Viscous Dissipation Effect On The MHD Flow Of A Third Grade Fluid Down: an Inclined Plane With Ohmic Heating

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Abstract

The thin film flow and heat transfer problem of a third grade fluid down an inclined plane is investigated. The fluid is incompressible and electrically conducting in the presence of a uniform magnetic field. The non-linear equation governing the flow and heat transfer are solved for the velocity and temperature profile by employing the regular perturbation technique as well as homotopy perturbation method and the results are presented graphically. The effect of magnetic parameter and Brinkman number are analyzed for velocity and temperature profile. It is noticed that increase in magnetic parameter reduced the velocity of the fluid and increases the temperature profile. Also, increase in Brinkman number increases the temperature profile.

Keywords: Third grade fluid, Brinkman number, Perturbation method, Homotopy perturbation method, Magnetohydrodynamics.

1. Introduction

The flow of non-Newtonian fluid has received attention considerably in the past decade due to its applications in science, engineering and Technology. Example of these application can be found in wire coating, ink-jet printing, materials manufactured by extraction process especially in polymer processing, micro fluids, geological flows within the earth's mantle as well as the flow of synovial fluid in human joints. One can make references to much literature on hydrodynamics visco-elastic flow with diverse physical effects. However, the works on the visco-elastic thin film Magnetohydrodynamics (MHD) fluid flow with heat transfer is not large.

For a list of references concerning this work, we refer to the articles by Makinde (2009), Hayat et al (2008), Chen et al (2003), Hsiao (2011) and Khan and Mahmood (2012).

In the present study, we extend the work of Siddiqui et al (2008) and investigate thin film flow of a third grade MHD fluid in an inclined plane. The heat transfer analysis is also carried out. The non-linear governing equation of this problem were solve using the two powerful semi analytic techniques namely, traditional regular perturbation method (Nayfah 1979) and homotopy perturbation method (He 2003 & 2009) which does not require the existence of small or large parameter. The effect of various parameter of interest is seen and discussed through graphs.

The organisations of this work are as follows. In section 2, we formulate the problem of thin film MHD fluid of third grade flowing in an inclined plane with no slip conditions. In section 3, we employ traditional perturbation method to obtain the solution of the problem. In section 4, the problem is again solved using the homotopy perturbation method. In section 5, graphical results are presented and discussed with respect to various parameters embedded in the system. And in section 6, some concluding remarks are given.

2. Problem Formulation

The fundamental equations governing the MHD flow of an incompressible electrically conducting fluid are the field equation:

$$\nabla \cdot v = 0 \tag{1}$$

$$\rho \frac{Dv}{Dt} = -\nabla p + div\,\tau + J \times B + \rho f \tag{2}$$

where ρ is the density f the fluid, v is the fluid velocity, B is the magnetic induction so that

$$B = B_0 + b \tag{3}$$

and

$$J = \sigma(E + v \times B) \tag{4}$$

is the current density, σ is the electrical conductivity, E is the electrical filed which is not considered (i.e $E = 0$), B_0 and b are applied and induced magnetic filed respectively, D/Dt denote the material derivative, p is the pressure, f is the external body force and T is the Cauchy stress tensor which for a third grade fluid satisfies the constitutive equation

$$\tau = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 \left(A_1 A_2 + A_2 A_1\right) + \beta_3 (trA_1^2) A_1 \tag{5}$$

$$A_n = \frac{DA_{n-1}}{Dt} + A_{n-1} \nabla v + (\nabla v)^{\perp} A_{n-1}, \qquad n \geq 1 \tag{6}$$

where pI is the isotropic stress due to constraint incompressibility, μ is the dynamics viscosity, $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ are the material constants; $\perp$ indicate the matrix transpose, A_1, A_2, A_3 are the first three Rivlin-Ericken tensors and $A_0 = I$ is the identity tensor.

We consider a thin film of an incompressible MHD fluid of a third grade flowing in an inclined plane. The ambient air is assumed stationary so that the flow is due to gravity alone.

By neglect the surface tension of the fluid and the film is of uniform thickness d , we seek a velocity field of the form

$$v = [u(y), 0, 0,] \tag{7}$$

In the absence of modified presence gradient, equation (1)-(4) along with equation (5)-(7) yields

$$\frac{\partial^2 u}{\partial y^2} + 6\beta \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + K - Mu = 0 \tag{8}$$

Subject to the boundary condition

$$u(y) = 0 \quad \text{at} \quad y = 0 \tag{9}$$

$$\frac{du}{dy} = 0 \quad \text{at} \quad y = 1 \tag{10}$$

where

$\beta = \dfrac{(\beta_2 + \beta_3)\mu}{d^4}$ is third grade fluid parameter

$K = f_1 \sin\alpha$ while $f_1 = \dfrac{d^3 \rho g}{\mu}$ is the gravitational parameter.

$M = \dfrac{d^2 \sigma B_0^2}{\mu}$ is the magnetic parameter.

Equation (9) is the no slip condition and equation (10) comes from $T_{yx} = 0$ at $y = 1$.

In the sequence, we take $\varepsilon = \beta$ and solve the system of equation (8)-(10) by the traditional perturbation method and also by the homotopy perturbation.

