

Transient Mixed Convective Heat and Mass Transfer Flow in a Vertical Channel with Travelling Thermal Wave and Heat Sources

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Abstract

In this paper, we investigate the transient mixed convective heat and mass transfer flow of a viscous, electrically conducting fluid confined in a vertical channel with traveling thermal wave. The concentration on the walls is maintained constant. The non-linear coupled equations governing the flow, heat and mass transfer have been solved by employing a regular perturbation technique with the aspect ratio δ as a perturbation parameter. The effect of various governing parameters of the flow characteristics have been discussed graphically. The rate of heat and mass transfer are evaluated numerically for different variations.

Keywords : Dissipation, chemical reaction and thermo-diffusion.

Nomenclature:

$R = \dfrac{UL}{\nu}$ (Reynolds Number), $G = \dfrac{\beta g \Delta T_e L^3}{\nu^2}$ (Grashof Number)

$P = \dfrac{\mu c_p}{k}$ (Prandtl Number), $D^{-1} = \dfrac{L^2}{k}$ (Darcy Parameter)

$\alpha = \dfrac{QL^2}{\lambda \Delta T}$ (Heat Source Parameter), $Ec = \dfrac{\beta g L^3}{C_p}$ (Eckert Number)

$\delta = mL$ (Aspect Ratio), $\gamma = \dfrac{n}{\nu m^2}$ (Non-dimensional thermal wave velocity)

$Sc = \dfrac{\nu}{D_1}$ (Schimdt Number), $N = \dfrac{\beta^* \Delta C}{\beta \Delta T}$ (*Buoyancy ratio*)

$S_0 = \dfrac{\beta^* k_{11}}{\nu \beta}$ (Soret Parameter), $M^2 = \dfrac{\sigma \mu_e^2 H_o^2 L^2}{\nu^2}$ (Hartman Number)

$K = \dfrac{krL^2}{\nu}$ (Chemical Reaction Parameter), $\nabla_1^2 = \delta^2 \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$

ρ_e density of the fluid in the equilibrium state,

T_e temperature in the equilibrium state, C_e Concentration in the equilibrium state,

u, v velocity components along x and y directions respectively,

p pressure, T fluid temperature, C fluid Concentration,

ρ density of the fluid, μ coefficient of viscosity, Cp specific heat at constant pressure,

λ coefficient of thermal conductivity, D_1 molecular diffusivity, k_{11} cross diffusivity,

β coefficient of thermal expansion, β^* volumetric coefficient of expansion

Q strength of the heat source

Introduction

The time dependent thermal convection flows have applications in chemical engineering, space technology, etc. These flows can be achieved by either time dependent movement of the boundary or unsteady temperature of the boundary. The unsteady temperature may be attributed to the free stream oscillations or oscillatory flux or temperature oscillations. The oscillatory convection problems are important from the technological point of view as the effect of surface temperature oscillations on skin friction and heat transfer from surface to the surrounding fluid has special interest in heat transfer engineering.

Flows which arise due to the interaction of the gravitational force and density differences caused by the simultaneous diffusion of thermal energy have many applications in geophysics and engineering. Such thermal and mass diffusion plays a dominant role in a number of technological and engineering systems. Obviously, the understanding of this transport process is desirable in order to effectively control the overall transport characteristics. The combined effect of thermal and mass diffusion in channel flows has been studied by a few authors in recent times [5, 7, 10, 18]. The problem of combined buoyancy driven thermal and mass diffusion has been studied in parallel plate geometries by a few authors, notably, Chen and Moutsoglou [4], Trevisan [32] and Angirasa et al. [2].

As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging – diverging geometries for improving the design of heat transfer equipment. Vajravelu and Neyfeh [33] have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastri [35] have analyzed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls in the presence of constant heat source. Later Vajravelu and Debnath [34] have extended this study to convective flow in a vertical wavy channel in four different geometrical configurations. This problem has been extended to the case of wavy walls by Deshikachar et al [6], Rao et al. [16] and Sree Ramachandra Murthy [29] have analyzed that the flow heat and mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Mahdy et al. [11] have studied the mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media (PST/PSE). Jafarunnisa et al. [9] have discussed unsteady hydromagnetic mixed convection flow in a vertical channel on whose walls travelling thermal wave is imposed. Jer-Huan Jang et al. [10] have analyzed that the Mixed convection heat and mass transfer along a vertical wavy surface.

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. A common area of interest in the field of aerodynamics is the analysis of thermal boundary layer problems for two dimensional steady and incompressible laminar flow passing a wedge. Simultaneous heat and mass transfer from different geometrics embedded in porous media has many engineering and geophysical application such as geothermal reservoirs, drying of porous solids thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection/conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in no conventional energy sources, such as the use of salt-gradient solar ponds for energy collection and storage.

Unsteadiness in the flow can also be created by imposing traveling thermal waves on the boundaries. From a physical point of view, the motion induced by traveling thermal waves is quite interesting as a purely fluid dynamical problem and can be used as a possible explanation for the observed four-day retrograde zonal motion of the upper atmosphere of Venus. Keeping the above applications in view several authors [13, 17, 15, 18, 22, 30] have investigated convective Heat and Mass transfer flow in wavy channels with traveling thermal waves on the walls.

