

Confounding and Fractional Replication in Factorial Design

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Abstract

In factorial experiments when the number of factors or the levels of factors are increased the number of treatment combinations increased rapidly. Also, it becomes difficult to maintain the homogeneity between experimental units. To overcome the decrease of the experimental units, we need to decrease the number of those treatments by using a confounded design (complete and partial) and fractional replication design.

A factorial experiment for 2^4 in randomized complete block design with four blocks has been applied, for the aim of comparison among factorial randomized complete block design, confounded designs and fractional replication design in applied factorial experiments.

Key Words: Factorial Experiment, Complete Confounding, Partial Confounding, Half fractional Replication.

1.1. The Aim of Study

The study aims to comparison among the results of factorial experiment conducted in randomized complete block design, complete confounding, partial confounding and half fractional replication, using mean squares error to differentiate the results of this study.

1.2. Introduction

In factorial experiments when the number of factors or number of levels of the factors increase, the number of treatment combinations increase very rapidly and it is not possible to accommodate all these treatment combinations in a single homogeneous block. For example, a 2^5factorial would have 32 treatment combinations and blocks of 32 plots are quite big to ensure homogeneity within them. A new technique is there for necessary for designing experiments with a large number of treatments.

In order to keep the advantages of the factorial experimental error to a minimum, a device known as confounding or fractional factorial is adopted.

Fisher (1926) first suggested the confounded design. Fisher and Wishart (1930) gave the explanation of the numerical procedure of the analysis of randomized block and Latin square experiments; they also gave an example of a confounded experiment [6]. The use of experiments in factorial replication was proposed in (1945) by Finney.

He outlined methods of construction for $2^n$and 2^3 factorials and described a half- replicate of a 4×2^4, agricultural field experiment that had been conducted in 1942 [3].

1.3. Factorial Experiments

In a factorial experiment the treatments are combinations of two or more levels of two or more factors. A factor is a classification or categorical variable which can take one or more values called levels [2].

Factorial experiments provide an opportunity to study not only the individual effects of each factor but also their interactions. They have the further advantage of economizing on experimental resources [6].

The mathematical model for factorial RCBD is [7]:

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \rho_k + \varepsilon_{ijk} \begin{cases} i = 1,2,\ldots\ldots,a \\ j = 1,2,\ldots\ldots,b \\ k = 1,2,\ldots\ldots,r \end{cases} \quad \ldots\ldots (1)$$

Where μ is the overall mean, $\alpha_i$is the effect of the i_{th} level of factor A, β_j is the effect of the j_{th} level of factor B, $(\alpha\beta)_{ij}$ is the effect of the interaction between the i_{th} level of factor A and j_{th} level of factor B, , ρ_k is the effect of the k_{th} block, and ε_{ijk} is the random error associated with the k_{th} replication in cell (ij).

In the two factors fixed effects model, we are interested in the hypotheses:

A main effect:

$$\left.\begin{array}{l} H_0: \alpha_1 = \cdots = \alpha_a = 0 \\ H_1: at\ least\ one\ of\, \alpha_i \neq 0 \end{array}\right\} \quad \ldots\ldots (2)$$

B main effect:

$$\left.\begin{array}{l} H_0: \beta_1 = \cdots = \beta_b = 0 \\ H_1: at\ least\ one\ of\, \beta_j \neq 0 \end{array}\right\} \quad \ldots\ldots (3)$$

AB interaction effect:

$$\left.\begin{array}{l} H_0: (\alpha\beta)_{11} = \cdots = (\alpha\beta)_{ij} = 0 \\ H_1: at\ least\ one\ of\, (\alpha\beta)_{ij} \neq 0 \end{array}\right\} \quad \ldots\ldots (4)$$

