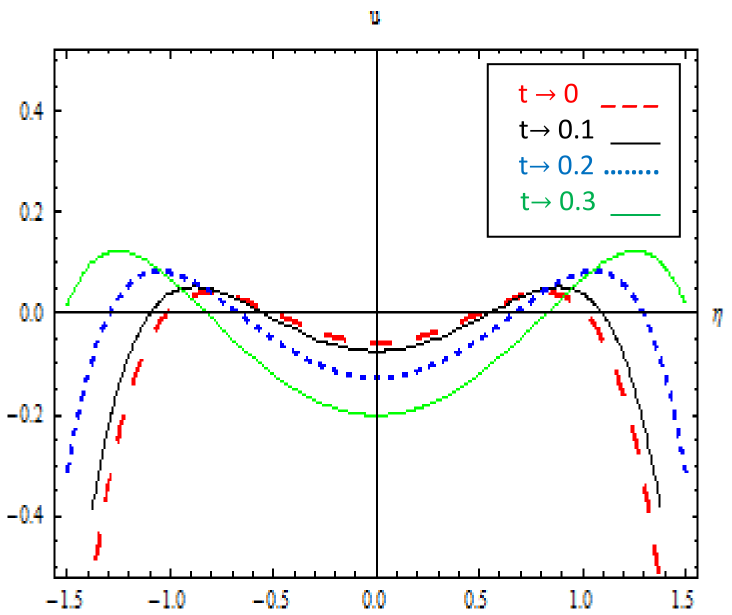


Fig 2. The axial velocity vs. transverse displacement for different values of t with $\xi = 1, \frac{\partial p}{\partial \xi} = 1$, M = 1, $\alpha = 3$, $\varphi = 0.5$, k = 5.

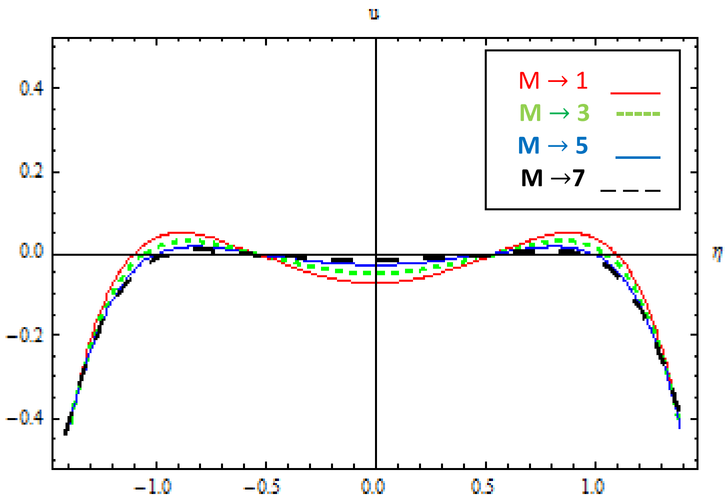


Fig 3. The axial velocity vs. transverse displacement for different values of M with $\xi = 1, \frac{\partial p}{\partial \xi} = 1$, t = 0.1, $\alpha = 3$, $\varphi = 0.5$, k =5.

