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SINGLE SAMPLING PLAN FOR VARIABLES UNDER MEASUREMENT ERROR FOR NON-NORMAL DISTRIBUTION

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Abstract In this paper, the single sampling plan for variables under measurement error for non-normal distributions represented by the first four terms of an Edgeworth series is studied for known σ case. The effect of using the normal theory sampling plan in a non-normal situation is studied by obtaining the distorted errors of the first and second kind. As one will be interested in having a suitable sampling plan under measurement error for non-normal variables, the values of n and k are determined.

Keywords: Non-Normal distribution, Edgeworth series, Measurement error, Single Sampling Plan

1. Introduction

The most important area for studying the effect of error due to measurement and misclassification is the sampling inspection plan. In general, imbedded within the design of acceptance sampling plans is the assumption that inspection procedure are free from error and normal distribution. In the area of sampling inspection plan by variables very little attention has paid so for to investigate the effect of measurement error. Walsh (1963) studied the effect of measurement error on the characteristic function of a single sampling inspection by variables with one-sided specification limit. The assumption made by them to investigate the effect of measurement error are (i) the error is unbiased and independent of the actual value of the characteristics measured (ii) both the true value of the characteristic measured and the measurement error follows normal distributions (iii) the ratio of the population variance to the error variance has a positive upper bound. Singh (1966) studied the effect of measurement on the operating characteristic function of a single sampling by variables for normal as well non-normal product distributions assuming one-sided specification limit, viz, the upper specification limit. His assumptions were almost same as those of Walsh (1963). Bennett et al. (1974) investigated the effect of Type-I and Type-II inspection errors on the economic design of a single sampling plan by attributes. Case et al. (1975) developed the formula for evaluating the average outgoing quality function when attributes examination is subject to Type-I and Type-II inspection errors. The following hyperbolic remarks of Geary (1947b) strike out: "Normality is a myth, there never was, and never will be normal distribution". Now the question arises whether such optimistic assumption of normality is likely to be seriously misleading under non-normal situations. In other words, to what extent the standard statistical procedures developed for normal universes are applicable, when samples, in real sense, come from other than normal populations. A statistical procedure which is insensitive to departures from the assumptions, which underlie it is called "Robust" an apt term introduced by Box (1953) and now widely used. An early survey of the literature on non-normality and robustness was given by Box and Anderson (1955). In recent years, distribution-free or non-parametric methods have become quite popular because they are readily computable and permit freedom from worry about the classical assumptions of the standard normal theory. It should, however, be pointed out that in cases where classical assumptions hold entirely or even approximately, the analogous standard normal procedures are generally more efficient for detecting departures from the null hypothesis.

2. Sampling Distribution of Observed Mean

Assuming that the true measurement x and the random error of measurement e are additive, we can write the observed measurement X as :

$$X = x + e, \qquad (2.1)$$

where x and e are independent.

We now assume the density function of x to be specified by the first four terms of the Edgeworth series as follows:

$$f(x) = \frac{1}{\sigma_p}\left[\phi\left(\frac{x-\mu}{\sigma_p}\right) - \frac{\lambda_3}{6}\phi^{(3)}\left(\frac{x-\mu}{\sigma_p}\right) + \frac{\lambda_4}{24}\phi^{(4)}\left(\frac{x-\mu}{\sigma_p}\right) + \frac{\lambda_3^2}{72}\phi^{(6)}\left(\frac{x-\mu}{\sigma_p}\right)\right], \qquad (2.2)$$

where,

$$\phi(t) = \frac{1}{\sqrt{2\pi}}\exp\left[-t^2/2\right] \text{ and } \phi^{r}_{(t)} = \frac{d^r}{dt^r}\phi(t).$$

Then the density function of X can be written from equation (2.2) as

$$f(X) = \frac{1}{\sigma_X}\left[\phi\left(\frac{X-\mu}{\sigma_X}\right) - \rho^3\frac{\lambda_3}{6}\phi^{(3)}\left(\frac{X-\mu}{\sigma_X}\right) + \rho^4\frac{\lambda_4}{24}\phi^{(4)}\left(\frac{X-\mu}{\sigma_X}\right) + \rho^6\frac{\lambda_3^2}{72}\phi^{(6)}\left(\frac{X-\mu}{\sigma_X}\right)\right], \qquad (2.3)$$

which is again an Edgeworth series.