Table1: ANOVA for the factorial RCBD

(S.O.V.)	(d.f.)	(S.S.)	(M.S.)	(F.Cal)
Blocks	r-1	$\frac{\sum Y_{..k}^2}{ab} - \frac{(Y_{...})^2}{abr}$	$\frac{SSbl.}{r-1}$	$F_{cal} = \frac{Mbl}{MSE}$
A	a-1	$\frac{\sum Y_{i..}^2}{rb} - \frac{(Y_{...})^2}{abr}$	$\frac{SSA}{a-1}$	$F_{cal} = \frac{MSA}{MSE}$
B	b-1	$\frac{\sum Y_{.j.}^2}{ra} - \frac{(Y_{...})^2}{abr}$	$\frac{SSB}{b-1}$	$F_{cal} = \frac{MSB}{MSE}$
AB	(a-1)(b-1)	$\frac{\sum Y_{ij..}^2}{r} - \frac{(Y_{...})^2}{abr} - SSA - SSB$	$\frac{SS(AB)}{(a-1)(b-1)}$	$F_{cal} = \frac{MS(AB)}{MSE}$
Error	(r-1)(ab-1)	$SST - SSbl. - SSA - SSB - SS(AB)$	$\frac{SSE}{(r-1)(ab-1)}$	
Total	abr-1	$SST = \sum Y_{ijk}^2 - \frac{Y_{...}^2}{abr}$		

1.4. Confounding

Confounding is a technique for designing experiments with a large number of treatments in factorial experiments. The treatment combinations are divided into as many groups as the number of blocks per replication. The different groups of treatments are allocated to the blocks. The grouping of treatments combinations must be done in such a way that only the unimportant effects are confused with the block effects and other import anted effects could be evolved compare significantly[4]. There are two types of confounding [2], [3]: complete confounding and partial confounding.

If the same effect confounded in all the other replications, then the interaction is said to be completely confounded. And all the information on confounded interactions are lost.

When an interaction is confounded in one replicate and not in another, the experiment is said to be partially confounded. The confounded interactions can be recovered from these replications in which they are not confounded. The table (2) of positives and negatives signs for the 2^4 design. The signs in the columns of this table can be used to estimate the factor effects.

Table2: Table of positive and negative signs for the 2^4

Trea. Com.	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD
1	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
a	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
b	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-
ab	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
c	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-
ac	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
bc	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
abc	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
d	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-
ad	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
bd	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
abd	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
cd	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
acd	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
bcd	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
abcd	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

1.5. Fractional Factorial Designs

As the number of factors in a 2^k factorial design increases, the number of trials required for a full replicate of the design rapidly outgrows the resources available for many experiments. In such cases, one cannot perform a full replicate of the design and a fractional factorial design has to be run [8].

Such an experiment contains one- half fraction of a 2^4 experiment and is called 2^{4-1} factorial experiment. Similarly, $\frac{1}{2^3}$ fraction of 2^4 factorial experiment requires only 8 runs and contains $\frac{1}{2^2}$ fraction of 2^4 factorial experiment and called as 2^{4-2} factorial experiment. In general, contains $\frac{1}{2^p}$ fraction of a 2^k factorial experiment requires only 2^{k-p} runs and is denoted as 2^{k-p} factorial experiment [9]. A $\frac{1}{2}$ fractional can be generated from any interaction, but using the highest - order interaction is the standard. The interaction used to generate $\frac{1}{2}$ fraction is called the generator of the fractional factorial design. When there are 4 factors, use ABCD as the generator of the 2^{4-1} design.

Based on the signs (positive or negative) as shown in table (2), attached to the treatments in this expression, two groups of treatments can be formed out of the complete factorial set. Retaining only one set with either negative or positive signs, we get a half fractional of the 2^4 factorial experiment. The two sets of treatments are shown below.

Treatments with negative signs

a	b	c	abc	d	abd	acd	bcd

Treatments with positive signs

1	ab	ac	bc	ad	bd	cd	abcd

The alias structure for this design is found by using the defining relation $I = ABCD$. Multiplying any effect by the defining relation yields the aliases for that effect. The alias of A is

$$A = A.I = A.ABCD = A^2BCD = BCD$$

Aliases are two factorial effects that are represented by the same comparisons. Thus A and BCD are aliases. Similarly, we have other aliases:

$$B = ACD, \quad C = ABD, \quad D = ABC$$
$$AB = CD, \quad AC = BD, \quad AD = BC$$