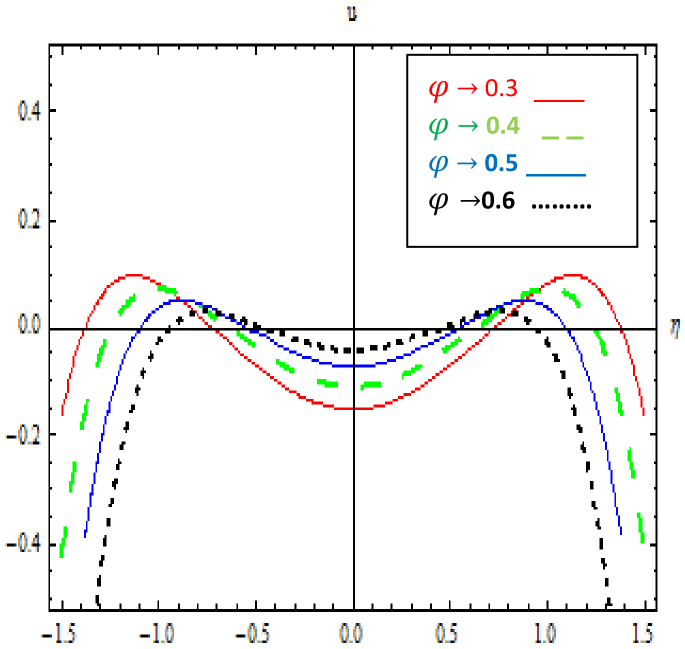


Fig 4.The axial velocity vs. transverse displacement for different values of φ with $\xi = 1$, $\frac{\partial p}{\partial \xi} = 1$, t = 0.1 , $\alpha = 3$, M = 1 , k =5.

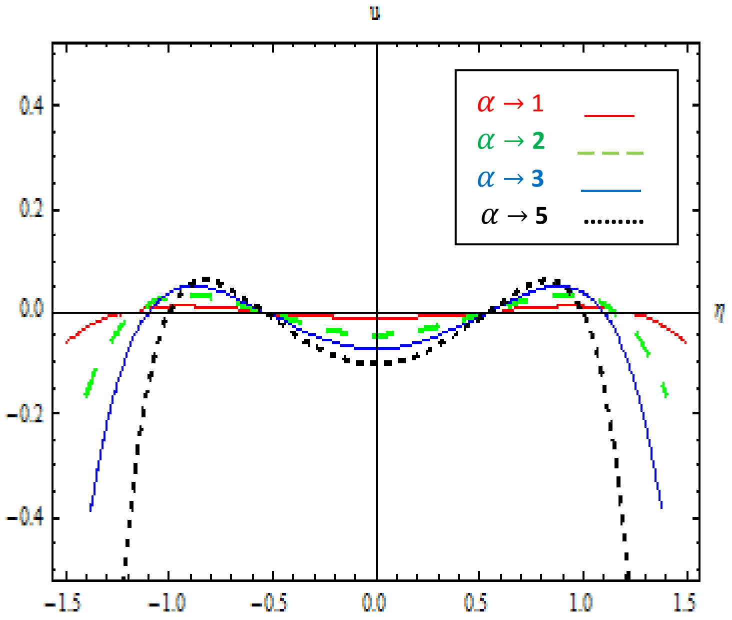


Fig 5. The axial velocity vs. transverse displacement for different values of α with $\xi = 1$, $\frac{\partial p}{\partial \xi} = 1$, t = 0.1 , M = 1 , $\varphi = 0.5$, k =5 .

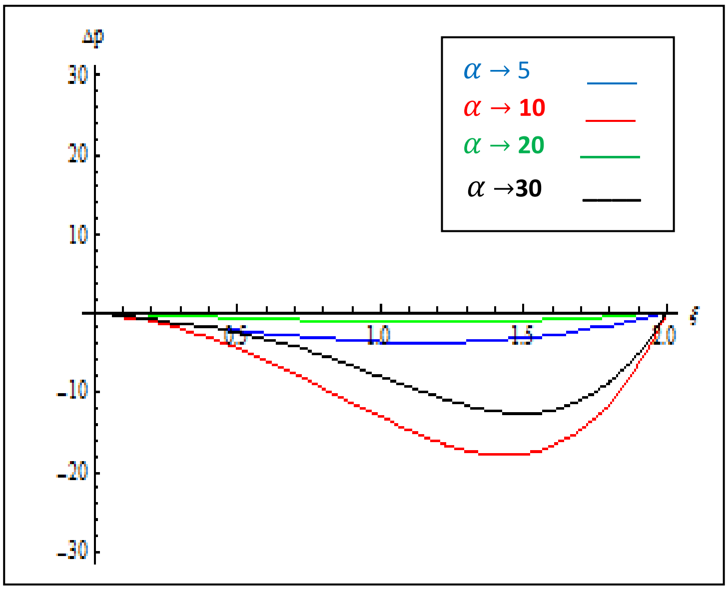


Fig 6. The pressure difference vs. axial distance for different values of α with Various instants(t = 0.25,0.5,0.75,1) , $l = 1$, M = 1 , $\varphi = 0.9$, k =5.