The distribution of $\bar{X}$ for the observed samples of size n drawn from the population (2.3) is found by following Gayen (1952) as,

$$f(\bar{X}) = \frac{\sqrt{n}}{\sigma_X}\left[\phi\left(\frac{\bar{X}-\mu}{\sigma_X/\sqrt{n}}\right) - \rho^3\frac{\lambda_3}{6\sqrt{n}}\phi^{(3)}\left(\frac{\bar{X}-\mu}{\sigma_X/\sqrt{n}}\right) + \rho^4\frac{\lambda_4}{24n}\phi^{(4)}\left(\frac{\bar{X}-\mu}{\sigma_X/\sqrt{n}}\right) + \rho^6\frac{\lambda_3^2}{72n}\phi^{(6)}\left(\frac{\bar{X}-\mu}{\sigma_X/\sqrt{n}}\right)\right], \qquad (2.4)$$

where $$r = \frac{\sigma_p}{\sigma_e} = \frac{\rho}{\sqrt{(1-\rho^2)}}.$$

We now examine the effect of the measurement error on the usual test criterion of single sampling plan described below:

Accept the lot if $\bar{x} + k\sigma \leq U,$

And reject otherwise,

for a given set of values of the producer's risk , consumer's risk , Acceptable Quality Level (AQL) p_1 and Lot Tolerance Proportion Defective (LTPD) p_2, the values of n and k are determined by the formulae

$$n = \left[\frac{(K_\alpha + K_\beta)}{(K_{p_1} - K_{p_2})} \right]^2, \tag{2.5}$$

$$k = \left[\frac{(K_\alpha K_{p_2} + K_\beta K_{p_1})}{(K_\alpha + K_\beta)} \right] \tag{2.6}$$

where K_{p_1}, K_{p_2} K and K are determined by the equation

$$\int_{k_\theta}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{t^2}{2} \right] dt = \theta, \tag{2.7}$$

for different choices of fraction defective θ. If θ is the proportion defective in the lot, we know that

$$\frac{U - \mu}{\sigma} = K_\theta. \tag{2.8}$$

The OC function L_p corresponding to a fraction defective p is found out as follows. Under the assumption of normality, a lot having p percent defective items will be accepted, if

$$\bar{x} + k\sigma \le U = \mu + K_p\sigma$$

where K_p is given by equation (2.8) for $\theta = p$. The expression for probability of acceptance under measurement error

$$L_p = \Pr ob.\ [\bar{x} + k\sigma \le \mu + K_p\sigma]$$

is derived by recalling the normality of the statistic ($\bar{x} + k\sigma$). The above probability after some simplification works out to be

$$L_p = \Phi[\sqrt{n}\ \rho\ (K_p - k)], \tag{2.9}$$

where $\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2} \right] dx$.

3. Known σ Plan Under Measurement Error for Non-Normal Situation

Let the true quality characteristic x follows the first four term of an Edgeworth series under measurement error. The OC function of the plan is given by

$$L'_p = \Pr ob\,[\bar{x} + k\sigma \le U = \mu = \mu'],$$

where $\mu' = U - K'_p\ \sigma,$

K'_p being the upper 100p percent point of the standardized Edgeworthian population. i.e., K'_p is given by

$$\int_{-\infty}^{K'_p}\left\{\phi(x)-\frac{\lambda_3}{6}\phi^{(3)}(x)+\frac{\lambda_4}{24}\phi^{(4)}(x)+\frac{\lambda_3^2}{72}\phi^{(6)}(x)\right\}dx=1-p. \qquad (3.1)$$

where $x=\frac{X-\mu}{\sigma}$, $\overline{x}=\frac{(\overline{X}-\mu)}{\sigma/\sqrt{n}}\rho$, $\phi(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$.

Using the distribution of $\overline{x}$ from equation (2.4), the OC function of the known σ plan is obtained as

$$L'_p=\Phi(z)-\frac{\lambda_3}{6\sqrt{n}}\rho^3\phi^{(2)}(z)+\frac{\lambda_4}{24n}\rho^4\phi^{(3)}(z)+\frac{\lambda_3^2}{72n}\rho^6\phi^{(5)}(z). \qquad (3.2)$$

where $z=\sqrt{n}\rho(K'_p-k_0)$.

The equation for determining the value of the plan parameters n and k are

$$L'(p_1)=1-\alpha, \qquad (3.3)$$

and

$$L'(p_2)=\beta. \qquad (3.4)$$

Explicit expression for n and k cannot be obtained under measurement error and non-normal situations. Equation (3.3) and (3.4) can however, be solved numerically. If n_o and k_o are the initial solutions than the improved solution can be obtained as ($n_0+\delta n_0$) and ($k_0+\delta k_0$) where δn_0 and δk_0 are the solution of linear equations

$$A(p_1)\,\delta n_0-B(p_1)\,\delta k_0=1-\alpha-c(p_1) \qquad (3.5)$$

and

$$A(p_2)\,\delta n_0-B(p_2)\,\delta k_0=\beta-c(p_2), \qquad (3.6)$$

where

$$A(p)=\frac{\rho}{2n_0}[z_0\Phi(z_0)-\frac{\lambda_3}{6\sqrt{n_0}}\rho^3\,(z_0\phi^{(3)}(z_0)-\phi^{(2)}(z_0))+\frac{\lambda_4}{24n_0}\rho^4$$

$$(z_0\phi^{(4)}(z_0)-2\phi^{(3)}(z_0))+\frac{\lambda_3^2}{72n_0}\rho^6\,(z_0\phi^{(6)}(z_0)-2\phi^{(5)}(z_0)] \qquad (3.7)$$