The treatment combinations in the 2^{4-1} design yields four degrees of freedom associated with the main effects. From the upper half of table, we obtain the estimates of the main effects as linear combinations of the observations,

$$A = \frac{1}{4}[ad + ab + ac + abcd - 1 - bd - cd - bc] \quad \ldots\ldots (5)$$

$$B = \frac{1}{4}[bd + ab + bc + abcd - 1 - ad - cd - ac] \quad \ldots\ldots (6)$$

$$C = \frac{1}{4}[cd + ac + bc + abcd - 1 - ad - bd - ab] \quad \ldots\ldots (7)$$

$$D = \frac{1}{4}[ad + bd + cd + abcd - 1 - ab - ac - bc] \quad \ldots\ldots (8)$$

$$AB = \frac{1}{4}[1 + ab + cd + abcd - ad - bd - ac - bc] \quad \ldots\ldots (9)$$

$$AC = \frac{1}{4}[1 + bd + ac + abcd - ad - ab - cd - bc] \qquad \ldots\ldots (10)$$

$$BC = \frac{1}{4}[1 + ad + bc + abcd - bd - ad - cd - ac] \qquad \ldots\ldots (11)$$

2. Applications

This section tackles the practical application of the factorial experiment for 2^4 in randomized complete block design with four blocks given in Cochran and Cox (1957) has been applied, for the aim of comparison among factorial randomized complete block design, confounded designs and fractional replication design. The minitab 16 is used. Then the resulting data is as follows:

Table 3: Data experiment

Treatment combination	Rep. 1	Rep. 2	Rep. 3	Rep. 4	Total
1	32	43	27	19	121
a	47	41	48	45	181
b	26	36	24	18	104
ab	61	76	56	64	257
c	29	39	27	28	123
ac	51	34	40	48	173
bc	36	31	32	30	129
abc	76	65	70	63	274
d	35	42	56	35	168
ad	63	41	60	53	217
bd	80	68	75	67	290
abd	100	68	87	66	321
cd	40	44	53	36	173
acd	64	39	75	72	250
bcd	105	99	74	73	351
abcd	90	82	89	101	362
Total	935	848	893	818	3494

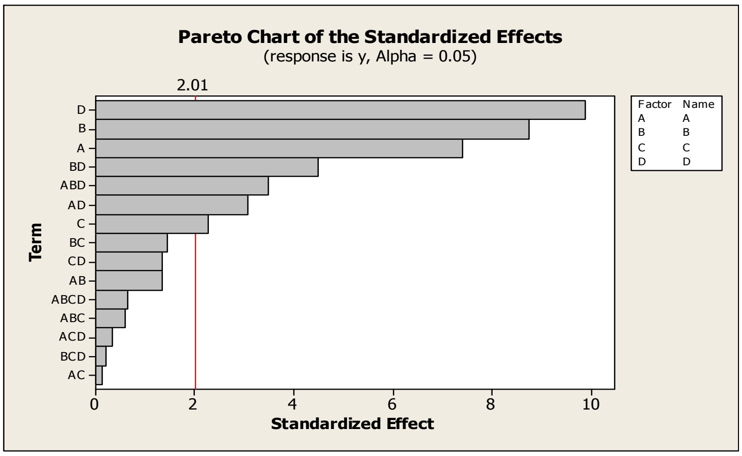
2.1. Full Factorial

Table 4: ANOVA for full factorial RCBD

S.O.V	D.F	SS	MS	F	P- value
Replication	3	493.32	164.43	1.82	0.158
A	1	5184	5184	57.26	0.000*
B	1	7267.56	7267.56	80.27	0.000*
C	1	484	484	5.35	0.025*
D	1	9264.06	9264.06	102.32	0.00*
AB	1	169	169	1.87	0.179
AC	1	1.56	1.56	0.02	0.896
AD	1	900	900	9.94	0.003*
BC	1	196	196	2.16	0.148
BD	1	1914.06	1914.06	21.14	0.000*
CD	1	169	169	1.87	0.179
ABC	1	33.06	33.06	0.37	0.549
ABD	1	1156	1156	12.77	0.001*
ACD	1	10.56	10.56	0.12	0.734
BCD	1	4	4	0.044	0.834
ABCD	1	39.06	39.06	0.43	0.515
Error	45	4074.2	90.54		
Total	63	31359.44			

*significant at level (0.05)

Figure 1: Pareto plot for full factorial RCBD



In the analysis, the results show those main effects A, B, C and D and the two factor interactions AD, BD and three factor interaction ABD are significant and the interactions AB, AC, BC,CD, ABC,ACD, BCD, ABCD are non significant at the level of significant (α=0.05). And the Pareto plot looks at the effects and orders them from largest to smallest as shown in figure 1.

