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Application of Electromagnetic Field Tensors in Special Relativity Theory

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ABSTRACT
Maxwell's Field Equations (MFE's) for the propagation of electromagnetic waves were found to be invariant under Lorentz transformation (LT) and could be derived using assumptions different from what Einstein used. Here, we start with electromagnetic field tensors, obtain the MFE's and apply the relativistic principle to them. With this approach, Special Relativity Theory (SRT) is reformulated in a simple form without using the LT and it's kinematical contradictions. Our results are in agreement with the existing literature.
KEYWORDS
Maxwell's field equations, Relativity principle, Lorentz and Galilean transformation.

1. INTRODUCTION

Galilean relativity shows that the laws of mechanics are the same for a body at rest and a body moving at constant velocity. Newton also developed his laws of motion and his concept of relativity which states that, "the laws of mechanics must be the same in all inertial frames."[1]. Due to Galileo and Newton, the concept of absolute space became redundant but absolute time was retained, the development of electromagnetic theory in the nineteenth century demonstrated a problem with Newtonian relativity. It became inconceivable to physicist that electromagnetic wave could propagate without a medium (the ether) [2]. But as a consequence of Newtonian relativity, an observer moving through the ether with velocity u would measure the velocity the velocity of a light beam as $(c+u)$, hence the Michelson-Morley experiment showed that no ether (absolute reference frame) existed for electromagnetic phenomena [3].

This result opened a way for a new approach which is Einstein relativity [4]. He postulated that the speed of light is invariant in all inertial frames which lead to a new relationship between space and time (Lorentz transformation). MFE's for the propagation of electromagnetic waves were not invariant under Galilean transformation (GT), but were invariant under LT. For invariant under LT we deal with three quantities: space, time and light speed. In Einstein's approach for deriving the LT, he connected the three quantities through a new velocity v along x [5].

$$v_x{}' = \frac{v_x}{1-\frac{uv_x}{c^2}} \tag{1}$$

He used the basic definition for any velocity in frame S or S' as:

$$v_x = \frac{dx}{dt}; \quad v_x{}' = \frac{dx'}{dt'} \tag{2}$$

And his second postulate for light speed as:

$$\frac{dx}{dt} = \frac{dx'}{dt'} = c \tag{3}$$

This is understood as the "measurement rule."

For the invariant of light speed, time itself has to slow down and space must contract to give almost the same value in Eq.(3) [6]. Thus Einstein introduced relativity of simultaneity to physics. But the interpretation of LT and it's kinematical effects has long been questioned and misunderstood. Today, paradox [7,8], criticism [9,10] still continue to receive attention, as a result many physicist believe that a new interpretation or even a theory alternative to SRT may be needed [11].
In this research we use electromagnetic field tensors to generate MFE's and apply the relativistic principle to obtain the LT equations. The MFE's stands in place of Einstein's relativity of simultaneity. The LT produced by our alternative method is simply a neutral transformation and it's results are the same as that obtained initially [12, 13].

2. ELECTROMAGNETIC FIELD TENSORS

Electromagnetic field tensors is a mathematical objective that describes the electromagnetic field of a physical system[14]. The field tensor was first used after the 4-dimentional tensor formulation of SRT (Minkowski space).

It was known that the 4-vectors for the electromagnetic field $\vec{E}, \vec{B}$ is represented by the scalar and vector potential $\phi, \vec{A}$ as follows:

$$\vec{B} = \nabla \times \vec{A}\,; \quad \vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t} \qquad 4$$

$F_{\mu\nu}$ is defined as an antisymmetric rank-2 tensor which is written in terms of 4-dimentional vectors as[15]:

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}; \quad (\mu\nu = 1,2,3,4) \qquad 5$$

Where

$$x_\nu = (x, y, z, ict); \quad A_\mu = \left(A_x, A_y, A_z, \frac{i\phi}{c} \right) \qquad 6$$