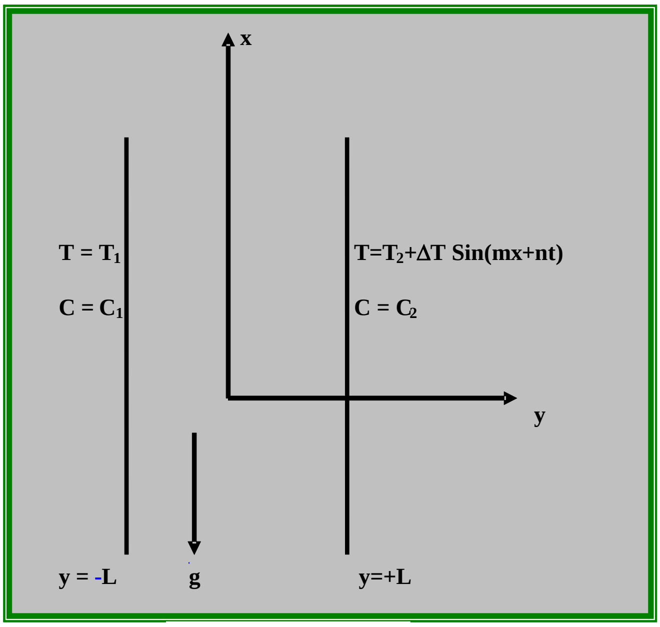
When heat and mass transfer occur simultaneously in a moving fluid, the relation between the fluxes and the driving potentials are of more intricate nature. Mass fluxes can be created by temperature gradients and this is the Soret effect or thermo-diffusion effect. Adrian Postelnicu[1], Emmanuel Osalusi et al. [7], Mohamed Abd-El-Aziz [12] have studied thermo-diffusion and diffusion thermo effects on combined heat and mass transfer through a porous medium under different conditions. Sreevani et al. [31] have studied the unsteady free convective heat and mass transfer flow through porous medium dissipative effect in rotating channel. All the above mentioned studies are based on the hypothesis that the effect of dissipation is neglected. This is possible in case of ordinary fluid flow like air and water under gravitational force. But this effect is expected to be relevant for fluids with high values of the dynamic viscosity force. On the other hand, Barletta [3] and Zanchini [36] pointed out that relevant effects of viscous dissipation on the temperature profiles and the Nusselt number may occur in the fully developed convection in tubes. In view of this, several authors, notably Soundalgekar and Pop [26], Barletta [3] and Zanchini [36], Sreevani [30] has studied the effect of viscous dissipation on the convective flows past an infinite vertical plate and through vertical channels and ducts. Sivaiah et al [24] have studied the Thermo-Diffusion effects on convective heat and mass transfer through a porous medium in Ducts. Indudhar et al. [8] have investigated the effect of thermo-diffusion and radiation on unsteady convective heat and mass transfer flow in a vertical channel with quadratic density –temperature relation. Rajasekhar et al. [19] have discussed the effect of chemical reaction and radiation absorption on unsteady convective heat and mass transfer flow of a viscous electrically conducting fluid in a vertical wavy channel with traveling thermal waves and Hall effects. Muthuraj et al. [13] have studied the mixed convective heat and mass transfer in a vertical wavy channel with traveling thermal waves and porous medium. Recently Sreeranga Vani et al. [28] analyzed the effect of chemical reaction and dissipation on unsteady convective heat and mass transfer flow in a vertical channel with traveling thermal waves imposed on the wall.

In this paper, we deal with the transient mixed convective heat and mass transfer flow of a viscous, electrically conducting fluid confined in a vertical channel with traveling thermal wave. The concentration on the walls is maintained constant. The coupled equations governing the flow, heat and mass transfer are solved by using the perturbation technique with δ, the aspect ratio as a perturbation parameter. The combined influence of Soret and dissipation effects on the velocity, temperature, concentration and rate of heat and mass transfer are discussed in detail.

FORMULATION OF THE PROBLEM

We consider the motion of viscous, incompressible, electrically conducting fluid in a vertical channel bounded by flat walls. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at y=L while the boundary at y = -L is maintained at constant temperature T_1. The walls are maintained at constant concentrations. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are taken into account to the transport of heat by conduction and convection in the energy equation. We take Soret effect into account in the diffusion equation. Also the kinematic viscosity ν, the thermal conductivity k are treated as constants. We choose a rectangular Cartesian system 0 (x ,y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at $y = \pm L$.

Configuration of the Problem



The equations governing the unsteady flow, heat and mass transfer are Equation of linear momentum

$$\rho_e\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-(\sigma\mu_e^2 H_0^2)u-\left(\frac{\mu}{k}\right)u-\rho g \quad (1)$$

$$\rho_e\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\left(\frac{\mu}{k}\right) \quad (2)$$

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (3)$$

Equation of energy

$$\rho_e C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q+ \\ +\mu\left(\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right)+\left(\frac{\mu}{k}+\sigma\mu_e^2H_0^2\right)(u^2+v^2) \quad (4)$$

Equation of Diffusion

$$\left(\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_1\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+k_{11}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+krC \quad (5)$$

Equation of state

$$\rho-\rho_e=-\beta\rho_e(T-T_e)-\beta^*\rho_e(C-C_e) \quad (6)$$

In the equilibrium state

$$0=-\frac{\partial p_e}{\partial x}-\rho_e g \quad (7)$$

where $p=p_e+p_D$, p_D being the hydrodynamic pressure.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$Q=\frac{1}{2L}\int_{-L}^{L}u\,dy. \quad (8)$$

The boundary conditions for the velocity and temperature fields are

$u=0$, $v=0$, $T=T_1$, $C=C_1$ on $y=-L$

$u=0$, $v=0$, $T=T_2+\Delta T_e\,Sin(mx+nt)$, $C=C_2$ on $y=L$ (9)

where $\Delta T_e=T_1-T_2$ and $Sin(mx+nt)$ is the imposed traveling thermal wave

In view of the continuity equation we define the stream function ψ as

$$u=-\psi_y,\; v=\psi_x \quad (10)$$

Eliminating pressure p from equations (2) & (3) and using the equations governing the flow in terms of ψ are

$$[(\nabla^2\psi)_t+\psi_x(\nabla^2\psi)_y-\psi_y(\nabla^2\psi)_x]=\nu\nabla^4\psi-\beta g(T-T_0)_y+ \\ -\beta^* g(C-C_0)_y-\left(\frac{\sigma\mu_e^2H_0^2}{\rho_0}\right)\frac{\partial^2\psi}{\partial y^2}-\left(\frac{\mu}{k}\right)\nabla^2\psi \quad (11)$$

$$\rho_e C_p\left(\frac{\partial\theta}{\partial t}-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)=\lambda\nabla^2\theta+Q+\mu\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\left(\frac{\partial^2\psi}{\partial x^2}\right)^2\right)+ \\ +\left(\frac{\mu}{k}+\sigma\mu_e^2H_0^2\right)\left(\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right) \quad (12)$$

$$\left(\frac{\partial C}{\partial t}-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right)=D_1\nabla^2C+k_{11}\nabla^2T-krC \quad (13)$$

Introducing the non-dimensional variables in (11)- (13) as

$$x'=mx,\; y'=y/L,\; t'=t\nu m^2,\; \Psi'=\Psi/\nu,\; \theta=\frac{T-T_2}{T_1-T_2},\; C'=\frac{C-C_1}{C_2-C_1} \quad (14)$$

the governing equations in the non-dimensional form (after dropping the dashes) are

$$\delta R\left(\delta(\nabla_1^2\psi)_t+\frac{\partial(\psi,\nabla_1^2\psi)}{\partial(x,y)}\right)=\nabla_1^4\psi+\left(\frac{G}{R}\right)(\theta_y+NC_y)-M^2\frac{\partial^2\psi}{\partial y^2}-D^{-1}\nabla^2\psi \quad (15)$$

The energy equation in the non-dimensional form is

$$\delta P(\delta\frac{\partial\theta}{\partial t}-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y})=\nabla_1^2\theta+\alpha+(\frac{PR^2E_c}{G})((\frac{\partial^2\psi}{\partial y^2})^2+\delta^2(\frac{\partial^2\psi}{\partial x^2})^2)+ \\ +(M^2)(\delta^2(\frac{\partial\psi}{\partial x})^2+(\frac{\partial\psi}{\partial y})^2)) \quad (16)$$

The Diffusion equation is

$$\delta Sc(\delta\frac{\partial C}{\partial t}-\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y})=\nabla_1^2C+\frac{ScS_0}{N}\nabla_1^2\theta-KScC \quad (17)$$

The corresponding boundary conditions are

$$\psi(+1)-\psi(-1)=-1$$

$$\frac{\partial\psi}{\partial x}=0,\ \frac{\partial\psi}{\partial y}=0 \quad at\ y=\pm1 \quad (18)$$

$$\theta(x,y)=1\ ,\ C(x,y)=0 \qquad \text{on } y=-1$$

$$\theta(x,y)=Sin(x+\gamma t),\ C(x,y)=1 \quad \text{on } y=1$$

$$\frac{\partial\theta}{\partial y}=0,\frac{\partial C}{\partial y}=0 \quad at \quad y=0 \quad (19)$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis (8). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t.

ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio δ to be small.

We adopt the perturbation scheme and write

$$\psi(x,y,t)=\psi_0(x,y,t)+\delta\psi_1(x,y,t)+\delta^2\psi_2(x,y,t)+..............$$

$$\theta(x,y,t)=\theta_0(x,y,t)+\delta\theta_1(x,y,t)+\delta^2\theta_2(x,y,t)+.............$$

$$C(x,y,t)=C_0(x,y,t)+\delta C_1(x,y,t)+\delta^2C_2(x,y,t)+.............(20)$$

On substituting (20) in (15) - (17) and separating the like powers of δ, the equations and respective conditions to the zeroth order are

$$\psi_{0,yyyy}-M_1^2\psi_{0,yy}=-\frac{G}{R}(\theta_{0,y}+NC_{0,y}) \quad (21)$$

$$\theta_{0,yy}+\alpha+\frac{PEcR^2}{G}(\psi_{0,yy})^2+\frac{PEcM_1^2}{G}(\psi_{0,y})^2 \quad (22)$$

$$C_{0,yy}-KScC_0=-\frac{ScS_0}{N}\theta_{0,yy} \quad (23)$$

with

$$\psi_0(+1)-\psi_0(-1)=-1$$

$$\psi_{0,y}=0,\ \psi_{0,x}=0 \quad at\ \ y=\pm1 \quad (24)$$

$$\theta_o=1\ \ ,C_0=0 \quad on \quad y=-1$$

$$\theta_o=Sin(x+\gamma t),C_0=1\ \ on\ \ y=1 \quad (25)$$

and to the first order are

$$\psi_{1,yyyy}-M_1^2\psi_{1,yy}=-\frac{G}{R}(\theta_{1,y}+NC_{1,y})+(\psi_{0,y}\psi_{0,xyy}-\psi_{0,x}\psi_{0,yyy}) \quad (26)$$

$$\theta_{1,yy} = P(\psi_{0,x}\theta_{0,y} - \psi_{0,y}\theta_{0,x}) - \frac{2PEcR^2}{G}(\psi_{0,yy}\psi_{1,yy}) - \frac{2PEcM_1^2}{G}(\psi_{0,y}\psi_{1,y}) \quad (27)$$

$$C_{1,yy} - KScC_1 = -\frac{ScS_0}{N}\theta_{1,yy} + (\psi_{0,x}C_{0,y} - \psi_{0,y}C_{0,x}) \quad (28)$$

with

$$\psi_{1(+1)} - \psi_{1(-1)} = 0$$
$$\psi_{1,y} = 0, \ \psi_{1,x} = 0 \quad at \quad y = \pm 1 \quad (29)$$

$\theta_1(\pm1) = 0$, $C_1(\pm1) = 0$ at y= ± 1 (30)

Assuming Ec<<1 to be small we take the asymptotic expansions as

$$\Psi_0(x,y,t) = \Psi_{00}(x,y,t) + Ec\Psi_{01}(x,y,t) + \dots\dots\dots$$
$$\Psi_1(x,y,t) = \Psi_{10}(x,y,t) + Ec\Psi_{11}(x,y,t) + \dots\dots\dots$$
$$\theta_0(x,y,t) = \theta_{00}(x,y,t) + Ec\theta_{01}(x,y,t) + \dots\dots\dots$$
$$\theta_1(x,y,t) = \theta_{10}(x,y,t) + Ec\theta_{11}(x,y,t) + \dots\dots\dots$$
$$C_0(x,y,t) = C_{00}(x,y,t) + EcC_{01}(x,y,t) + \dots\dots\dots$$
$$C_1(x,y,t) = C_{10}(x,y,t) + EcC_{11}(x,y,t) + \dots\dots\dots \quad (31)$$

Substituting the expansions(31) in equations (21)-(23) and separating the like powers-of Ec we get the following

$$\theta_{00,yy} + \alpha = 0, \theta_{00}(-1) = 1, \theta_{00}(+1) = SinD_1 \quad (32)$$

$$C_{00,yy} - KScC_{00} = \frac{-ScS_0}{N}\theta_{00,yy}, \ C_{00}(-1) = 0, \ C_{00}(+1) = 1 \quad (33)$$

$$\Psi_{00,yyyy} - M_1^2\Psi_{00,yy} = \frac{-G}{R}(\theta_{00,y} + NC_{00,y})$$
$$\Psi_{00}(+1) - \Psi_{00}(-1) = -1,$$
$$\Psi_{00,y} = 0, \Psi_{00,x} = 0 \quad \text{at } y=\pm1 \quad (34)$$

$$\theta_{01,yy} = \frac{-PR^2Ec}{G}\Psi^2_{00,yy} - \frac{-PM_1^2Ec}{G}\Psi^2_{00,y} \quad , \quad \theta_{01}(\pm1) = 0 \quad (35)$$

$$C_{01,yy} - KScC_{01} = \frac{-ScS_0}{N}\theta_{01,yy} \quad , \quad C_{01}(\pm1) = 0 \quad (36)$$

$$\Psi_{01,yyyy} - M_1^2\Psi_{01,yy} = \frac{-G}{R}(\theta_{01,y} + NC_{01,y})$$
$$\Psi_{01}(+1) - \Psi_{01}(-1) = 0,$$
$$\Psi_{01,y} = 0, \Psi_{01,x} = 0 \quad \text{at } y = \pm1 \quad (37)$$

$$\theta_{10,yy} = P(\Psi_{00,y}\theta_{00,x} - \Psi_{00,x}\theta_{00,y}) \quad , \quad \theta_{10}(\pm1) = 0 \quad (38)$$

$$\mathrm{C}_{10,yy} - KScC_{10} = \frac{-ScS_0}{N}\theta_{10,yy} + Sc(\Psi_{00,y}C_{00,x} - \Psi_{00,x}C_{00,y}) \ , \quad (39)$$
$$C_{10}(\pm1) = 0$$