2.2.1 Complete Confounding

The 2^4 experiment with four factors A, B, C, and D, each at two levels. There are only 16 treatment combinations.

a. Suppose that each replicate in experiment is divided in to two blocks of eight units each, such that one block contains all treatment combinations that have on positive signs, while the other contains all negative signs. The interaction of highest order is the ABCD interaction. This interaction is estimated from the comparison. The plan would be as follows:

Table 5: Plan for 2^4factorial, blocks of 8 units, with ABCD confounded

Replicate 1		**Replicate 2**		**Replicate 3**		**Replicate 4**	
(1) 32	(a) 47	(1) 43	(a) 41	(1) 27	(a) 48	(1) 19	(a) 45
(ab) 61	(b) 26	(ab) 76	(b) 36	(ab) 56	(b) 24	(ab) 64	(b) 18
(ac) 51	(c) 29	(ac) 34	(c) 39	(ac) 40	(c) 27	(ac) 48	(c) 28
(bc) 36	(abc)76	(bc) 31	(abc) 65	(bc) 32	(abc)70	(bc) 30	(abc) 63
(ad) 63	(d) 35	(ad) 41	(d) 42	(ad) 60	(d)56	(ad) 53	(d) 35
(bd) 80	(abd)100	(bd) 68	(abd) 68	(bd) 75	(abd) 87	(bd) 67	(abd) 66
(cd) 40	(acd)64	(cd) 44	(acd) 39	(cd) 53	(acd) 75	(cd) 36	(acd) 72
(abcd) 90	(bcd)105	(abcd) 82	(bcd) 99	(abcd) 89	(bcd) 74	(abcd)101	(bcd) 73
453	482	419	429	432	461	418	400
935		848		893		818	

$Corect\ Factor(C.F) = 190750.56$

$SSTotal = 32^2 + 47^2 + \cdots + 101^2 - C.F = 31359.44$

$SSRepl. = 493.32$

$SSBlock = \frac{(453)^2+(482)^{2+\cdots+}(400)^2}{8} - C.F = 624.94$

$SS(Block/Rep) = SSBlock - SSRep. = 131.62$

The sums of squares for the main effects and interactions are calculated using the factorial effect totals which can be obtained by the Yates method as shown in table (6).

Table 6: Yates method for effect totals

Treat. comb.	**Total Treatments**	**Sum and different of pairs**				$SS=\frac{[IV]^2}{4*2^4}$
		I	**II**	**III**	**IV**	
1	121	302	663	1362	3494	-
a	181	361	699	2132	576	5184
b	104	296	996	408	682	7267.56
ab	257	403	1136	168	104	169
c	123	385	213	166	176	484
ac	173	611	195	516	-10	1.56
bc	129	423	80	188	112	196
abc	274	713	88	-84	-46	33.06
d	168	60	59	36	770	9264.06
ad	217	153	107	140	-240	900
bd	290	50	226	-18	350	1914.06
abd	321	145	290	8	-272	1156
cd	173	49	93	48	104	169
acd	250	31	95	64	26	10.56
bcd	351	77	-18	2	16	4
abcd	362	11	-66	-48	-50	39.06

