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Influence of non uniform heat source/sink on Powell-Eyring fluid past an inclined stretching sheet with suction/injection

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Abstract: Through this paper, we studied the effects of non uniform het source/sink on unsteady flow of Powell-Eyring fluid past an inclined stretching sheet with suction/injection effects. The governing equations are transformed into a system of ordinary differential equations by making use of self similarity transformations and then solved numerically using Runge-Kutta based shooting technique. Further we studied the influence of governing parameters on the flow, heat transfer, friction factor and local Nusselt number through graphs and tables, which are obtained using MATLAB package. From this study, it is found that increasing values of non uniform heat source/sink parameters enhances the temperature.

Key words: Unsteady flow, Powell-Eyring fluid, Non-uniform heat source/sink, Suction/injection.

Introduction:

The flow of non-Newtonian fluids has attained a greatest importance and increasing interest in the theory of fluid mechanics. A few examples of non-Newtonian fluids include tooth paste, food products, flow of blood etc. Many investigators studied electrically conducting non-Newtonian fluids of the two-dimensional magneto hydrodynamic boundary layer flows. It is assumed that non-Newtonian behaviour described by power-law model. The Powell-Eyring model is mathematically more complex, and it has advantages of power-law model.

The effect of thermal radiation on unsteady flow of a nanofluid over an infinite vertical plate was discussed by Sandeep et al. (2013). Further Krishna et al. (2014) studied the influence of magnetic field and heat source on nanofluid flow past a vertical plate. Ramana Reddy et al. (2014) investigated the unsteady MHD dusty viscous flow by considering the effects of chemical reaction and thermal diffusion. Ara et al. (2014) studied the boundary layer flow of Powell-Eyring fluid over an exponentially shrinking sheet. Similar type of study past a non linear stretching surface was investigated by panigrahi et al. (2014). Sugunamma et al. (2014) discussed the influence of thermal radiation and magnetic field on the flow of nanofluids in a rotating frame. Sandeep et al. (2015) studied the impact of non uniform heat source/sink on dusty nanofluid flow past a stretching/shrinking sheet. The heat and mass transfer effects on Powell-Eyring fluid past an exponentially stretching surface were discussed by Hayat et al. (2015).The free convection flow of a nanofluid past a slendering stretching sheet was analyzed by Sulochana and Sandeep (2015). With the help of this study, they found that thermal boundary layer thickness become high in the presence of velocity slip. The effects of thermal diffusion and diffusion thermo on the flow of Powell-Eyring fluid due to cone were studied by Khan and Sultan (2015). Recently, Sandeep et al. (2016) discussed the influence of non uniform heat source/sink on the flow of dusty nanofluid past a stretching /shrinking cylinder. Similar type of study past a slendering stretching sheet was reported by Ramana Reddy et al. (2016). Abbasi et al. (2016) have investigated the effects of heat generation/absorption on doubly stratified mixed convection flow of Maxwell nanofluid. Very recently, Raju and Sandeep (2016) analyzed the heat and mass transfer characteristics on MHD bio-convection flow over a rotating cone/plane with cross diffusion.

Mathematical Formulation

Consider an incompressible, two-dimensional unsteady flow of Power-Eyring fluid past an inclined stretching sheet. The sheet makes an angle α with the vertical direction. The *x*-axis is taken along the sheet and *y* axis is normal to it. In addition, we considered the effects non-uniform heat source/sink.