Heat Transfer Analysis

The thermal boundary layer equation for the thermodynamically compatible third grade fluid with viscous dissipation, work done due to deformation and joule heating is given as

$$k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 + 2(\beta_2+\beta_3)\left(\frac{du}{dy}\right)^4 + \sigma B_0^2 u^2 = 0 \tag{11}$$

with boundary condition

$$T(y) = T_w \quad \text{at} \quad y = 0 \tag{12}$$

$$T(y) = T_d \quad \text{at} \quad y = d \tag{13}$$

where k is the thermal conductivity, T is the temperature, and T_d is the temperature of the ambient fluid.

Introducing the following dimensionless variable

$$\bar{u} = \frac{ud}{\mu},\ \bar{T} = \frac{T - T_1}{T_2 - T_1} \tag{14}$$

where $T_1 = T_w$ and $T_2 = T_d$

The system of equation (11)-(13) and (14) after dropping the caps take the following form:

$$\frac{d^2T}{dy^2} + B_r\left(\frac{du}{dy}\right)^2 + 2B_r\beta\left(\frac{du}{dy}\right)^4 + B_r M u^2 = 0 \tag{15}$$

$$T(y) = 0 \quad \text{at} \quad y = 0 \tag{16}$$

$$T(y) = 1 \quad \text{at} \quad y = 1 \tag{17}$$

where $B_r = \dfrac{\mu^3}{kd^2(T_2 - T_1)}$ is the Brinkman number

$M = \dfrac{\sigma B_0^2 d^2}{\mu}$ is the magnetic parameter

$\beta = \dfrac{(\beta_2 + \beta_3)\mu}{d^4}$ is third grade fluid parameter

Again, in the sequence, we take $\varepsilon = \beta$ and solve the system of equation (15)-(17) by the traditional perturbation method and also by the homotopy perturbation.

3. Solution of the problem

Solution by regular perturbation method

Let us assume ε as a small parameter in order to solve equation (8) by this method, we expand

$$u(y,\varepsilon) = u_0(y) + \varepsilon u_1(y) + \varepsilon^2 u_2(y) + \cdots \tag{18}$$

Substituting equation (18) into equation (8) and rearranging based on powers of $\varepsilon -$terms.

We obtain the following problems of different order with their boundary conditions:

Zeroth – order problem

$$\frac{d^2u_0}{dy^2} - Mu + K = 0 \tag{19}$$

$$u_0(y) = 0 \text{ at } y = 0 \tag{20}$$

$$\frac{du_0}{dy} = 0 \quad \text{at} \quad \text{y=1} \tag{21}$$

First order problem

$$\frac{d^2u_1}{dy^2} + 6\frac{d^2u_0}{dy^2}\left(\frac{du_0}{dy}\right)^2 - Mu_1 = 0 \tag{22}$$

$$u_1(y) = 0 \text{ at } y = 0 \tag{23}$$

$$\frac{du_1}{dy} = 0 \quad \text{at} \quad \text{y=1} \tag{24}$$

Second order problem

$$\frac{d^2u_2}{dy^2} + 6\frac{d^2u_1}{dy^2}\left(\frac{du_0}{dy}\right)^2 + 12\frac{du_0}{dy}\frac{du_1}{dy}\frac{d^2u_0}{dy^2} - Mu_2 = 0 \tag{25}$$

$$u_2(y) = 0 \text{ at } y = 0 \tag{26}$$

$$\frac{du_2}{dy} = 0 \quad \text{at} \quad \text{y=1} \tag{27}$$

Now we solve this sequence of problems and generate the series solution

Zeroth – order problem solution

$$u_0(y) = c1e^{y\sqrt{M}} + c2e^{-y\sqrt{M}} + \frac{K}{M} \tag{28}$$

First order problem solution

$$u_1(y) = (c37 + c7y)e^{y\sqrt{M}} + (c38 + c6y)e^{-y\sqrt{M}} - c9e^{3y\sqrt{M}} - c10e^{-3y\sqrt{M}} \tag{29}$$

Second – order problem solution

$$\begin{aligned} u_2(y) = {} & (c35 + c26y + c27y^2)e^{y\sqrt{M}} + (c36 + c23y + c24y^2)e^{-y\sqrt{M}} \\ & + (c31 + c32y)e^{3y\sqrt{M}} + (c29 + c30y)e^{-3y\sqrt{M}} + c34e^{5y\sqrt{M}} + c35e^{-5y\sqrt{M}} \end{aligned} \tag{30}$$

Next, we find the approximate solution for temperature distribution, for which we write

$$T(y,\varepsilon)=T_0(y)+\varepsilon T_1(y)+\varepsilon^2 T_2(y)+\cdots\cdots \tag{31}$$

Substituting equation (31) into equation (15) –(17) and collecting the same power of ε, yields different order problems.