Table 7: ANOVA with ABCD Confounded

S.O.V	D.F	SS	MS	F	P- value
Blocks	r-1=3	493.32	164.44	1.73	
Block/Repl.	r = 4	131.62	32.91	0.35	
A	1	5184	5184	54.68	0.000*
B	1	7267.56	7267.56	76.66	0.000*
C	1	484	484	5.11	0.029*
D	1	9264.06	9264.06	97.72	0.000*
AB	1	169	169	1.78	0.189
AC	1	1.56	1.56	0.016	0.898
AD	1	900	900	9.49	0.004*
BC	1	196	196	2.07	0.158
BD	1	1914.06	1914.06	20.19	0.000*
CD	1	169	169	1.78	0.189
ABC	1	33.06	33.06	0.35	0.558
ABD	1	1156	1156	12.19	0.001*
ACD	1	10.56	10.56	0.11	0.740
BCD	1	4	4	0.042	0.838
Error	42	3981.64	94.8		
Total	63	31359.44			

*significant at level (0.05)

In the analysis, the results show those main effects A, B, C and D and the two factor interactions AD, BD and three factor interaction ABD are significant and the interactions AB, AC, BC,CD, ABC,ACD, BCD, ABCD are non significant at the level of significant (α=0.05). While the mean squares error is equal to (94.8) greater than the result of the analysis in the table (4) and that the mean squares error is equal to (90.54).

b. Each replicate in experiment is divided in to four blocks of four units each, the interactions of ABC, BCD and AD completely confounded,

$ABC\ BCD = AB^2C^2D = AD$

There will be $\frac{2^k}{2^p} = \frac{2^4}{2^2} = 4$ blocks per replicate.

Let $X_1, X_2, X_3, and\ X_4$ denoted the levels (0 or 1) of each of the 4 factors A, B, C and D. Solving the following equations would result in different blocks of the design:

For interaction ABC: $X_1 + X_2 + X_3 = 0,1$

For interaction BCD: $X_2 + X_3 + X_4 = 0,1$

Treatment combinations satisfying the following solutions of above equations will generate the required 4 blocks: (0,0), (0,1), (1,0), (1,1).

The solution (0,0) will give the key block(a key block is one that contains one of the treatment combination of factors, each at lower level)[4].similarly we can write the other blocks by taking the solutions of above equations as (0,1), (1,0) and (1,1). In this case that each replicate in experiments divided into4 blocks of 4 units each, with 4 replicates the plan would be as follows:

Table 8: Plan for 2^4factorial, blocks of 4 units, with ABC, BCD and AD confounded

Replicate 1					Replicate 2			
(b)26	(a)47	(d)35	(bc)36		(c) 39	(bd)68	(ab)76	(acd)39
(c) 29	(bd)80	(ab)61	(abd)100		(b) 36	(a)41	(d)42	(bc)31
(ad)63	(cd)40	(ac)51	(acd)64		(ad)41	(abc)65	(ac)34	(abd)68
(abcd)90	(abc)76	(bcd)105	(1)32		(abcd)82	(cd)44	(bcd)99	(1)43
208	243	252	232		198	218	251	181
935					848			

Replicate 3					Replicate 4			
(ad)60	(bd)75	(ab)56	(abd)87		abcd)101	(abc)63	(ac)48	(1)19
(abcd)89	(abc)70	(ac)40	(1)27		(b)18	(bd)67	(bcd)73	(acd)72
(c)27	(a)48	(acd)74	(bc)32		(ad)53	(cd)36	(d)35	(abd)66
(b) 24	(cd)53	(d)56	(acd)75		(c) 28	(a)45	(ab)64	(bc)30
200	246	226	221		200	211	220	187
893					818			

$$SSTotal = 32^2 + 47^2 + \cdots + 101^2 - C.F = 31359.44$$

$$SSRepl. = 493.32$$

$$SSBlock = \frac{(208)^2+(243)^{2+\cdots+(187)^2}}{4} - C.F = 1862.94$$

$$SS(Block/Rep) = SSBlock - SSRep. = 1369.62$$

The sums of squares for the main effects and interactions are calculated using the factorial effect totals which can be obtained by the Yates method as shown in table (6), and the analysis of variance as shown in table (9).