$$\Psi_{10,yyyy} - M_1^2\Psi_{10,yy} = \frac{-G}{R}(\theta_{10,y} + NC_{10,y}) + R(\Psi_{00,y}\Psi_{00,xyy} - \Psi_{00,x}\Psi_{00,yyy}) \quad (40)$$
$$\Psi_{10}(+1) - \Psi_{10}(-1) = 0,$$
$$\Psi_{10,y} = 0, \Psi_{10,x} = 0 \quad \text{at } y=\pm1$$

$$\theta_{11,yy} = \frac{-2PR^2Ec}{G}(\Psi_{00,yy}\Psi_{10,yy}) - \frac{2PM_1^2Ec}{G}(\Psi_{00,y}\Psi_{10,y}) \quad , \ \theta_{11}(\pm1) = 0 \quad (41)$$
$$+ P(\Psi_{00,y}\theta_{01,x} - \Psi_{01,y}\theta_{00,x} + \theta_{00,x}\Psi_{01,y} - \theta_{01,x}\Psi_{00,y})$$

$$C_{11,yy} - KScC_{11} = \frac{-ScS_0}{N}\theta_{11,yy} + Sc(\Psi_{00,y}C_{01,x} - \Psi_{01,x}C_{00,y} + C_{00,x}\Psi_{01,y} - C_{01,x}\Psi_{00,y})$$
$$C_{11}(\pm 1) = 0 \qquad (42)$$
$$\Psi_{11,yyyy} - M_1^2\Psi_{11,y} = \frac{-G}{R}(\theta_{11,y} + NC_{11,y}) + R(\Psi_{00,y}\Psi_{01,xyy} - \Psi_{00,x}\Psi_{01,yyy} + \Psi_{01,y}\Psi_{00,xyy} - \Psi_{01,x}\Psi_{00,yyy}) \qquad (43)$$
$$\Psi_{11}(+1) - \Psi_{11}(-1) = 0, \quad \Psi_{11,y} = 0, \Psi_{11,x} = 0 \quad \text{at } y=\pm 1$$

Nusselt Number and Sherwood Number

The local rate of heat transfer coefficient (Nusselt number) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w}\left(\frac{\partial\theta}{\partial y}\right)_{y=\pm 1} \qquad \text{where } \theta_m = 0.5\int_{-1}^{1}\theta\,dy$$

The local rate of mass transfer coefficient (Sherwood number) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w}\left(\frac{\partial C}{\partial y}\right)_{y=\pm 1} \qquad \text{where } C_m = 0.5\int_{-1}^{1}C\,dy$$

RESULTS AND DISCUSSIONS

In this analysis we investigate the effect of dissipation, chemical reaction and thermo-diffusion on unsteady convective heat and mass transfer flow of a viscous, electrically conducting fluid in a vertical channel on whose wall is travelling thermal wave is imposed. The non-linear coupled equations governing the flow heat and mass transfer have been solved by employing a regular perturbation technique with the aspect ratio δ as a perturbation parameter. Here we take Prandtl number P=0.71 and δ=0.01.

The axial velocity (u) is shown in figures 1-5 for different values of G, R, Sc, N, S_0, K and Ec. It is found that the actual axial flow is in the vertically upward direction and hence u<0 represents a reversal flow. Figure 1 represents the variation of u with Grashof number G. It is found from the profiles that the axial velocity reduces with increase in G>0 and enhances with G<0 which maximum attained at y=0. Figure 2 represents u with Sc and N. It is found that lesser the molecular diffusivity smaller u in the flow region. The variation of u with buoyancy ratio N shows that when the molecular buoyancy force dominates over the thermal buoyancy force the axial velocity enhances when the buoyancy forces are in the same direction and for the forces acting in opposite direction the velocity reduces. Figure 3 represents u with Reynolds number R and S_0. It is found that an increase in R and S_0>0 enhances the velocity u while it reduces with S_0<0. Figure 4 represents u with chemical reaction parameter K. it is found that the axial velocity reduces in the degenerating chemical reaction case. The effect of dissipation of u can be observed from fig.5 higher the dissipative heat smaller the velocity in the flow region.

The secondary velocity (v) is representing in figures 6-10 for different parametric values. It is found that the secondary velocity is towards zero in the mid region and it is towards boundary in the left half in all variations. Figure 6 represents v with Grashof number G. it is found that the magnitude of |v| reduces with increase in G>0 while it enhances in the region (-0.8, -0.2) and in the remaining region |v| reduces with |G|≤15.

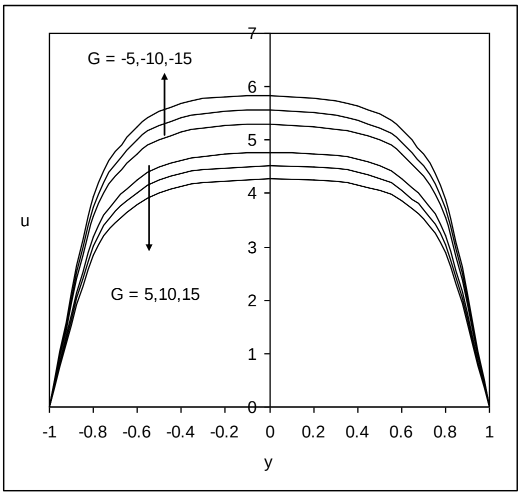


Fig. 1 : Variation of u with G
M=2, D^{-1}=2, α=2, Sc=1.3,N=1,
R=35, S_0=0.5, k=0.5, Ec=0.01

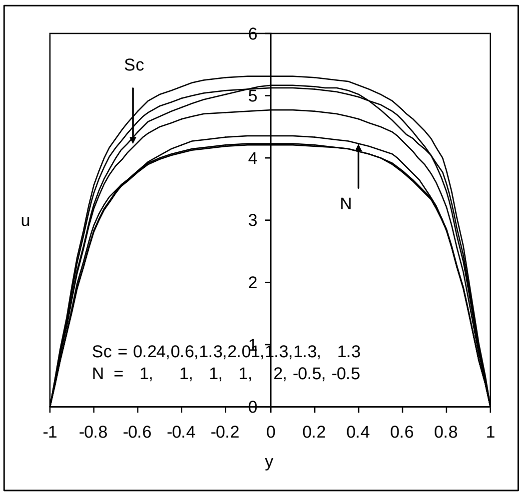


Fig. 2 : Variation of u with Sc, N
G = 5, M=2, D^{-1}=2, α=2,
R=35, S_0=0.5, k=0.5, Ec=0.01