We represent this set of equations in a 4×4 matrix form as;

$$F_{\mu\nu} = \begin{bmatrix} 0 & B_z & -B_y & -\frac{i}{c}E_x \\ -B_z & 0 & B_x & -\frac{i}{c}E_y \\ B_y & -B_x & 0 & -\frac{i}{c}E_z \\ \frac{i}{c}E_x & \frac{i}{c}E_y & \frac{i}{c}E_z & 0 \end{bmatrix} \qquad 7$$

This tensor simplifies and reduces Maxwell equations as 4-vector calculus equations into 2-vector field equations. In magnetostatics, Gauss law for magnetism and Maxwell-Faraday's equation are gotten from:

$$\frac{\partial F_{\alpha\beta}}{\partial x^\mu} + \frac{\partial F_{\beta\mu}}{\partial x^\alpha} + \frac{\partial F_{\mu\alpha}}{\partial x^\beta} = 0; \quad (\alpha, \beta, \mu = 1,2,3,4) \qquad 8$$

And applying the sets $\bar{1}: 1,2,3;\ \bar{2}: 4,2,3;\ \bar{3}: 4,3,1;\ \bar{4}: 4,1,2$, we can obtain the following

For $\bar{1}: 1,2,3;$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \qquad 9$$

This equation can be written as

$$\nabla . B = O \qquad 10$$

For $\bar{2}: 4,2,3;$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t} \qquad 11$$

For $\bar{3}: 4,3,1;$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t} \qquad 12$$

For $\bar{4}: 4,1,2;$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t} \qquad 13$$

Equations(11)-(13) are the x,y,z components respectively, multiplying them by appropriate unit vectors and adding we obtain;

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{14}$$

In electrodynamics, Gauss law for electricity and Maxwell-Ampere's equation are gotten from:

$$\sum_{\nu=1}^{4} \frac{\partial F_{\mu\nu}}{\partial x^{\nu}} = \mu_o J_{\mu}; \quad J_{\mu} = \left(J_x, J_y, J_z, ic\rho\right) \tag{15}$$

According to Einstein summation convection; when the same letter index appears as subscript as well as superscript, then summation will appear over that index. Hence Eq(11) becomes:

$$\frac{\partial F_{\mu 1}}{\partial x^1} + \frac{\partial F_{\mu 2}}{\partial x^2} + \frac{\partial F_{\mu 3}}{\partial x^3} + \frac{\partial F_{\mu 4}}{\partial x^4} = \mu_0 J_{\mu}; \quad (\mu = 1,2,3,4) \tag{16}$$

For $\mu = 1$;

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \frac{1}{c^2}\frac{\partial E_x}{\partial t} + \mu_o J_x \tag{17}$$

For $\mu = 2$;

$$\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial y} = \frac{1}{c^2}\frac{\partial E_y}{\partial t} + \mu_o J_y \tag{18}$$

For $\mu = 3$;

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \frac{1}{c^2}\frac{\partial E_z}{\partial t} + \mu_o J_z \tag{19}$$

Equations (17)-(19) are the x,y,z components respectively, multiplying them by appropriate unit vectors and adding we obtain;

$$\nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t} + \mu_o J \tag{20}$$

For $\mu = 4$;

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = c^2 \mu_0 \rho \tag{21}$$

The last equation can be expressed as

$$\nabla . E = \frac{\rho}{\varepsilon_o} \tag{22}$$

Where $c^2 = \dfrac{1}{\varepsilon_0 \mu_0}$

Equations (10),(14),(20),(22) are the familiar MFE's which describe the electric and magnetic fields arising from distributions of electric charges and currents, and how those fields change with time.
Hence the MFE's in frame S may be expressed as:

$$\left.\begin{aligned} \nabla . E &= \frac{\rho}{\varepsilon_o} \quad ; \quad \nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t} + \mu_o J \\ \nabla . B &= 0 \quad ; \quad \nabla \times E = -\frac{\partial B}{\partial t} \end{aligned}\right\} \tag{23}$$