Table 9: ANOVA with ABC, BCD and AD Confounded

S.O.V	D.F	SS	MS	F	P- value
Blocks	r-1=3	493.32	164.44	1.62	0.221
Block/Rep	r (b-1)=12	1369.62	114.135	1.13	0.300
A	1	5184	5184	51.12	0.000*
B	1	7267.56	7267.56	71.67	0.000*
C	1	484	484	4.77	0.024*
D	1	9264.06	9264.06	91.35	0.000*
AB	1	169	169	1.67	0.200
AC	1	1.56	1.56	0.02	0.896
BC	1	196	196	1.93	0.176
BD	1	1914.06	1914.06	18.87	0.000*
CD	1	169	169	1.67	0.200
ABD	1	1156	1156	11.4	0.002*
ACD	1	10.56	10.56	0.1	0.740
ABCD	1	39.06	39.06	0.38	0.540
Error	36	3650.64	101.41		
Total	63	31359.44			

*significant at level (0.05)

In the analysis, the results show those main effects A, B,C, and D and the two factor interactions BD and three factor interaction ABD are significant and the interactions AB, AC, BC,CD, ACD , ABCD are non significant at the level of significant (α=0.05), And the mean squares error is equal to (101.41).

2.2. 2.Partial Confounding

Consider again 2^4 experiment with each replicate divided into two blocks of 8 units each. It is not necessary to confound the same interaction in all the replicates and several factorial effects may be confounded in one single experiment. The following plan confounds the interaction ABCD, ABC, ACD and BCD in replicates 1, 2, 3 and 4 respectively.

Table 10: Plan for 2^4 factorial, blocks of 8 units, with ABCD, ABC, ACD and BCD partially confounded

Replicate 1 Confound ABCD		Replicate 2 Confound ABD		Replicate 3 Confound ACD		Replicate 4 Confound BCD	
(1) 32	(a) 47	(a) 41	(1) 43	(a) 48	(1) 27	(b) 18	(1) 19
(ab) 61	(b) 26	(b) 36	(ab) 76	(ab) 56	(b) 24	(ab) 64	(a)45
(ac) 51	(c) 29	(c) 39	(ac) 34	(c) 27	(ac) 40	(c) 28	(bc) 30
(bc) 36	(abc)76	(abc) 65	(bc) 31	(bc) 32	(abc) 70	(ac) 48	(abc) 63
(ad) 63	(d) 35	(ad) 41	(d) 42	(d) 56	(ad) 60	(d) 35	(bd) 53
(bd) 80	(abd)100	(bd) 68	(abd) 68	(bd) 75	(abd) 87	(ad) 67	(abd) 66
(cd) 40	(acd)64	(cd) 44	(acd) 39	(acd) 75	(cd) 53	(bcd) 73	(cd) 36
(abcd) 90	(bcd)105	(abcd) 82	(bcd) 99	(abcd)89	(bcd) 74	(abcd)101	(acd)72
453	482	416	432	458	435	434	384
935		848		893		818	

The sums of squares for blocks and for the not confounded effects are found in the usual way (see table Yates method).

$SSRepl. = 493.32$

$$SSBlock = \frac{(453)^2 + \ldots + (384)^2}{8} - C.F = 751.19$$

$$SS(Block/Rep) = SSBlock - SSRep. = 257.87$$

The sum of squares for ABCD is calculated from replicates (2, 3, 4), similarly it is possible to recover information on the other confounded interactions ABC (from 1, 3, 4), ACD (from 1, 2, 4) and BCD (1, 2, 3) as shown in table (11). The sum of squares for partially confounded are calculated as follows:

$$SSABCD = \frac{1}{(r-1)2^4}\begin{bmatrix}(I + ab + ac + bc + ad + bd + cd + abcd) - \\ (a + b + c + abc + d + abd + acd + bcd)\end{bmatrix}^2 \quad \ldots\ldots. (12)$$

$$= \frac{1}{48}[-21]^2 = 9.188$$

$$SSABC = \frac{1}{(r-1)2^4}\begin{bmatrix}(a + b + c + abc + ad + bd + cd + abcd) - \\ (I + ab + ac + bc + d + abd + acd + bcd)\end{bmatrix}^2 \quad \ldots\ldots (13)$$