Zeroth –order problem with boundary conditions:

$$\frac{d^2T_0}{dy^2}+B_r\left[\left(\frac{du_0}{dy}\right)^2+Mu_0\right]=0 \tag{32}$$

$$T_0(0)=0 \quad \text{at} \quad y=0 \tag{33}$$

$$T_0(1)=1 \quad \text{at} \quad y=1 \tag{34}$$

with the solution

$$T_0(y)=c42e^{y\sqrt{M}}+c43e^{-y\sqrt{M}}+c44e^{2y\sqrt{M}}+c45e^{-2y\sqrt{M}}+c46e^{4y\sqrt{M}} +c47e^{-4y\sqrt{M}}+c39y^2+c40y+c41 \tag{35}$$

First –order problem with boundary conditions:

$$\frac{d^2T_1}{dy^2}+2B_r\left[\frac{du_0}{dy}\frac{du_1}{dy}+\left(\frac{du_0}{dy}\right)^4+Mu_0u_1\right]=0 \tag{36}$$

$$T_1(0)=0 \quad \text{at} \quad y=0 \tag{37}$$

$$T_1(1)=1 \quad \text{at} \quad y=1 \tag{38}$$

solution

$$\begin{aligned} T_1(y)=&\left(\frac{2c51}{M^{\frac{3}{2}}}-\frac{c48}{M}-\frac{c51\,y}{M}\right)e^{y\sqrt{M}}+\left(-\frac{c49}{M^{\frac{3}{2}}}-\frac{2c50}{M^{\frac{3}{2}}}-\frac{c50\,y}{M}\right)e^{-y\sqrt{M}} \\ &+\left(-\frac{c52}{4M}+\frac{c54}{4M^{\frac{3}{2}}}-\frac{c54\,y}{4M}\right)e^{2y\sqrt{M}}+\left(-\frac{c55}{4M^{\frac{3}{2}}}-\frac{c53}{4M}-\frac{c55\,y}{4M}\right)e^{-2y\sqrt{M}} \\ &+\left(-\frac{c56}{9M}\right)e^{3y\sqrt{M}}++\left(-\frac{c57}{9M}\right)e^{-3y\sqrt{M}}+\left(-\frac{c58}{16M}\right)e^{4y\sqrt{M}}+\left(-\frac{c59}{16M}\right)e^{-4y\sqrt{M}} \\ &-\frac{c60\,y^2}{2}+c61\,y+c62 \end{aligned} \tag{39}$$

First –order problem with boundary conditions:

$$\frac{d^2T_2}{dy^2}+B_r\left[2\frac{du_0}{dy}\frac{du_2}{dy}+\left(\frac{du_1}{dy}\right)^2+8\left(\frac{du_0}{dy}\right)^3\frac{du_1}{dy}+2Mu_0u_2+Mu_1^2\right]=0 \tag{40}$$

$$T_2(0)=0 \quad \text{at} \quad y=0 \tag{41}$$

$$T_2(1)=1 \quad \text{at} \quad y=1 \tag{42}$$

with the solution

$$\begin{aligned}T_2(y)=&\left(c65+c66y+c67y^2\right)e^{y\sqrt{M}}+\left(c68+c69y+c70y^2\right)e^{-y\sqrt{M}}\\&+\left(c71+c72y+c73y^2\right)e^{2y\sqrt{M}}+\left(c74+c75y+c76y^2\right)e^{-2y\sqrt{M}}\\&+\left(c77+c78y\right)e^{3y\sqrt{M}}+\left(c79+c80y\right)e^{-3y\sqrt{M}}+\left(c81+c82y\right)e^{4y\sqrt{M}}\\&+\left(c83+c84y\right)e^{-4y\sqrt{M}}+c85e^{5y\sqrt{M}}+c86e^{-5y\sqrt{M}}+c87e^{6y\sqrt{M}}+c88e^{-6y\sqrt{M}}\\&+c63y+c89y^2+c90y^3+c64\end{aligned} \tag{43}$$

4. Solution by Homotopy perturbation method

The problem under consideration i.e equations (8)-(10) can be written as

$$L(v)-L(u_0)+qL(u_0)+q\left[6\frac{\beta}{\mu}\left(\frac{dv}{dy}\right)^2\frac{d^2v}{dy}-Mv+K\right]=0 \tag{44}$$

where $L=\dfrac{d^2}{dy^2}$ and equation (28) is the initial guess approximation.

Let $v=v_0+qv_1+q^2v_2+\cdots$ (45)

Substitute equation (45) into (44) and equating the coefficient of like powers of q, we have

Zeroth –order problem with boundary conditions:

$$\frac{d^2v_0}{dy^2}-\frac{d^2u_0}{dy^2}=0 \tag{46}$$

$$v_0(y)=0 \quad \text{at} \quad y=0 \tag{47}$$

$$\frac{dv_0}{dy}=0 \quad \text{at} \quad y=1 \tag{47}$$

with the solution

$$v_0(y)=c1e^{y\sqrt{M}}+c2e^{-y\sqrt{M}}+c91y-c1-c2 \tag{48}$$

First –order problem with boundary conditions:

$$\frac{d^2v_1}{dy^2}+\frac{d^2u_0}{dy^2}+6\frac{\beta}{\mu}\frac{d^2v_0}{dy^2}\left(\frac{dv_0}{dy}\right)^2-Mv_0+K=0 \quad (49)$$

$$v_1(y)=0 \quad \text{at} \quad y=0 \quad (47)$$

$$\frac{dv_1}{dy}=0 \quad \text{at} \quad y=1 \quad (48)$$

with the solution

$$\begin{aligned} v_1(y)=&\,c64e^{y\sqrt{M}}+c65e^{-y\sqrt{M}}+c66e^{2y\sqrt{M}}+c67e^{-2y\sqrt{M}}+c68e^{3y\sqrt{M}}+c69e^{-3y\sqrt{M}}\\ &+c70y^3+c71y^2+c72y+c73 \end{aligned} \quad (49)$$