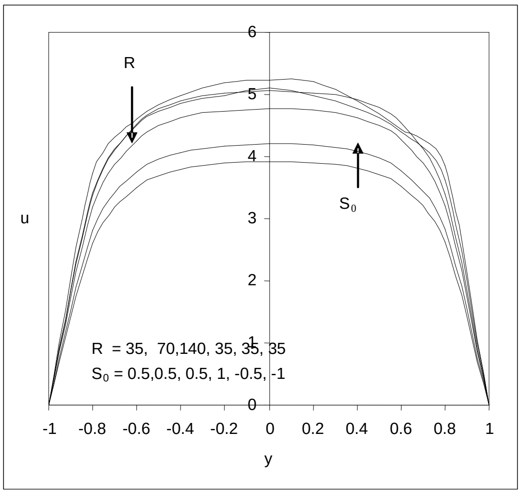


Fig. 3 : Variation ofu with R , S_0
G = 5, M=2, D^1=2, α=2, Sc=1.3,N=1,
K=0.5, Ec=0.01

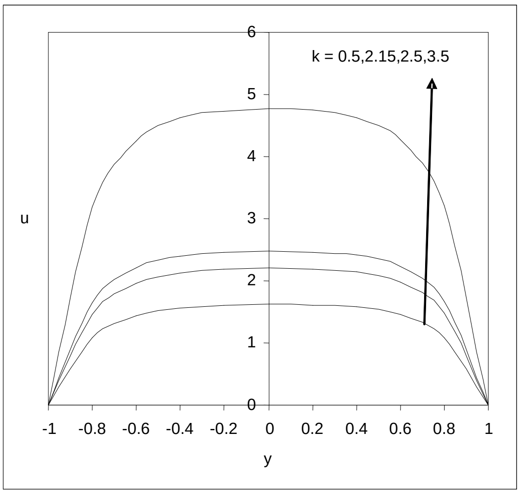


Fig. 4: Variation ofu with K
G = 5, M=2, D^1=2, α=2, Sc=1.3,N=1,
R=35, S_0=0.5, Ec=0.01

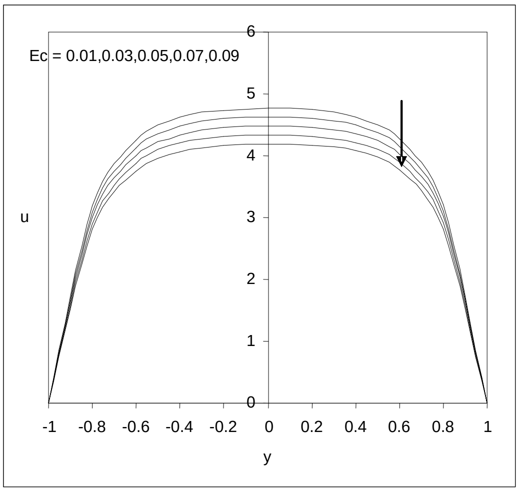


Fig. 5 : Variation ofu with Ec
G = 5, M=2, D^{1}=2, α=2, Sc=1.3,N=1,
R=35, S_0=0.5, K=0.5

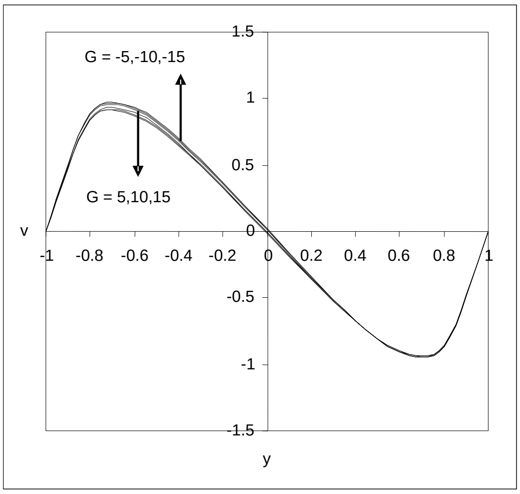


Fig. 6 : Variation ofv with G
M=2, D^{-1}=2, α=2, Sc=1.3,N=1,
R=35, S_0=0.5, K=0.5, Ec=0.01

With respect to Sc we notice depreciation in |v| with increase in Sc. The variation of v with buoyancy ratio N shows that |v| enhances in the left half and reduces in the right half with increase in N>0 while for an increase in |N|, |v| reduces in the left half and enhances in the right half (Fig.7). Figure 8 represents v with R and S_0. An increase in the Reynolds number R enhances |v| in the entire flow region. With respect to Soret parameter S_0 we notice an enhancement in |v| in the left half and depreciation in the right half while S_0 <0, a reversed effect is noticed in the behavior of |v|. Figure 9 represents v with chemical reaction parameter K. |v| reduces with increase in K. Figure 10 represents v with Eckert number Ec. It is found that higher the dissipative heat smaller |v| in the region.

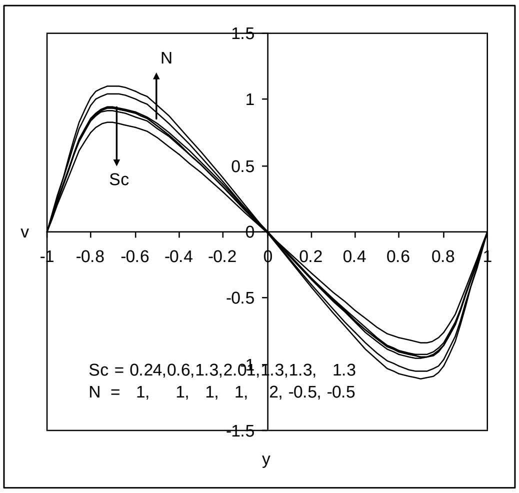


Fig. 7 : Variation of v with Sc, N
G = 5, M=2, D^{-1}=2, α=2,
R=35, S_0=0.5, k=0.5, Ec=0.01

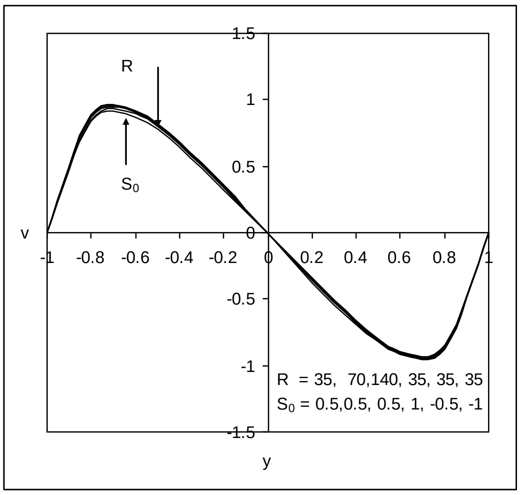


Fig. 8 : Variation of v with R, S_0
G = 5, M=2, D^{-1}=2, α=2, Sc=1.3,N=1,
k=0.5, Ec=0.01