$$= \frac{1}{48}[-30]^2 = 18.75$$

$$SSACD = \frac{1}{(r-1)2^4}\begin{bmatrix}(a + ab + c + bc + d + bd + acd + abcd) - \\ (I + b + ac + abc + ad + abd + cd + bcd)\end{bmatrix}^2 \quad \ldots\ldots (14)$$

$$= \frac{1}{48}[3]^2 = 0.188$$

$$SSBCD = \frac{1}{(r-1)2^4}\begin{bmatrix}(b + ab + c + ac + d + ad + bcd + abcd) - \\ (I + a + bc + abc + bd + abd + cd + acd)\end{bmatrix}^2 \quad \ldots\ldots. (15)$$

$$= \frac{1}{48}[-6]^2 = 0.75$$

Table 11: ANOVA for partial confounded

S.O.V	D.F	SS	MS	F	P- value
Replications	r-1= 3	493.32	164.43	1.74	
Block/Repl.	r = 4	257.87	64.47	0.68	0.79
A	1	5184	5184	54.86	0.00*
B	1	7267.56	7267.56	76.91	0.00*
C	1	484	484	5.12	0.029
D	1	9264.06	9264.06	98.04	0.00*
AB	1	169	169	1.78	0.189
AC	1	1.56	1.56	0.02	0.89
AD	1	900	900	9.52	0.004*
BC	1	196	196	2.07	0.58
BD	1	1914.06	1914.06	20.25	0.00*
CD	1	169	169	1.78	0.189
(ABC)´	1	18.75	18.75	0.19	0.177
ABD	1	1156	1156	12.23	0.002*
(ACD)´	1	0.188	0.188	0.001	0.91
(BCD)´	1	0.75	0.75	0.007	0.93
(ABCD)´	1	9.188	9.188	0.09	0.75
Error	41	3874.134	94.49		
Total	63	31359.44			

*significant at level (0.05)

In the analysis, the results show those main effects A, B, and C and the two factor interactions AD, BD and three factor interaction ABD are significant and main effect D and the interactions AB, AC, BC,CD, ABC,ACD, BCD, ABCD are non significant at the level of significant (α=0.05). While the mean squares error is equal to (94.49) less than the results of the analysis for complete confounding with 2 blocks and complete confounding with 4 blocks and that the mean squares errors are equal to (94.8) and (101.41) respectively.

2.3. Fractional Replication

There are 4 factors, use ABCD as the generator of the 2^{4-1} design. Based on the signs (positive or negative) as shown in table (2), attached to the treatments in this expression, two groups of treatments can be formed out of the complete factorial set. Retaining only one set with either negative or positive signs, we get a half fractional of the 2^4 factorial experiments.

The alias structure for this design is found by using the defining relation $I = ABCD$. Multiplying any effect by the defining relation yields the aliases for that effect. The alias of A is

$A = A.I = A.ABCD = A^2BCD = BCD$

Aliases are two factorial effects that are represented by the same comparisons. Thus A and BCD are aliases. Similarly, we have other aliases:

$B = ACD,\ \ C = ABD\ ,\ \ D = ABC$

$C.F = \frac{(G.Total)^2}{rt}$

$C.F = \frac{(1722)^2}{4(8)} = 92665.125$

$SSTotal = 32^2 + 61^2 + \cdots + 101^2 - C.F = 13652.875$

$SSRepl. = 99.625$

For the four factors tested, a $\frac{1}{2}$ fractional factorial design is a Resolution IV design. The resolution of the design is based on the number of the letters in the generator. The main effects are aliased with three way interactions and the two way interactions are aliased with each other [1]. Therefore, we cannot determine from this type of design which of the two way interactions are important because they are confounded or aliased with each other.

The sums of squares for the main effects and interactions are calculated as shown in table (12).