Second –order problem with boundary conditions:

$$\frac{d^2v_1}{dy^2}+6\frac{\beta}{\mu}\left[2\frac{d^2v_0}{dy^2}\frac{dv_0}{dy}\frac{dv_1}{dy}+\left(\frac{dv_0}{dy}\right)^2\frac{d^2v_1}{dy^2}\right]-Mv_1=0 \quad (50)$$

$$v_2(y)=0 \quad \text{at} \quad y=0 \quad (51)$$

$$\frac{dv_2}{dy}=0 \quad \text{at} \quad y=1 \quad (52)$$

with the solution

$$\begin{aligned} v2(y)=&\left(c92+c93y+c94y^2\right)e^{y\sqrt{M}}+\left(c97+c95y+c96y^2\right)e^{-y\sqrt{M}}\\ &+\left(c100+c98y+c99y^2\right)e^{2y\sqrt{M}}+\left(c103+c101y+c102y^2\right)e^{-2y\sqrt{M}}\\ &+c104e^{3y\sqrt{M}}+c105e^{-3y\sqrt{M}}+c106e^{4y\sqrt{M}}+c107e^{-4y\sqrt{M}}+c108e^{5y\sqrt{M}}\\ &+c109e^{-5y\sqrt{M}}+c113y^5+c112y^4+c110y^3+c111y^2+c74y+c114 \end{aligned} \quad (53)$$

Next, we find approximate solution of temperature profile using homotopy perturbation by written equation (15) as

$$L(\overline{T})-L(\theta_0)+qL(\theta_0)+q\left[B_r\left(\frac{dv}{dy}\right)^2+2B_r\beta\left(\frac{dv}{dy}\right)^4+B_rMv^2\right]=0 \quad (54)$$

Let $\overline{T}=\overline{T}_0+q\overline{T}_1+q^2\overline{T}_2+\cdots\cdots$ (55)

Substitute equation (55) into (54) and equating the coefficient of like powers of q, we have

Zeroth –order problem with boundary conditions:

$$\frac{d^2\overline{T}_0}{dy^2}-\frac{d^2\theta_0}{dy^2}=0 \tag{56}$$

$$\overline{T}_0(0)=0 \quad \text{at} \quad y=0 \tag{57}$$
$$\overline{T}_0(1)=1 \quad \text{at} \quad y=1 \tag{58}$$

Using equation (35) as θ_0 to serve as initial guess approximation, we have

$$\begin{aligned}\overline{T}_0(y)=c115e^{y\sqrt{M}}+c116e^{-y\sqrt{M}}+c117e^{2y\sqrt{M}}+c118e^{-2y\sqrt{M}}+c119e^{4y\sqrt{M}}+c120e^{-4y\sqrt{M}}\\+c122y^2+c121y+c123\end{aligned} \tag{59}$$

First-order problem with boundary conditions:

$$\frac{d^2\overline{T}_1}{dy^2}-\frac{d^2\theta_0}{dy^2}+B_r\left(\frac{dv_0}{dy}\right)^2+B_rMv_0^2=0 \tag{60}$$

$$\overline{T}_1(0)=0 \quad \text{at} \quad y=0 \tag{61}$$

$$\overline{T}_1(1)=1 \quad \text{at} \quad y=1 \tag{62}$$

with the solution

$$\begin{aligned}\overline{T}_1(y)=(c125+c124y)e^{y\sqrt{M}}+(c127+c126y)e^{-y\sqrt{M}}+c128e^{2y\sqrt{M}}+c129e^{-2y\sqrt{M}}\\+c130e^{4y\sqrt{M}}+c131e^{-4y\sqrt{M}}+c132y^4+c133y^3+c134y^2+c70y+c77\end{aligned} \tag{63}$$

Second order problem with boundary conditions:

$$\frac{d^2\overline{T}}{dy^2}+2B_r\frac{dv_0}{dy}\frac{dv_1}{dy}+8B_r\beta\left(\frac{dv_0}{dy}\right)^3\frac{dv_1}{dy}+2B_rMv_0v_1=0 \tag{64}$$

$$\overline{T}_2(0)=0 \quad \text{at} \quad y=0 \tag{65}$$

$$\overline{T}_2(1)=1 \quad \text{at} \quad y=1 \tag{66}$$

with solution

$$\begin{aligned}\overline{T}_2(y) = &\left(c135 + c136y + c137y^2 + c138y^3\right)e^{y\sqrt{M}} \\ &+\left(c139 + c140y + c141y^2 + c142y^3\right)e^{-y\sqrt{M}} \\ &+\left(c143 + c144y + c145y^2\right)e^{2y\sqrt{M}} \\ &+\left(c146 + c147y + c148y^2\right)e^{-2y\sqrt{M}} \\ &+\left(c149 + c150y + c151y^2\right)e^{3y\sqrt{M}} + \left(c152 + c153y + c154y^2\right)e^{-3y\sqrt{M}} \\ &+ c155e^{4y\sqrt{M}} + c156e^{-4y\sqrt{M}} + c157e^{5y\sqrt{M}} + c158e^{-5y\sqrt{M}} + c159e^{6y\sqrt{M}} \\ &+ c160e^{-6y\sqrt{M}} + c165y^6 + c164y^5 + c163y^4 + c162y^3 + c161y^2 + c78y + c79\end{aligned} \tag{67}$$