The boundary layer equations comprising the balance laws of mass, linear momentum and energy can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\upsilon+\frac{1}{\rho\tilde{\beta}\gamma}\right)\frac{\partial^2 u}{\partial y^2}-\frac{1}{2\rho\tilde{\beta}\gamma^3}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+g\beta_T\left(T-T_\infty\right)\cos\alpha, \tag{2}$$

$$\rho c_p\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=k\frac{\partial^2 T}{\partial y^2}+q''', \tag{3}$$

q''' is the non-uniform heat source (q'''>0) or sink (q'''<0) per unit volume. The non-uniform heat source/sink, q''' is modelled by the following expression

$$q''' = \frac{k u_s(\mathrm{x},\mathrm{t})}{x\upsilon}\left[A^*\left(T_s - T_\infty\right) f' + B^*\left(T - T_\infty\right)\right], \tag{4}$$

in which A^* and B^* are the coefficients of space and temperature dependent heat source/sink, respectively. The surface velocity is denoted by $u_s\left(x,t\right) = \frac{bx}{\left(1-at\right)}$, whereas the surface temperature $T_s\left(x,t\right) = T_\infty + T_{ref}\frac{bx^2}{2\upsilon}\left(1-at\right)^{-3/2}$. Here b (stretching rate) and a are positive constants having dimension time. Also T_{ref} is constant reference to temperature.

The boundary conditions are taken as follows:

$$u = u_s\left(x,t\right), v = v_s, T = T_s\left(x,t\right) at \ y = 0, and\, u \to 0, T \to T_{\infty,} as \ \ y \to \infty, \tag{5}$$

By introducing the similarity transformations

$$u = \frac{bx}{\left(1-at\right)} f'\left(\eta\right), v = -\sqrt{\frac{\upsilon b}{\left(1-at\right)}} f\left(\eta\right), \theta = \frac{T - T_\infty}{T_s - T_\infty}, \eta = \sqrt{\frac{b}{\upsilon\left(1-at\right)}} y, \tag{6}$$

Equation (1) is identically satisfied and equations (2)-(3) become

$$\left(1+\Gamma\right) f''' - \left(f'\right)^2 + ff'' - \Gamma\beta f''^2 f''' - \varepsilon\left(f' + \frac{1}{2}\eta f''\right) + Gr\theta\cos\alpha = 0, \tag{7}$$

$$\theta'' + \Pr\left(f\theta' - 2f'\theta - \frac{1}{2}\varepsilon\left(3\theta + \eta\theta'\right)\right) + A^* f' + B^*\theta = 0, \tag{8}$$

Boundary conditions are

$$f = S, f' = 1, \theta = 1, at \ \eta = 0, and \ f' \to 0, \theta \to 0, as \ \ \eta \to \infty, \tag{9}$$

where prime denotes differentiation with respect to η, f is the dimensionless stream function, θ is the dimensionless temperature and the dimensionless numbers are

$$\Gamma = \frac{1}{\mu\tilde{\beta}\gamma}, \beta = \frac{\rho u_s^3}{\mu x\gamma^2}, Gr = \frac{g_0\beta_T\left(T_s - T_\infty\right)x^3/\upsilon^2}{u_s^2 x^2/\upsilon^2} = \frac{Gr_x}{\mathrm{Re}_x^2}, \mathrm{S} = -\frac{v_0}{\sqrt{vb}}, \varepsilon = \frac{a}{b}, \Pr = \frac{\mu c_p}{k}, \tag{10}$$

where Γ and β are the dimensionless material fluid parameters, Gr is the thermal Grashof number, ε is the unsteadiness parameter, Pr is the Prandtl number and S is the suction/injection parameter, where $S > 0$ for suction and $S < 0$ for injection.

For engineering interest the coefficient of skin friction and local Nusselt number are defined as,

$$C_f \mathrm{Re}_x^{1/2} = f''(0), \tag{11}$$

$$Nu_x \mathrm{Re}_x^{-1/2} = -\theta'(0), \tag{12}$$

where $\mathrm{Re}_x = \frac{u_s x}{\nu}$ is the local Reynolds number.