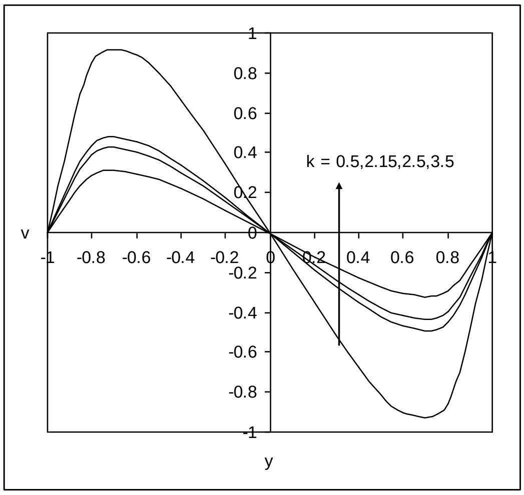


Fig. 9 : Variation of v with k
G = 5, M=2, D^{-1}=2, α=2, Sc=1.3,N=1,
R=35, S_0=0.5, Ec=0.01

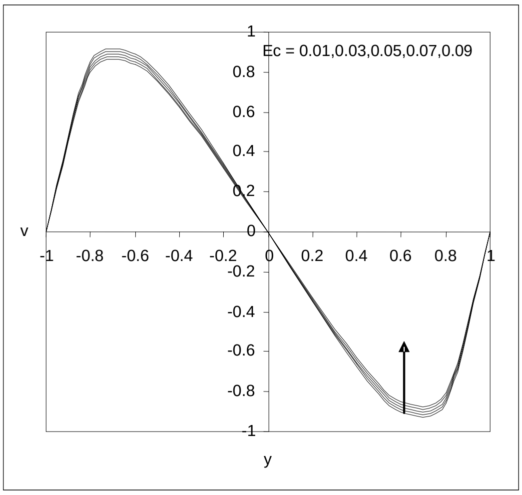


Fig. 10 : Variation of v with Ec
G = 5, M=2, D^{-1}=2, α=2, Sc=1.3,N=1,
R=35, S_0=0.5, K=0.5

The non-dimensional temperature (θ) is exhibited in figures 11-14 for different parametric values. It is found that the non-dimensional temperature (θ) is positive for all variations. This implies that the actual temperature is greater than T_e, the equilibrium temperature. Figure 11 represents θ with Grashof number G. it is found that the actual temperature reduces with increase in G>0 and enhances with increase in G<0 which maximum attained at y=0. Figure 12 represents θ with Sc and N. Lesser the molecular diffusivity smaller the actual temperature. With respect to buoyancy ratio N we find that when the molecular buoyancy force dominates over the thermal buoyancy force the actual temperature enhances when the buoyancy forces are in the same direction and for the forces acting in opposite direction it reduces in the flow region. Figure 13 represents θ with Soret parameter S_0. It can be seen from the profile that the actual temperature enhances with S_0 >0 and reduces with S_0<0. Figure 14 represents θ with Ec. Higher the dissipative heat larger the actual temperature in the entire flow region.

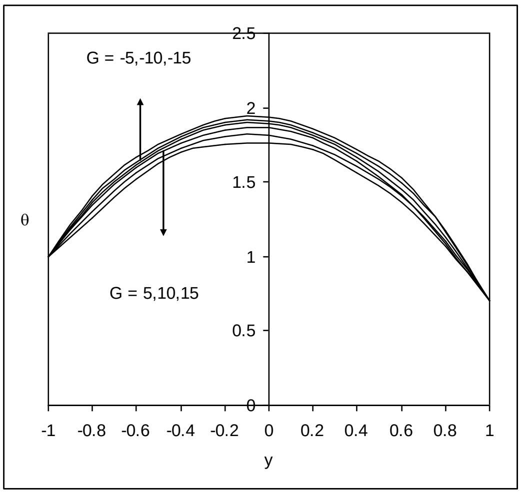


Fig. 11 : Variation of θ with G
M=2, D^{-1}=2, α=2, Sc=1.3,N=1,
S_0=0.5, x+γt=π/4, Ec=0.01

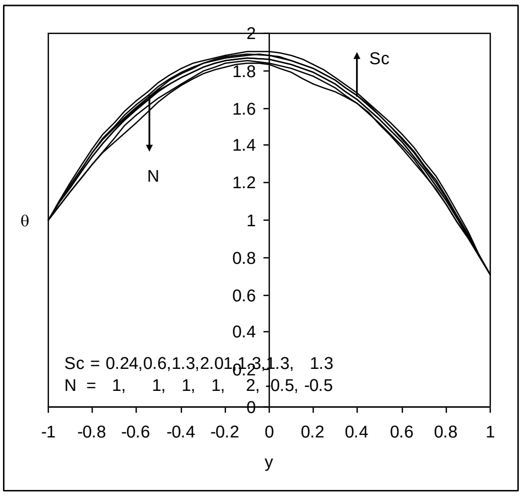


Fig. 12 : Variation of θ with Sc, N
G = 5, M=2, D^{-1}=2, α=2,
S_0=0.5, Ec=0.01, x+γt=π/4

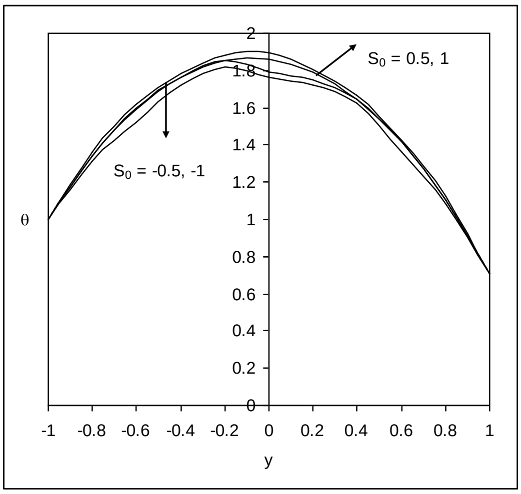


Fig. 13 : Variation of θ with S_0
G = 5, M=2, D^{-1}=2, α=2, Sc=1.3,N=1,
x+γt=π/4, Ec=0.01

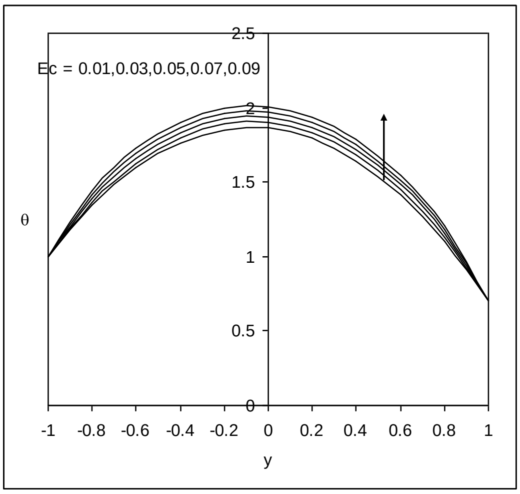


Fig. 14 : Variation of θ with Ec
G = 5, M=2, D^{-1}=2, α=2, Sc=1.3,N=1,
S_0=0.5, x+γt=π/4