5. Results And Discussion

The solution for the velocity and temperature distribution of a third grade MHD fluid flow through an inclined plane with heat transfer was approximated analytically. The governing non-linear ordinary differential equations are solved using the regular perturbation method as well as homotopy perturbation method and the results are compared. It is found in figure 1 that results obtained from the two methods are in good agreement for small values of β. This shows that the two methods have the same solution, hence for further study, we discuss the effects of various physical parameters on the velocity and temperature distribution when K is positive and negative (i.e. $K = +1$ or $K = -1$) by placing their graphs side by side in figure 2-6. It is found from the figures that the solutions obtained by the regular perturbation and homotopy perturbation method are same for various values of magnetic parameter (M) and Brinkman number (B_r). Figure 2-3 illustrates the effect of magnetic parameter M on the velocity distribution of the fluid when K is positive and negative. It is noticed from the figures that the velocity distribution decrease when K is positive and increases when K is negative as magnetic parameter increases. The effects of increasing values of magnetic parameter are to reduce the fluid velocity when K is positive as well as increase the fluid velocity when K is negative and thereby reducing and increasing respectively the boundary layer thickness.

Therefore, the rate of transport is considerably reduce with increase of magnetic field parameter when the fluid is flowing down an inclined plane and increasing with increase of magnetic parameter when the fluid is flowing up an inclined plane. This clearly shows that the transverse magnetic field opposes the transverse phenomenon.

Figure 4-5 shows the graphical representation of temperature distribution $T(y)$ with y for various values of physical parameters. It is observed from the figures that as magnetic parameter increases, the temperature distribution increases and decreases when the value of K is positive and negative respectively by keeping B_r and β fixed.

Also, from figure 6-7 we observe that the temperature distribution is lower for negative values of K and higher for positive values of K when keeping magnetic and viscoelastic parameter are fixed with increase in the value of B_r

6. Conclusion

The steady laminar flow and heat transfer of an incompressible and electrically conducting third grade fluid through an inclined plane subjected to a transverse uniform magnetic field have examined. The effects of physical parameters such as magnetic field parameter, viscoelastic parameter and Brinkman number on velocity and temperature distribution is noticed. Analytical expression for both the velocity and temperature distribution are obtained.

The following conclusions can be drawn from the computed results.

(i) The result for fluid flow through an inclined plane obtained by Regular perturbation and homotopy perturbation method are the same.

(ii). The effects of magnetic field parameter is to decrease velocity distribution and increase temperature distribution in the boundary layer as the fluid is flowing down an inclined plane.

(iii) As the fluid move up through an inclined plane, the velocity distribution increases and temperature distribution decreases when magnetic field parameter increases.

(iv) The temperature distribution is decreases as a result of increases in the values of B_r when the fluid is flowing up an inclined plane.

(v) The temperature distribution is increases as a result of increasing in the values of B_r when the fluid is flowing down an inclined plane.

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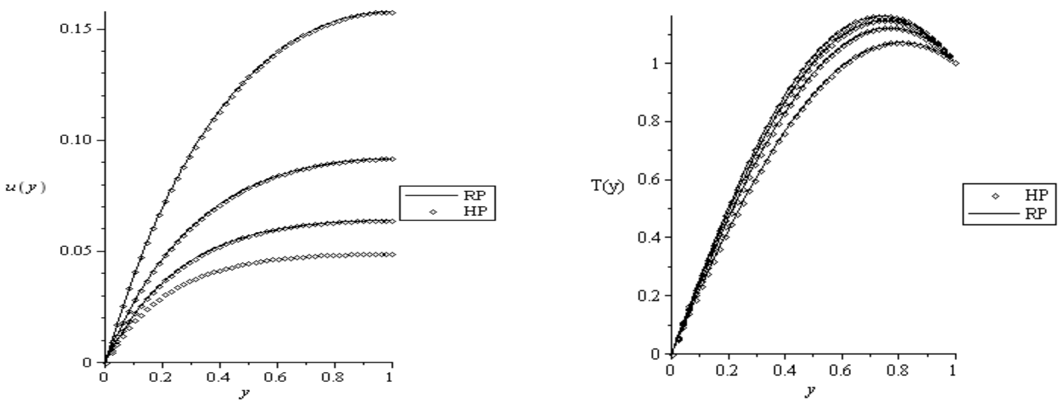

Figure 1: Comparison of velocity and temperature profile using Regular Perturbation (RP) and Homotopy Perturbation (HP) for various values of $M = 5.0, 10.0, 15.0, 20.0$ when $K = +1$, $d = 1.0$, $Br = 5.0$ and $\beta = 0.001$