By applying the relativistic principle to Eq.(23) they will preserve their form in frame S' and they are expressed as follows:

$$\left.\begin{aligned} \nabla'.E' = \frac{\rho'}{\varepsilon_o} \quad &; \quad \nabla' \times B' = \frac{1}{c^2}\frac{\partial E'}{\partial t'} + \mu_o J' \\ \nabla'.B' = 0 \quad &; \quad \nabla' \times E' = -\frac{\partial B'}{\partial t'} \end{aligned}\right\} \qquad 24$$

where ρ, J, ρ', J' are the relativistic charge and current density in frame S and S' respectively. The light speed c is defined in terms of pure electromagnetic constant as $c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$. But we know that ε_o and μ_o have same value in all reference frames so $c' = c$ which implies invariance of light speed.

3. DERIVING THE LORENTZ TRANSFORMATION

Taking the x-component of Eq.(23) and writing Eq.(4) in terms of cathesian component.

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \frac{1}{c^2}\frac{\partial E_x}{\partial t} + \mu_o J_x \qquad 25$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_o} \qquad 26$$

Multiply Eq.(25) by γ and Eq.(26) by $\frac{u\gamma}{c^2}$ and then subtracting we have:

$$\frac{\partial}{\partial y}\gamma\left(B_z - \frac{u}{c^2}E_y\right) - \frac{\partial}{\partial z}\gamma\left(B_y - \frac{u}{c^2}E_z\right) = \frac{\gamma}{c^2}\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)E_x + \mu_o\gamma\left[J_x - u\rho\right] \qquad 27$$

Taking the x-component of Eq.(24)

$$\frac{\partial B_z'}{\partial y'} - \frac{\partial B_y'}{\partial z'} = \frac{1}{c^2}\frac{\partial E_x'}{\partial t'} + \mu_o J_x' \qquad 28$$

Comparing Eq.(27) and Eq.(28)

$$\left.\begin{aligned} & E_x' = E_x\,; \quad B_y' = \gamma\left(B_y - \frac{u}{c^2}E_z\right); \quad B_z' = \gamma\left(B_z - \frac{u}{c^2}E_y\right) \\ & \frac{\partial}{\partial t'} = \frac{\gamma}{c^2}\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right); \qquad \frac{\partial}{\partial y'} = \frac{\partial}{\partial y}; \qquad \frac{\partial}{\partial z'} = \frac{\partial}{\partial z} \\ & J_x' = \gamma\left[J_x - u\rho\right] \end{aligned}\right\} \qquad 29$$

Now multiply Eq.(25) by $u\gamma$ and Eq.(26) by γ and then subtracting we have

$$\gamma\left(\frac{\partial}{\partial x} + \frac{u}{c^2}\frac{\partial}{\partial t}\right)E_x + \frac{\partial}{\partial y}\gamma\left(E_y - uB_z\right) + \frac{\partial}{\partial z}\gamma\left(E_z - uB_y\right) = \frac{\gamma}{\varepsilon_o}\left(\rho - \frac{u}{c^2}J_x\right) \qquad 30$$

Taking the cathesian component of Eq.(24)

$$\frac{\partial E_x'}{\partial x'} + \frac{\partial E_y'}{\partial y'} + \frac{\partial E_z'}{\partial z'} = \frac{\rho'}{\varepsilon_o} \qquad 31$$

Comparing Eq.(30) with Eq.(31) and noting that $E_x' = E_x$, we have:

$$\left.\begin{aligned} & E_y' = \gamma\left(E_y - uB_z\right); \quad E_z' = \gamma\left(E_z - uB_y\right); \\ & \frac{\partial}{\partial x'} = \gamma\left(\frac{\partial}{\partial x} + \frac{u}{c^2}\frac{\partial}{\partial t}\right) \\ & \rho' = \gamma\left(\rho - \frac{u}{c^2}J_x\right) \end{aligned}\right\} \quad 32$$

The Eqs .(29),(32) are the differential Lorentz transformation. We can fix the scalar factor γ by applying the relativistic principle (interchanging primed and the unprimed variables and letting $u = -u$.) on the z-part of Eq.(29).