$$B(p)=\sqrt{n_0}\rho\left[\Phi(z_0)-\frac{\lambda_3}{6\sqrt{n_0}}\rho^3\phi^{(3)}(z_0)+\frac{\lambda_4}{24n_0}\rho^4\phi^{(4)}(z_0)\right.$$

$$+\frac{\lambda_3^2}{72n_0}\rho^4\phi^{(6)}(z_0)\Bigg] \qquad (3.8)$$

$$C(p)=\Phi(z_0)-\frac{\lambda_3}{6\sqrt{n_0}}\rho^3\,\phi^{(2)}(z_0)+\frac{\lambda_4}{24n_0}\rho^4\phi^{(3)}(z_0)$$

$$+\frac{\lambda_3^2}{72n_0}\rho^6\phi^{(5)}(z_0) \qquad (3.9)$$

and $z_0=\sqrt{n_0}\,\rho\,(K'_p-k_0)$.

The required value of n and k can be obtained by taking the normal theory values as the initial solution and repeating the process of interaction for equation (3.5) and (3.6) till the desired accuracy is obtained.

4.**Discussion of Numerical Results and Conclusions**

For the purpose of illustrating the effect of measurement error and non- normality on the error of the first and second kind and the plan parameters n and k, we have determined the values of these quantities for few chosen sets of values of p_1, p_2, α β, λ_3, λ_4 and for different error sizes. The values of n and k are determined from the equations (2.5) and (2.6). The actual error of the first and second kind are given by

$$\alpha'=1-L'(p_1)\text{ and }\beta'=L'(p_2).$$

The actual errors of the first kind when normal theory known σ plan is used under measurement error and non-normal situations are determined and presented in the Table (1). As is evident from the Table, for leptokurtic, platykurtic population under different error sizes the marked difference is seen in α' and β'. The effect of skewness, kurtosis and measurement error are serious. For few non-normal situations and measurement error the value of the plan parameters n and k are given in Table (2). The values of n are rounded up and the values of k are given up to 4 decimal places which is correct up to the third place of decimal. As can be seen from the Table the negative skewness and negative kurtosis increases n where as the positive skewness and positive kurtosis decreases the value of n. The value of k is also considerably affected by measurement error and the non-normality of the population.

It may inferred that the use of normal theory sampling plan is not valid for the non-normal situations and measurement error problems. Even when there is slight departure from normality and different error sizes it is advisable to take into account the measurement error and the non-normality of the parent population while choosing the sampling plan parameters n and k.

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Table 1

Value of α' and β' (Underline) for known σ plan

$\alpha = 0.05, \beta = 0.10, p_1 = 0.05, p_2 = 0.30$

λ3/λ4	r = 2		r = 4		r = 6		r = ∞	
	0	**0.5**	**0**	**0.5**	**0**	**0.5**	**0**	**0.5**
0.0	0.0695	0.039	0.0537	0.0291	0.0506	0.0274	0.0969	0.0258
	0.1231	0.1034	0.1033	0.0867	0.1003	0.084	0.0102	0.0814
-0.5	0.0659	0.0373	0.0507	0.027	0.0486	0.0253	0.0447	0.0238
	0.1369	0.1147	0.118	0.0981	0.1141	0.0934	0.1103	0.0904
0.05	0.0695	0.0398	0.0537	0.0291	0.0506	0.0274	0.0485	0.0258
	0.1232	0.1019	0.104	0.0854	0.1004	0.0827	0.097	0.0800
1.5	0.0778	0.0458	0.0621	0.0349	0.0585	0.0331	0.0563	0.0313
	0.0871	0.074	0.0708	0.059	0.0668	0.0558	0.0642	0.0538
2.5	0.0825	0.0491	0.0673	0.0386	0.0636	0.0367	0.0613	0.0349
	0.0672	0.0576	0.0513	0.044	0.0490	0.0415	0.0459	0.0390

Table 2

Values of n (integer) and k for known σ plan

$\alpha = 0.05$, $\beta = 0.10$, $p_1 = 0.05$, $p_2 = 0.30$

λ3/λ4	r = 2		r = 4		r = 6		r = ∞	
	0	0.5	0	0.5	0	0.5	0	0.5
0.0	7	6	8	7	7	8	7	6
	1.0090	0.9610	1.0110	1.0244	1.0098	0.9790	1.0115	0.9711
-0.5	7	6	9	8	8	7	8	7
	1.0368	0.9919	1.0786	1.0533	1.0500	1.0130	1.0429	1.0013
0.05	7	6	8	7	7	6	7	6
	1.0098	0.9564	1.0551	1.0201	1.0226	0.9752	1.016	0.9670
1.5	5	5	8	5	5	3	5	11
	0.9070	0.9338	0.9759	0.9016	0.9307	0.8209	0.9169	0.8037
2.5	2	4	5	3	3	1	2	1
	0.8033	0.7007	0.9101	0.8365	0.8387	0.7426	0.8265	0.7229