The non dimensional concentration (C) is shown in figures 15-18 for different parametric values. We follow the convention that the non-dimensional concentration positive or negative according as the actual concentration is greater or lesser than the equilibrium concentration. Figure 15 represents the concentration with G. it is found that the actual concentration reduces with increase in |G|. From fig.16 we notice that lesser the molecular diffusivity larger the actual concentration. Also the actual concentration reduces with increase in |N| irrespective of the directions of the buoyancy forces. The actual concentration reduces with increase in S_0>0 and enhances with S_0<0 (Fig.17). Figure 18 represents C with Ec. It can be seen from the profiles that higher the dissipative heat larger the actual concentration

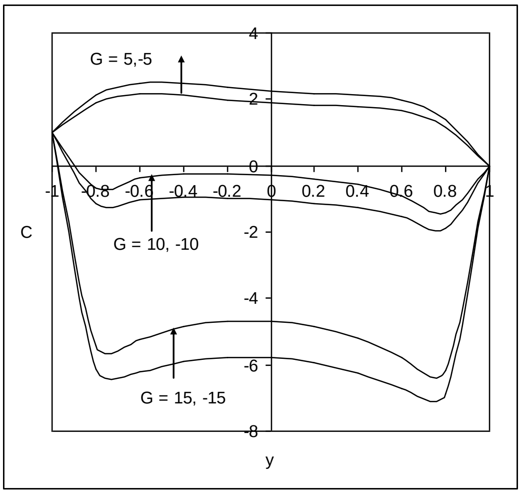


Fig. 15 : Variation of C with G
M=2, D^{-1}=2, α=2, Sc=1.3,N=1,
S_0=0.5, Ec=0.01

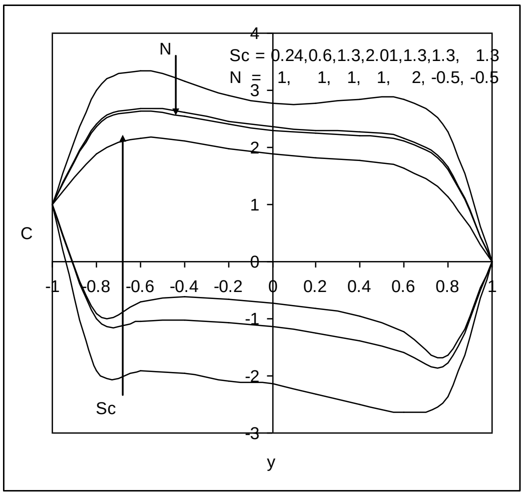


Fig. 16 : Variation of C with Sc, N
G = 5, M=2, D^{-1}=2, α=2,
S_0=0.5, Ec=0.01

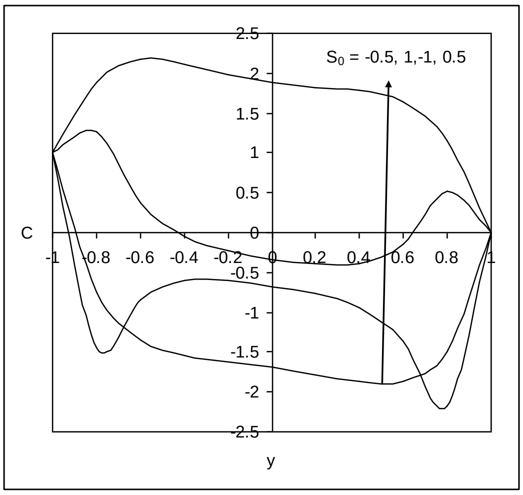


Fig. 17 : Variation of C with S_0
G = 5, M=2, D^{-1}=2, α=2, Sc=1.3,
N=1, Ec=0.01

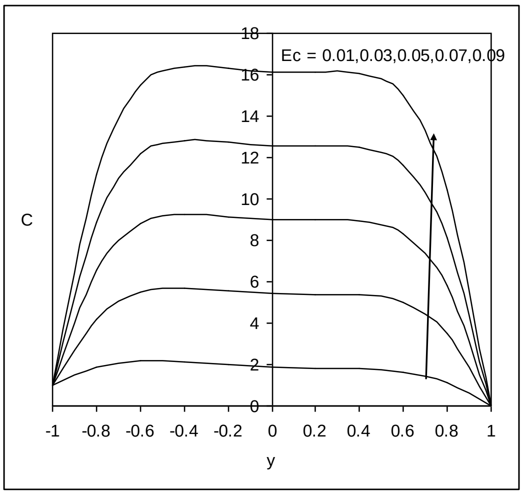


Fig. 18 : Variation of C with Ec
G = 5, M=2, D^{-1}=2, α=2, Sc=1.3,
N=1, S_0=0.5

The rate of heat transfer (Nusselt number) is exhibited in tables 1-6 for different values of Sc, N, S_0, K and Ec. It is found that an increase in |G| enhances |Nu| at y=+1 and reduces at y=-1. The variation of Nu with Sc shows that lesser the molecular diffusivity smaller |Nu| at y=+1 while at y=-1, |Nu| reduces with Sc≤0.6 and enhances with higher Sc≥1.3. When the molecular buoyancy force dominates over the thermal buoyancy force |Nu| enhances at y=+1 and reduces at y=-1 when the forces are in the same direction and for the forces acting in opposite direction a reversed effect is observed in the behavior of |Nu| at y±1 (Tables 1 & 4). With respect to Soret parameter S_0 we find that |Nu| enhances for G>0 and reduces for G<0 with increase in S_0>0 and reversed effect is observed with S_0<0 at y=+1. At y=-1 |Nu| reduces with S_0 >0 and enhances with S_0<0 (Table 2 & 4). With respect to Ec we find that higher the dissipative heat larger |Nu| at y=+1 and smaller at y=-1 (Tables 3 & 6).