$$B_z = \gamma\left(B_z' + \frac{u}{c^2}E_y'\right) \quad 33$$

Substituting the values of B_z' and E_y' from Eqs.(29),(32) we have:

$$B_z = \gamma\left[\gamma\left(B_z - \frac{u}{c^2}E_y\right) + \frac{u}{c^2}\gamma\left(E_z - uB_y\right)\right] \quad 34$$

$$B_z = \gamma^2\left(1 - \frac{u^2}{c^2}\right)B_z$$

where

$$\gamma^2\left(1 - \frac{u^2}{c^2}\right) = 1 \text{ or } \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \quad 35$$

Starting now from the y-component of Eq(23) and applying Eq.(35) we have:

$$\frac{\partial B_x}{\partial z} - \gamma^2\left(1 - \frac{u^2}{c^2}\right)\frac{\partial B_z}{\partial x} = \gamma^2\left(1 - \frac{u^2}{c^2}\right)\frac{1}{c^2}\frac{\partial E_y}{\partial t} + \mu_o J_y \quad 36$$

Adding and subtracting $\frac{u}{c^2}\gamma^2\frac{\partial E_y}{\partial x}$ on the L.H.S and $\frac{u}{c^2}\gamma^2\frac{\partial B_z}{\partial t}$ on the R.H.S of Eq.(36) we have:

$$\frac{\partial B_x}{\partial z} - \gamma^2\left(\frac{\partial}{\partial x} + \frac{u}{c^2}\frac{\partial}{\partial t}\right)\left(B_z - \frac{u}{c^2}E_y\right) = \frac{\gamma^2}{c^2}\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\left(E_y - uB_z\right) + \mu_o J_y \quad 37$$

Taking the y-component of Eq.(24)

$$\frac{\partial B_x'}{\partial z'} - \frac{\partial B_z'}{\partial x'} = \frac{1}{c^2}\frac{\partial E_y'}{\partial t'} + \mu_o J_y' \quad 38$$

Comparing Eq.(37) and Eq.(38)

$$B_x' = B_x; \quad J_y' = J_y \quad 39$$

In a similar way starting from the z-component of Eq(23) we obtain:

$$J_z' = J_z \quad 40$$

But the electromagnetic charge density ρ for a particle of charge q is $\rho = q\delta^3\left(x - x'\right)$, and the definition of current density J in frame S is:

$$J = qv\delta^3\left(x - x'\right) = \rho v \quad 41$$

By applying the relativistic principle the current density J in frame S' is:

$$J' = \rho' v' \quad 42$$

Applying Eqs.(41),(42) into Eqs(29),(32) we have

$$v_x{}' = \frac{v_x - u}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \qquad 43$$

Also from Eq.(39) and making use of Eq.(32) we have

$$v_y{}' = \frac{v_y}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \qquad 44$$

Finally from Eq.(40) and making use of Eq(32) we have

$$v_z{}' = \frac{v_z}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \qquad 45$$

The Eqs.(43)-(45) are the relativistic Lorentz velocity transformation.

Many researchers [12, 13, 16] have demonstrated the invariant of MFE's under LT, but have not used our approach. In our approach we begin with electromagnetic field tensors and obtain the MFE's in frames S and S', we apply them using relativistic principle to obtain the differential LT and hence deduce the scalar factor γ. We also obtained the relativistic Lorentz velocity transformation and the results are consistent with [17]. Thus none of Einstein's results changes; it is only the approach that changes.

4. CONCLUSION

Our results show that there is no physical distinction between Lorentz force law (LFL) and MFE's, so MFE's should govern the relativistic electromagnetic phenomena exactly as LFL does. Here we extended the relativistic principle to hold true for MFE's as held by LFL and obtain the same results as [18, 19]. Here, we presented the LT in its differential form, deduced the scalar factor γ and hence obtained the relativistic Lorentz velocity transformation equations.

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