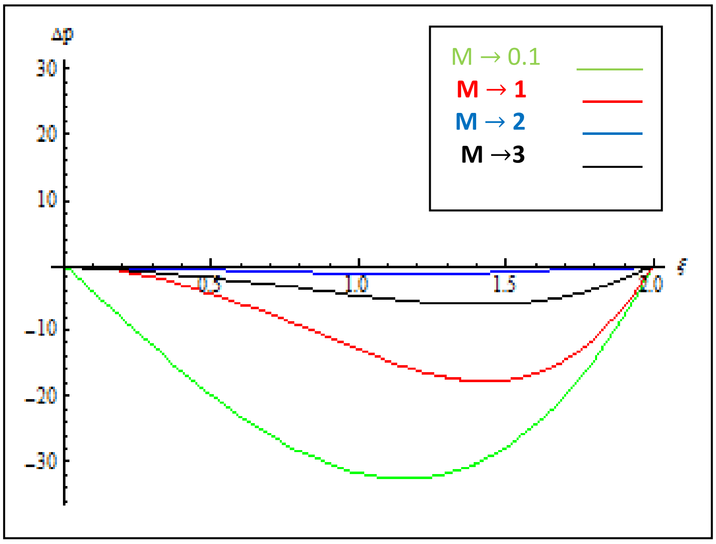


Fig 7. The pressure difference vs. axial distance for different values of M with Various instants(t = 0.25,0.5,0.75,1) , l = 2, α = 10 , φ = 0.9 , k =5.

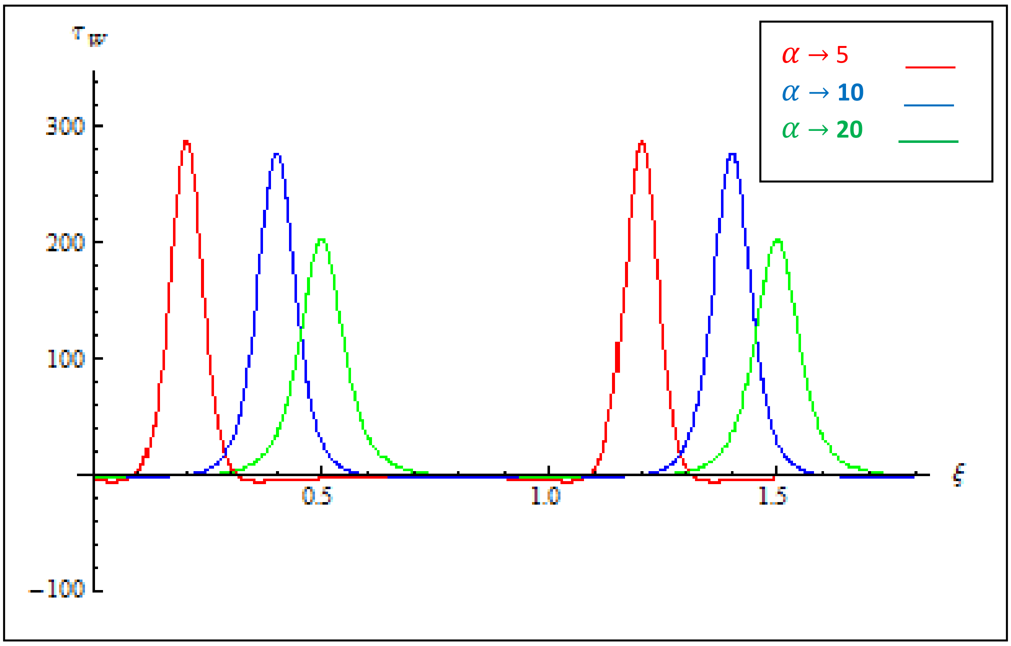


Fig 8. Local wall shear stress vs. axial distance for different values of α with Various instants(t = 0.25,0.5,0.75,1) , l = 2, M= 1 , φ = 0.9 , k =5.