Table 12: ANOVA for fractional replication

S.O.V	D.F	SS	MS	F	P- value
Replications	r-1= 3	99.625	33.2	0.51	0.679
A	1	3916.1	3916.1	60.34	0.000*
B	1	72	72	1.11	0.304
C	1	4095.1	4095.1	63.1	0.000*
D	1	2738	2738	42.19	0.000*
AB	1	128	128	1.97	0.175
AC	1	903.1	903.1	13.92	0.001*
BC	1	338	338	5.21	0.033*
Error	21	1362.9	64.9		
Total	31	13652.875			

*significant at level (0.05)

Figure 2: Pareto plot for fractional replication

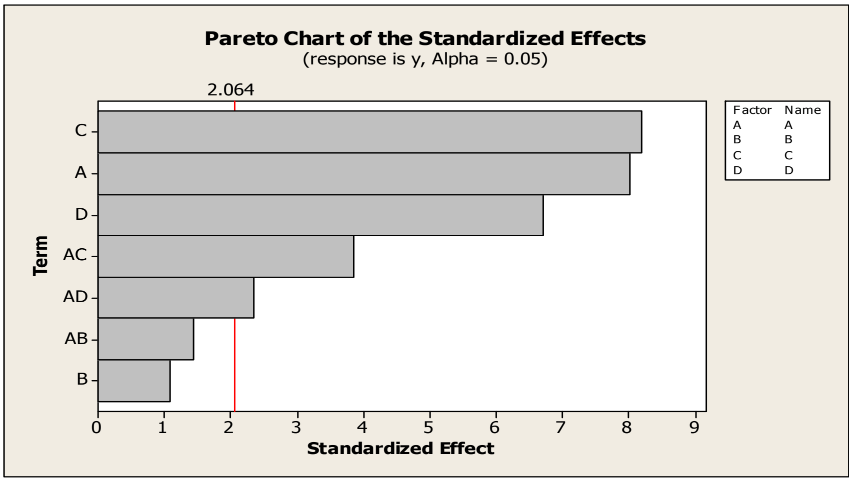
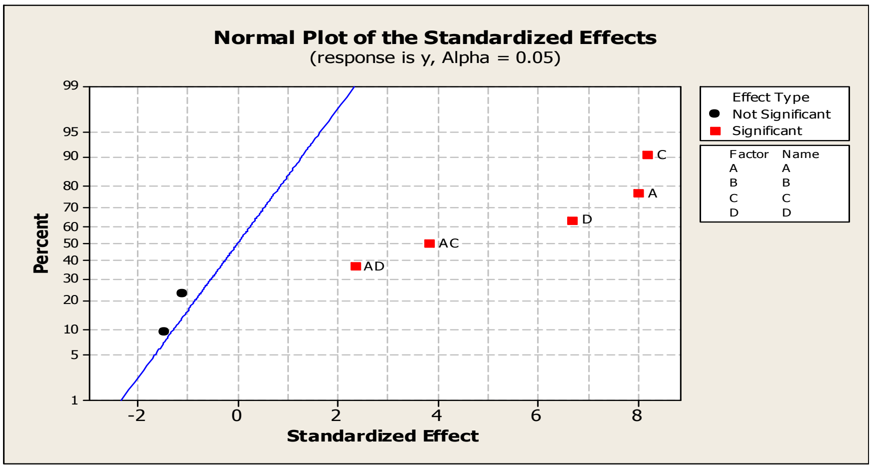


Figure 3: Normal probability plot of the effects



The result in table (12) shows those main effects A, C, and D and the two factor interactions AD, AC are significant and main effect B and the interactions AB are non significant at the level of significant (α=0.05), and the normal probability plot is very useful in assessing the significance of effects from a fractional factorial design, particularly when many effects are to be estimated. Figure (3) presents the normal probability plot of the effects. Notice that the A, C, D, AC and AD effects stand out clearly in this graph.

Conclusions

1. The result of analysis of variance for factorial randomized complete block design showed that the mean squares error is equal to (90.54).
2. When each replicate in experiment contains two blocks of eight units each and the interaction of ABCD completely confounded, the mean squares error is equal to (94.8). While, each replicate in experiment contains four blocks of four units each, and the interactions are completely confounded, the mean squares error is equal to (101.41) greater than the result of the analysis in the full factorial.
3. Partially confounding has been most efficient, the value of mean squares error is (94.49) less than the result of the analysis in completely confounded.
4. The result of analysis of variance showed that the fractional factorial design is the highest accuracy in estimating the effects and was the best in saving time and cost.

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