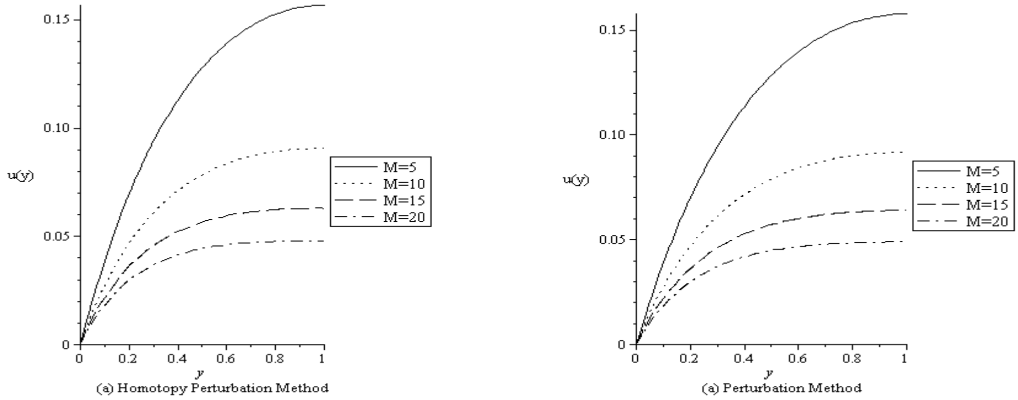

Figure 2: Velocity profile for different values of M when K= +1, $d = 1.0$ and $\beta = 0.001$

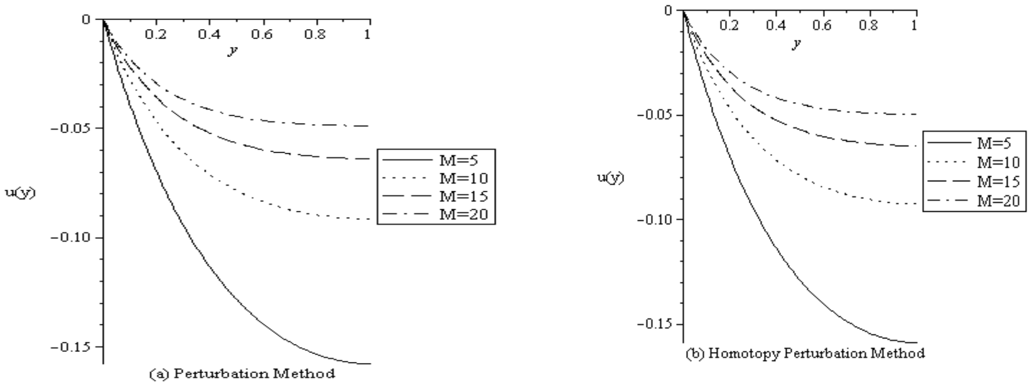

Figure 3: Velocity profile for different values of M when K= -1, $d = 1$ and $\beta = 0.001$

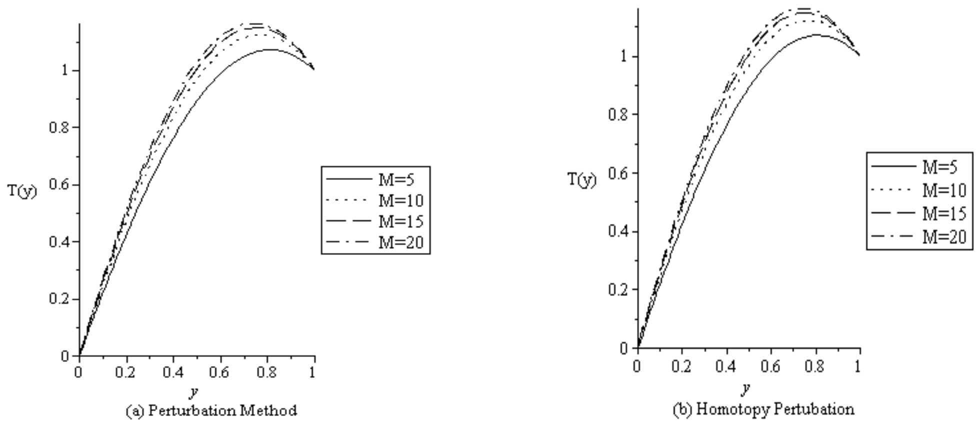

Figure 4: Temperature profile for different values of M when K= +1, $d = 1$ $Br = 5.0$ and $\beta = 0.001$

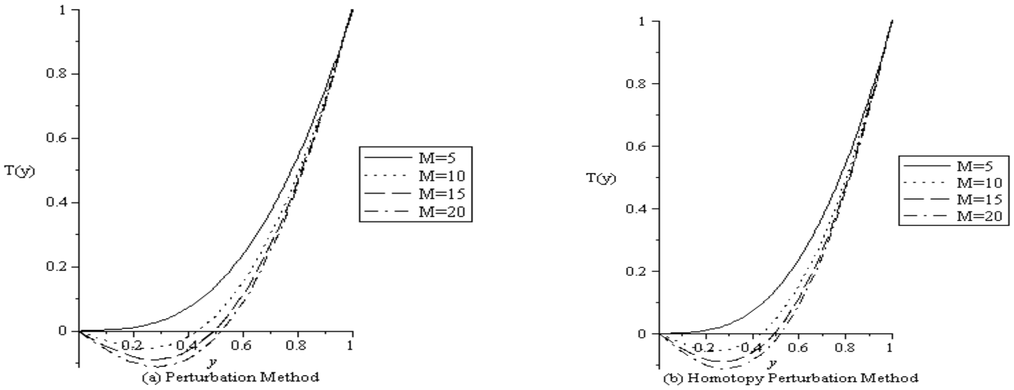

Figure 5: Temperature profile for different values of M when K= -1, $d = 1.0$, $Br = 5.0$ and $\beta = 0.001$