Results and Discussion

Eqs. (7) - (8) with the boundary conditions (9) have been solved numerically using Runge-Kutta based shooting technique. The results obtained shows the influence of the non-dimensional governing parameters, Material fluid parameter Γ, unsteadiness parameter ε, Inclined angle α and non-uniform heat source/sink parameters A^* and B^* on velocity, temperature, friction factor and Nusselt number. For numerical results we considered $\Gamma = \beta = 0.2, \varepsilon = 0.5, \alpha = \pi/4$, $\mathrm{Pr} = Gr = 1, \eta = 3, A^* = B^* = 0.1$. These values are kept as common in entire study except the varied values as displayed in the respective figures and tables.

Figures 1 depicts the influence of inclined angle on velocity profiles for suction and injection cases. It is observed that a raise the values of inclined angle depreciates the velocity profiles. This is due to the fact that at $\alpha = 0$ the sheet is in vertical direction and maximum gravitational force acts on the flow. As $\alpha \to \pi/2$ the sheet takes horizontal direction, the strength of buoyancy forces decreases and hence reduces the velocity boundary layer.

Figures 2 and 3 depict the effect of material fluid parameter on velocity and temperature profiles of the flow for both suction and injection cases. It is clear from the figures that the increase in the value of Γ enhances the velocity boundary layer and depreciates the thermal boundary layer thickness. It is also noticed that the enhancement in the velocity filed is more on injection case compared with suction case.

Figure 4 represents the influence of unsteadiness parameter on velocity profiles of the flow for both suction and injection cases. We noticed an interesting result that the enhancement in the value of unsteadiness parameter depreciates the velocity profiles of the flow in injection case and increases the velocity of the flow in suction case. This is due to the fact that increase in unsteadiness parameter reduces the velocity boundary layer in injection case but in suction case it works in opposite manner.

Figures 5-8 illustrate the effect of non-uniform heat source/sink parameters on the velocity and temperature profiles of the flow. It is evident from the figures that an increase in the values of A^* and B^* enhances the velocity and temperature profiles of the flow. This may happen due to the fact that the positive values of A^* and B^* acts like heat generators.

Generating the heat means releases the heat energy to the flow. This help to enhance the velocity and thermal boundary layer thickness.