Table-1 : Nusselt Number at y=+1

G	I	II	III	IV	V	VI	VII
5	-3.6820	-3.6740	-3.6515	-3.6277	-3.6743	-3.6167	-3.6096
10	-3.6551	-3.6540	-3.6435	-3.6283	-3.6896	-3.5720	-3.5573
-5	-3.7373	-3.7154	-3.6690	-3.6278	-3.6468	-3.7021	-3.7087
-10	-3.7652	-3.7365	-3.6785	-3.6287	-3.6347	-3.7430	-3.7557
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3
N	1	1	1	1	2	-0.5	-0.8

Table-2 : Nusselt Number at y=+1

G	I	II	III	IV
5	-3.6515	-3.6690	-3.6162	-3.5985
10	-3.6435	-3.6787	-3.5717	-3.5360
-5	-3.6690	-3.6520	-3.7029	-3.7197
-10	-3.6785	-3.6451	-3.7448	-3.7776
S_0	0.5	1	-0.5	-1

Table-3 : Nusselt Number at y=+1

G	I	II	III	IV	V
5	-3.6515	-3.6518	-3.6519	-3.6522	-3.6529
10	-3.6435	-3.6441	-3.6445	-3.6450	-3.6460
-5	-3.6690	-3.6706	-3.6710	-3.6711	-3.6716
-10	-3.6785	-3.6807	-3.6811	-3.6813	-3.6816
Ec	0.01	0.03	0.05	0.07	0.09

Table-4 : Nusselt Number at y=-1

G	I	II	III	IV	V	VI	VII
5	0.8163	0.6864	0.7046	0.7629	0.6599	0.7858	0.8046
10	0.9023	0.7445	0.7346	0.7711	0.6439	0.9398	0.9969
-5	0.6890	0.5943	0.6513	0.7470	0.6952	0.5959	0.5860
-10	0.6405	0.5571	0.6276	0.7392	0.7147	0.5331	0.5178
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3
N	1	1	1	1	2	-0.5	-0.8

Table-5 : Nusselt Number at y=-1

G	I	II	III	IV
5	0.7046	0.6666	0.7964	0.8524
10	0.7346	0.6565	0.9693	0.1158
-5	0.6513	0.6878	0.5895	0.5629
-10	0.6276	0.6990	0.5226	0.4828
S_0	0.5	1	-0.5	-1

Table-6 : Nusselt Number at y=-1

G	I	II	III	IV	V
5	0.7046	0.2900	0.2070	0.1714	0.1516
10	0.7346	0.3011	0.2143	0.1771	0.1565
-5	0.6513	0.2702	0.1939	0.1611	0.1429
-10	0.6276	0.2613	0.1880	0.1565	0.1391
Ec	0.01	0.03	0.05	0.07	0.09

The rate of mass transfer (Sherwood number) at $y=\pm1$ is exhibited in tables 7-12 for different parametric values. It is found that an increase in |G| reduces |Sh| at y=+1 and enhances y=-1. An increase in $Sc\le0.6$ enhances |Sh| and reduces with higher $Sc\ge1.3$ at y=+1 while at y=-1, it reduces for all G. When molecular buoyancy force dominates over the thermal buoyancy force the rate of mass transfer at y=+1 enhances for G>0 and reduces for G<0 irrespective of the directions of the buoyancy forces. At y=-1, |Sh| reduces with N>0 and enhances with |N| for all G (Tables 7 & 10). With respect to S_0 we find that |Sh| reduces with $S_0>0$ and enhances with $S_0<0$ at both the walls (Tables 8 & 11). The variation of Sh with Ec indicates that the rate of mass transfer experiences depreciation with Ec for all G at both the walls (Tables 9 & 12).

Table-7 : Sherwood Number at y=+1

G	I	II	III	IV	V	VI	VII
5	3.7728	17.4696	-11.5863	-6.1787	-11.3793	-11.9800	-12.0743
10	2.4222	10.2283	-13.6880	-6.3854	-12.9066	-15.8187	-16.5198
-5	9.9239	-17.5510	-8.8781	-5.7125	-8.8950	-8.8579	-8.8545
-10	20.0005	-32.3687	-7.9672	-5.4700	-7.9068	-8.0400	-8.0027
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3
N	1	1	1	1	2	-0.5	-0.8

Table-8 : Sherwood Number at y=+1

G	I	II	III	IV
5	-11.5863	-7.4479	-3.3905	-3.9599
10	-13.6880	-8.1117	-3.1442	-3.5482
-5	-8.8781	-6.2919	-3.7169	-4.4169
-10	-7.9672	-5.7998	-3.8195	-4.5349
S_0	0.5	1	-0.5	-1

Table-9 : Sherwood Number at y=+1

G	I	II	III	IV	V
5	-11.5863	-5.9096	-5.4000	-5.2091	-5.1092
10	-13.6880	-6.2341	-5.6432	-5.4248	-5.3111
-5	-8.8781	-5.3424	-4.9605	-4.8142	-4.7369
-10	-7.9672	-5.0977	-4.7654	-4.6368	-4.5686
Ec	0.01	0.03	0.05	0.07	0.09

Table-10 : Sherwood Number at y=-1

G	I	II	III	IV	V	VI	VII
5	42.1115	20.2298	8.1316	5.7718	8.1741	8.0603	8.0448
10	54.3783	35.3593	8.8011	5.9174	8.8425	8.7251	8.7071
-5	12.3623	11.7606	7.0325	5.4331	6.9618	7.1286	7.1466
-10	9.8623	9.9765	6.5838	5.2507	6.4259	6.7807	6.8155
Sc	0.24	0.6	1.3	2.01	1.3	1.3	1.3
N	1	1	1	1	2	-0.5	-0.8

Table-11 : Sherwood Number at y=-1

G	I	II	III	IV
5	8.1316	7.0960	4.6450	5.1165
10	8.8011	7.6097	4.5073	4.8778
-5	7.0325	6.1645	4.7607	5.2984
-10	6.5838	5.7520	4.7686	5.3102
S_0	0.5	1	-0.5	-1

Table-12: Sherwood Number at y=-1

G	I	II	III	IV	V
5	8.1316	5.6353	5.2738	5.1288	5.0506
10	8.8011	5.8962	5.4895	5.3274	5.2403
-5	7.0325	5.1635	4.8768	4.7606	4.6977
-10	6.5838	4.9537	4.6976	4.5933	4.5368
Ec	0.01	0.03	0.05	0.07	0.09

CONCLUSIONS:

The conclusions of the study are as follows:

1. Grashof number (G) has the effect of accelerating the primary velocity (u), the magnitude of the secondary velocity (v) profile and temperature for G<0 and decreases the velocity components u, v and temperature θ for G>0. An increase in |G| enhances |Nu|, reduces |Sh| at y=+1 and reduces |Nu|, enhances |Sh| at y=-1.
2. An increase in the Reynolds number (R) reduces both the velocity components u and v in the entire flow region.
3. The Schmidt number (Sc) decreases u, magnitude of v, while it enhances the temperature and concentration.
4. Increasing values of chemical reaction parameter (K) enhances the magnitude of the velocity components u and v in the flow region.
5. Increase in the Eckert number (Ec) enhances the temperature, concentration and the magnitude of the secondary velocity and reduces the primary velocity.
6. The maginitude of the velocity components u, v and concentration C enhances with increase in S_0, temperature θ enhances and |Sh| reduces with S_0>0 and for S_0<0.

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