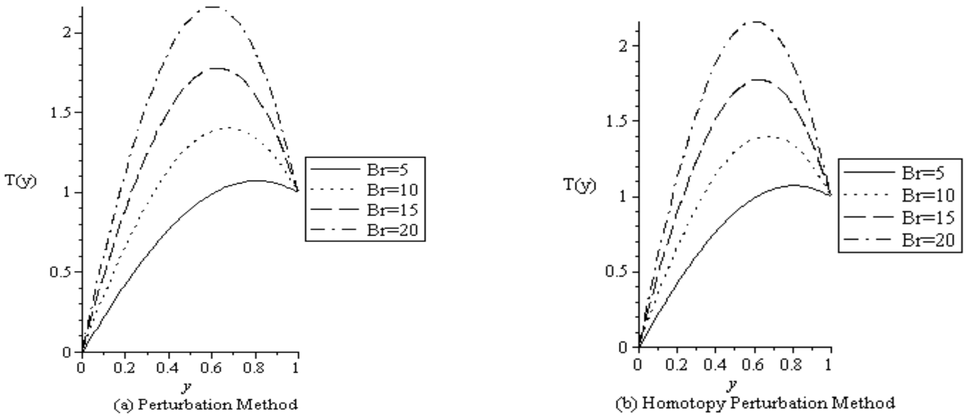

Figure 6: Temperature profile for different values of Br when M=5 and K=+1, $d = 1$ and $\beta = 0.001$

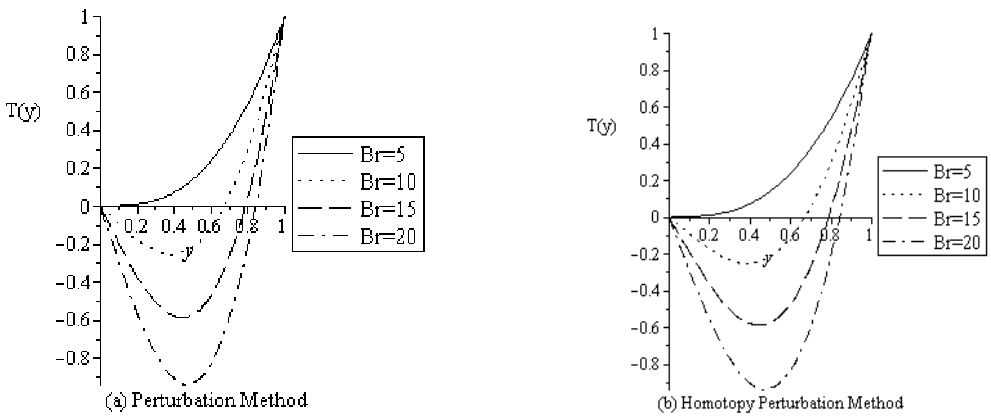

Figure 7: Temperature profile for different values of Br when M=5 and K= -1, $d = 1$ and $\beta = 0.001$

Appendix

$$c_1 = -\frac{e^{-\sqrt{M}}k}{M\left(e^{\sqrt{M}} + e^{-\sqrt{M}}\right)}$$

$$c_2 = -\frac{Ke^{\sqrt{M}}}{M\left(e^{\sqrt{M}} + e^{-\sqrt{M}}\right)}$$

$$c_3 = -\frac{3}{4}\frac{1}{\sqrt{M}\left(e^{\sqrt{M}} + e^{-\sqrt{M}}\right)}\Big(4M^2 e^{-3\sqrt{M}} c_1 c_2^2 e^{2\sqrt{M}} - M^{3/2} e^{-\sqrt{M}} c_2^3 - 2M^{3/2} e^{-\sqrt{M}} c_2^2 c_1$$
$$- 2M^{3/2} e^{-\sqrt{M}} c_1^2 c_2 - M^{3/2} e^{-\sqrt{M}} c_1^3 + 2M^{3/2} e^{-3\sqrt{M}} c_1^2 c_2 e^{4\sqrt{M}} - 2M^{3/2} e^{-3\sqrt{M}} c_1 c_2^2 e^{2\sqrt{M}}$$
$$+ 3M^{3/2} e^{-3\sqrt{M}} c_2^3 + 4M^2 e^{-3\sqrt{M}} c_1^2 c_2 e^{4\sqrt{M}} - 3M^{3/2} e^{-3\sqrt{M}} c_1^3 e^{6\sqrt{M}}\Big)$$