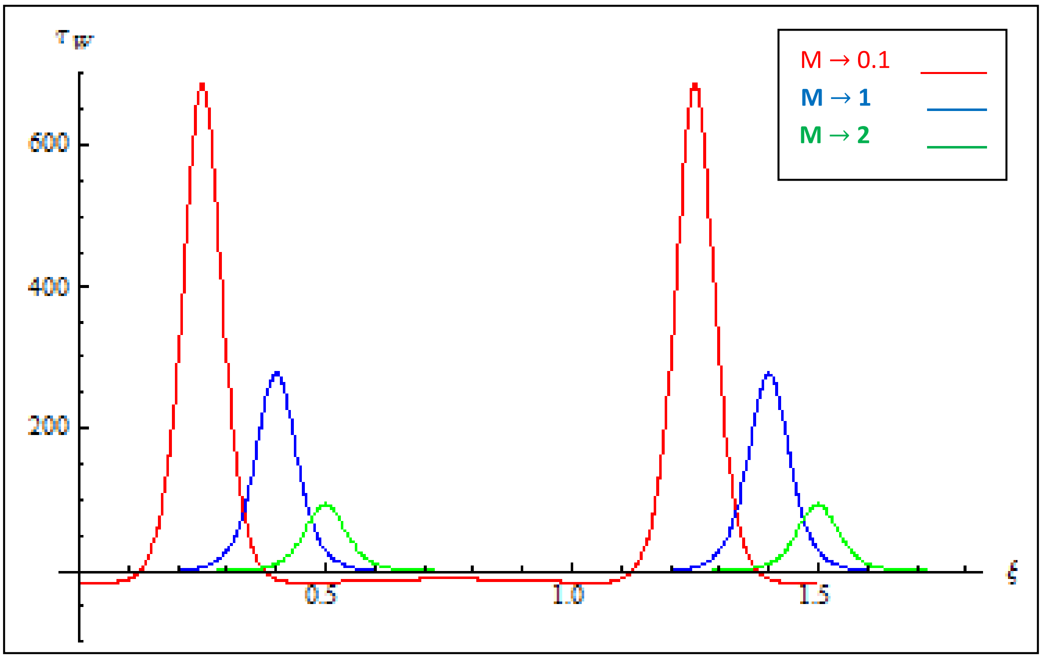


Fig 9.Local wall shear stress vs. axial distance for different values of M with Various instants(t = 0.25,0.5,0.4) , l = 2, α= 10 , φ = 0.9 , k =5.