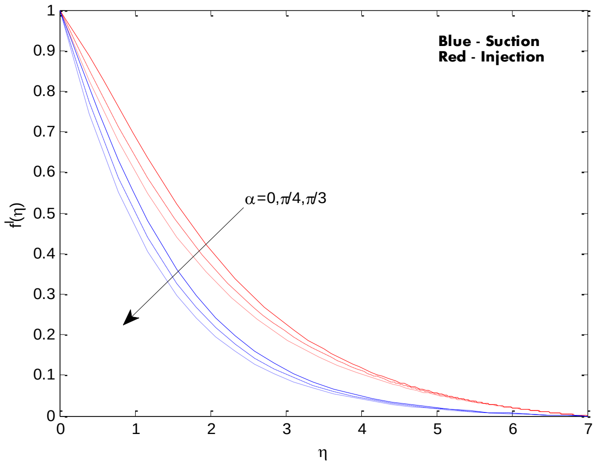

Fig.1 Variation of velocity with α

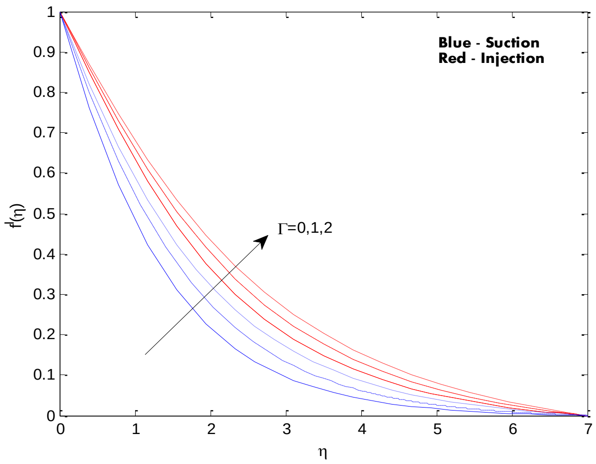

Fig.2 Variation of velocity with Γ

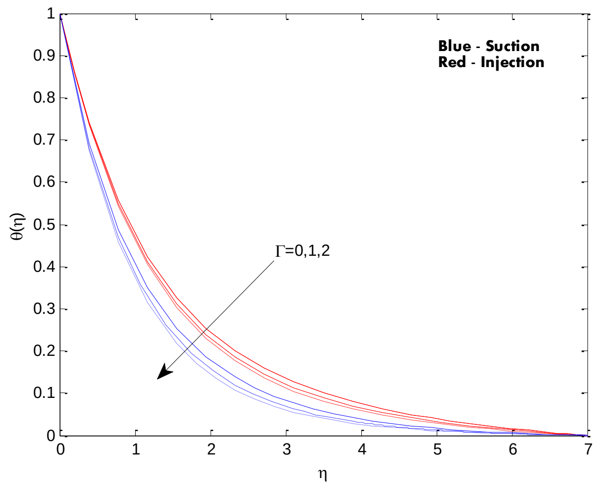

Fig.3 Variation of temperature with Γ

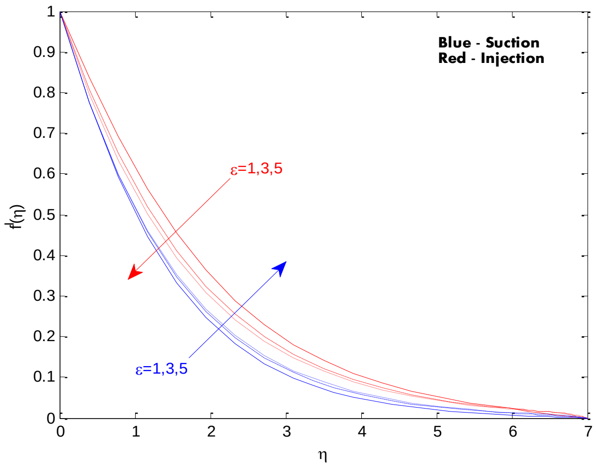

Fig.4 Variation of velocity with ε

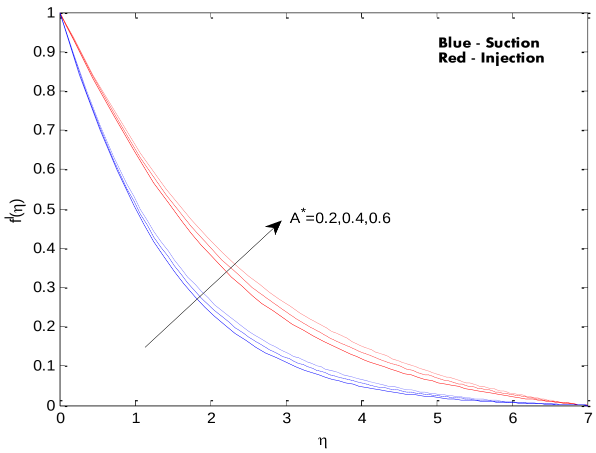

Fig.4 Variation of velocity with A^*

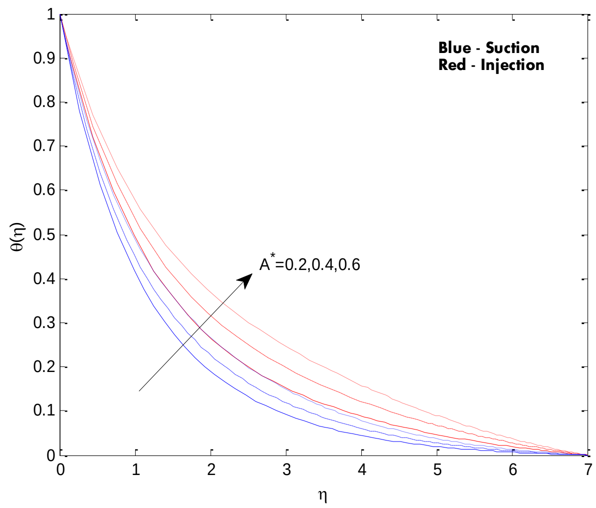

Fig.6 Variation of temperature with A^*

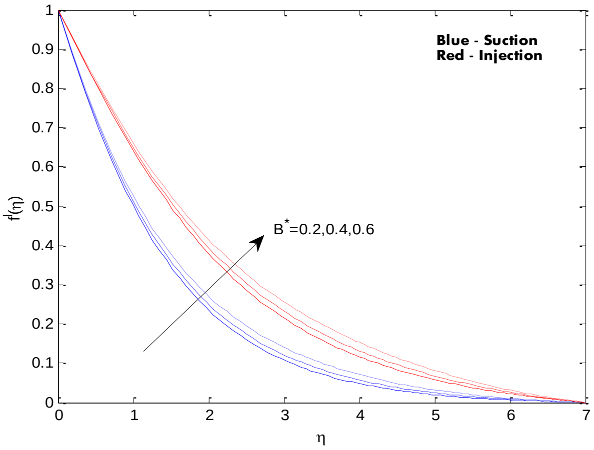

Fig.7 Variation of velocity with B^{*}

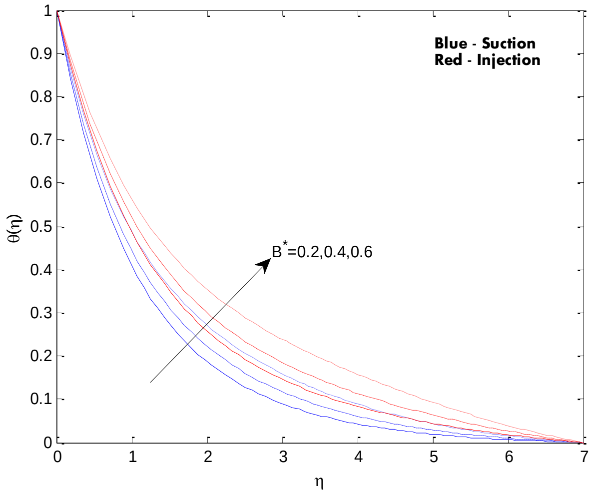

Fig.8 Variation of temperature with B^{*}

Conclusions:

- Increase in non uniform heat source/sink parameters helps to enhance the velocity and raises the temperature profiles.
- Increase in the value of material fluid parameter enhances the velocity but reduces the temperature of the fluid.
- An ehancement in the value of unsteadiness parameter depreciates the velocity profiles of the flow in injection case and increases the velocity of the flow in suction case.
- Increase in the value of material fluid parameter enhances the rate of heat transfer.

Table 1 displays the influence of non-dimensional governing parameters on friction factor and Nusselt number. From this table, it is clear that increasing values of non uniform heat source/sink parameters reduces the heat transfer rate in both suction and injection cases where as material fluid parameter enhances the heat transfer rate.

Table 1 Variation in $f''(0), and -\theta'(0)$ at different non-dimensional parameters.

S	Γ	α	A^*/B^*	$f''(0)$	$-\theta'(0)$
1	0			-0.6659	0.9778
	1			-0.5490	1.0060
	2			-0.4833	1.0227
-1	0			-0.3828	0.7769
	1			-0.3603	0.7856
	2			-0.3397	0.7921
1		0		-0.4877	1.0078
		$\pi/4$		-0.6350	0.9850
		$\pi/3$		-0.7434	0.9666
-1		0		-0.2485	0.7983
		$\pi/4$		-0.3785	0.7789
		$\pi/3$		-0.4731	0.7635
1			0.2	-0.6288	0.9480
			0.4	-0.6159	0.8726
			0.6	-0.6024	0.7954
-1			0.2	-0.3743	0.7449
			0.4	-0.3657	0.6763
			0.6	-0.3570	0.6067

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