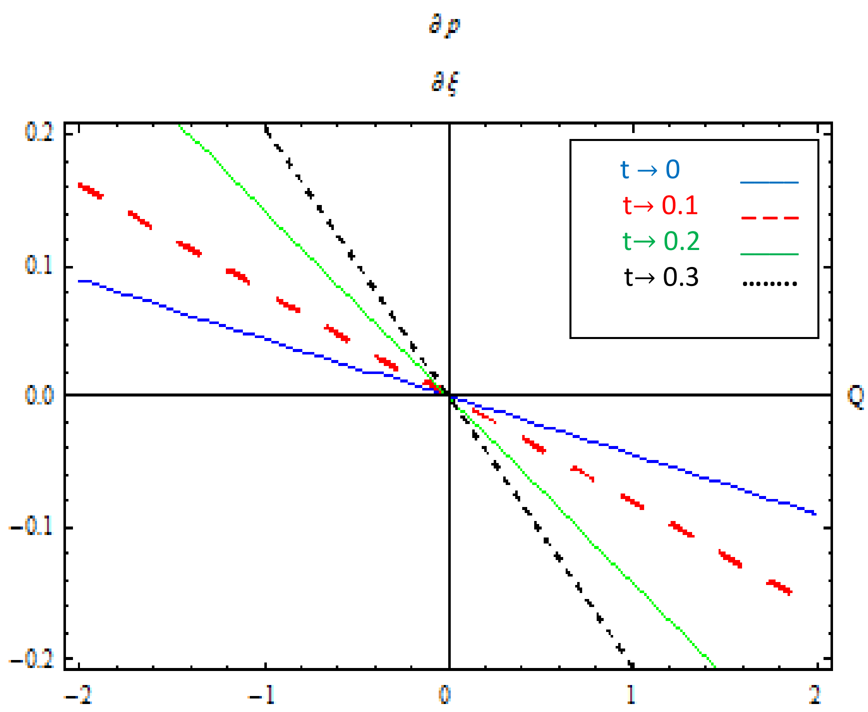


Fig 10. The pressure gradient vs. volume flow rate for different values of t with ξ = 1, α= 10 , M = 1 , φ = 0.5 , k =5.

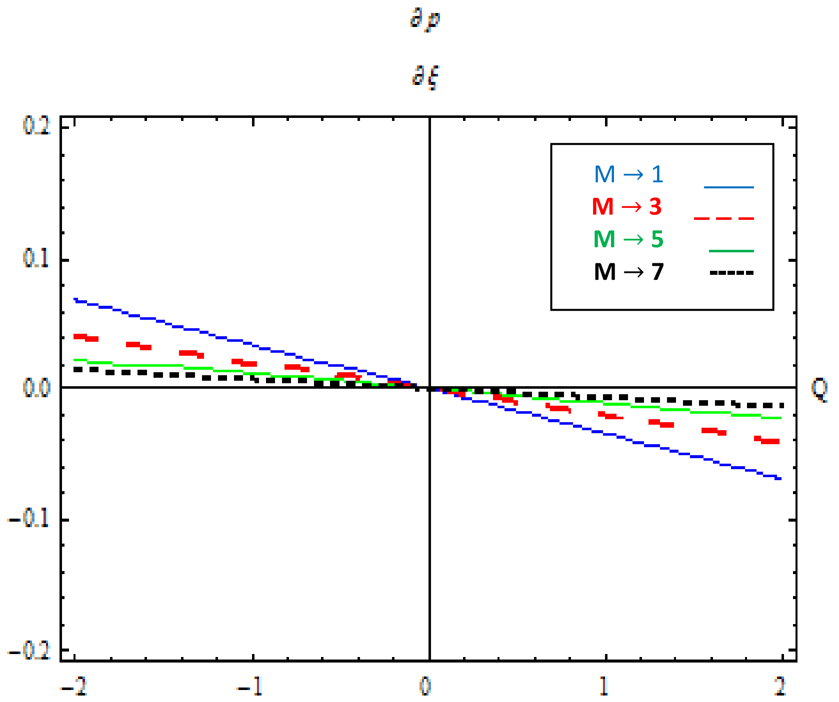


Fig 11. The pressure gradient vs. volume flow rate for different values of M with $\xi = 1, \alpha = 10$, t = 1 , $\varphi = 0.5$, k =5.

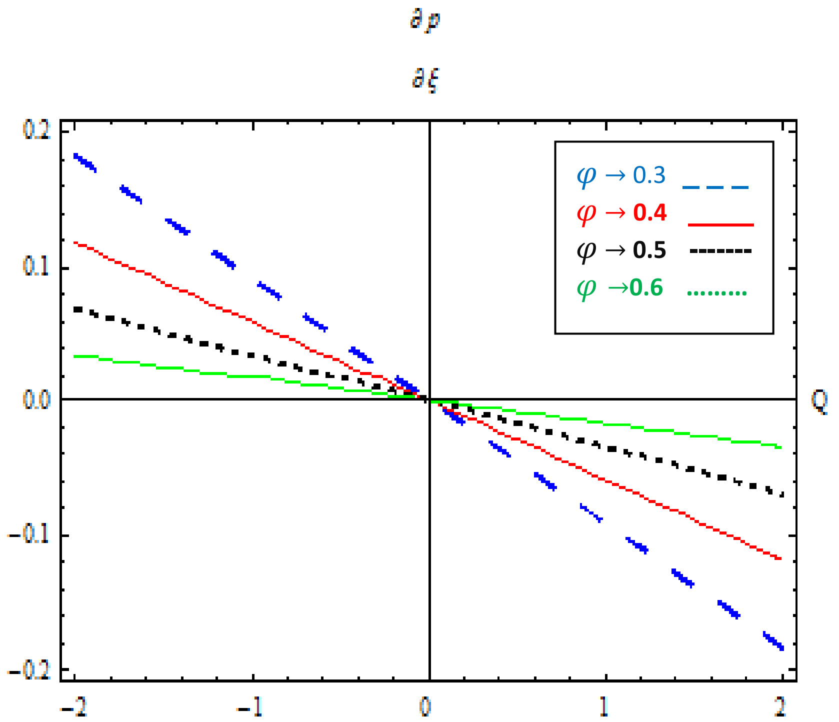


Fig 12. The pressure gradient vs. volume flow rate for different values of φ with $\xi = 1, \alpha = 10$, t = 1 , M = 1 , k =5.

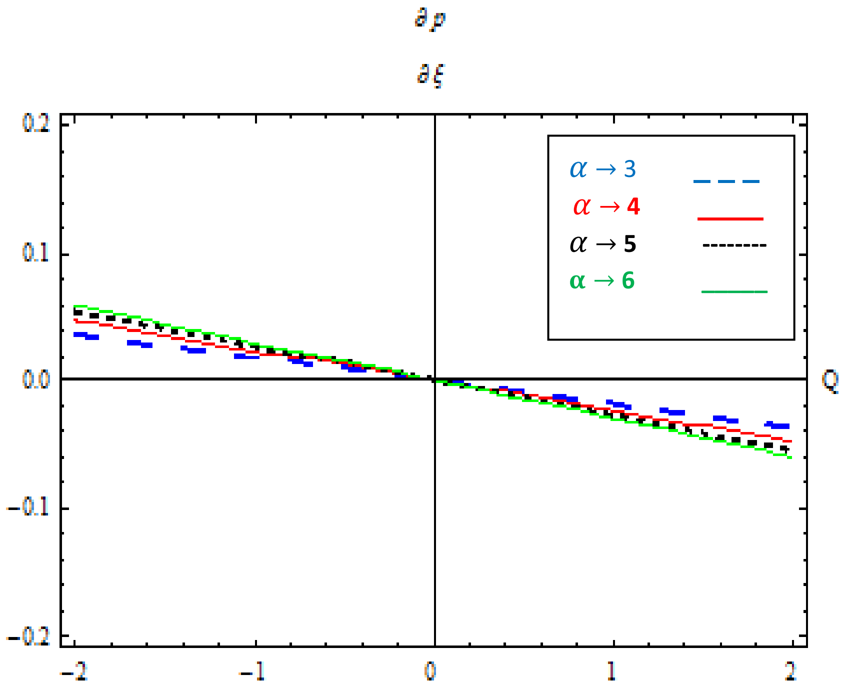


Fig 13. The pressure gradient vs. volume flow rate for different values of α with $\xi = 1, \varphi = 0.5$, t = 1 , M = 1 , k =5.

6- Concluding remarks:

we have discussed the influence of couple stress and porous medium on the transient magneto peristaltic flow of a non-Newtonian fluid in a finite length channel under a transverse magnetic field . The results are discussed through graphs. We have concluded the following observations:

1. The axial velocities increase with the increase in t , α and decrease with the increase in M , Ø.

2. The pressure difference decreases with the increases in α whereas it displaced further along the channel axis with increasing in t .

3. The pressure difference increases with the increases in M .

4. the decreasing in length of the finite channel has a major effect on it and in some cases on magnitudes .

5. The local wall shear stress decrease with increasing in M and α .

6. The increasing of couple stress (α) reduces local wall shear stress at all distances a long the channel axis .

7. the pressure gradient decrease with increasing in t and α where as it increase with increasing in M .

8. A linear relationship is computed between axial pressure gradient and volume flow rate.

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