$$c_4 = \frac{3}{4}\frac{1}{\sqrt{M}\left(e^{\sqrt{M}} + e^{-\sqrt{M}}\right)}\Big(4M^2 e^{-3\sqrt{M}} c_1 c_2^2 e^{2\sqrt{M}} + 2M^{3/2} e^{-3\sqrt{M}} c_1^2 c_2 e^{4\sqrt{M}} - 2M^{3/2} e^{-3\sqrt{M}}$$
$$c_1 c_2^2 e^{2\sqrt{M}} + 3M^{3/2} e^{-3\sqrt{M}} c_2^3 + 4M^2 e^{-3\sqrt{M}} c_1^2 c_2 e^{4\sqrt{M}} - 3M^{3/2} e^{-3\sqrt{M}} c_1^3 e^{6\sqrt{M}} + 2M^{3/2} c_2^2 c_1 e^{\sqrt{M}}$$
$$+ 2M^{3/2} c_1^2 c_2 e^{\sqrt{M}} + M^{3/2} c_1^3 e^{\sqrt{M}} + M^{3/2} c_2^3 e^{\sqrt{M}}\Big)$$

$$c_5 = \frac{3}{2} M\, c_1 c_2^2$$

$$c_6 = 3M^{3/2} c_2^2 c_1$$

$$c_7 = 3M^{3/2} c_1^2 c_2$$

$$c_8 = \frac{3}{2} M\, c_1^2 c_2$$

$$c_9 = \frac{3}{4} M\, c_1^3$$

$$c_{10} = \frac{3}{4} M\, c_2^3$$

$$c_{11} = -6c_1^2 M^2 c_4 + 6c_1^2 M^2 c_5 - 18c_2^2 M^2 c_9 - 24c_1 M^{3/2} c_2 c_7 - 12c_1 M^2 c_2 c_3 + 12c_1 M^2 c_2 c_8$$

$$c_{12} = 12c_1 M^2 c_2 c_7 + 6c_1^2 M^2 c_6$$

$$c_{13} = -18c_1^2 M^2 c_{10} - 6c_2^2 M^2 c_3 + 6c_2^2 M^2 c_8 - 12c_1 M^2 c_2 c_4 + 12c_1 M^2 c_5$$
$$- 24c_1 M^{3/2} c_2 c_6$$

$$c_{14} = 12c_1 M^2 c_2 c_6 - 6c_2^2 M^2 c_7$$

$$c_{15} = 24c_1^2 M^{3/2} c_7 + 18c_1^2 M^2 c_3 - 18c_1^2 M^2 c_8 + 108c_1 M^2 c_2 c_9$$

$$c_{16} = 18c_1^2 M^2 c_7$$

$$c_{17} = 24c_2^2 M^{3/2} c_6 + 18c_2^2 M^2 c_4 - 18c_2^2 M^2 c_5 + 108c_1 M^2 c_2 c_{10}$$

$$c_{18} = -18c_2^2 M^2 c_6$$

$$c_{19} = -90c_1^2 M^2 c_9$$

$$c_{20} = -90c_2^2 M^2 c_{10}$$

$$c_{21} = -\frac{1}{96}\frac{1}{M^4\left(e^{\sqrt{M}} + e^{-\sqrt{M}}\right)}\Big(-36e^{-5\sqrt{M}}M^3e^{8\sqrt{M}}c_{16} + 24e^{-5\sqrt{M}}M^3e^{4\sqrt{M}}c_{14} - 24e^{-5\sqrt{M}}$$
$$M^3e^{6\sqrt{M}}c_{12} + 36e^{-5\sqrt{M}}M^3e^{2\sqrt{M}}c_{18} - 24e^{-5\sqrt{M}}M^{7/2}e^{4\sqrt{M}}c_{14} - 48e^{-5\sqrt{M}}M^{7/2}e^{4\sqrt{M}}c_{13}$$
$$-48e^{-5\sqrt{M}}M^{7/2}e^{6\sqrt{M}}c_{11} - 24e^{-5\sqrt{M}}M^{7/2}e^{6\sqrt{M}}c_{12} - 36e^{-5\sqrt{M}}M^3e^{8\sqrt{M}}c_{15} - 20e^{-5\sqrt{M}}$$
$$M^3c_{19}e^{10\sqrt{M}} + 36e^{-5\sqrt{M}}M^3e^{2\sqrt{M}}c_{17} - 24e^{-5\sqrt{M}}M^3e^{6\sqrt{M}}c_{11} + 24e^{-5\sqrt{M}}M^3e^{4\sqrt{M}}c_{13}$$
$$+15e^{-5\sqrt{M}}M^{5/2}e^{8\sqrt{M}}c_{16} + 15e^{-5\sqrt{M}}M^{5/2}e^{2\sqrt{M}}c_{18} + 12e^{-5\sqrt{M}}M^{5/2}e^{6\sqrt{M}}c_{12} + 12e^{-5\sqrt{M}}$$
$$M^{5/2}e^{4\sqrt{M}}c_{14} + 24M^3e^{-\sqrt{M}}c_{11} - 4M^3e^{-\sqrt{M}}c_{20} - 12M^3e^{-\sqrt{M}}c_{15} + 12M^{5/2}e^{-\sqrt{M}}c_{14}$$
$$+24M^3e^{-\sqrt{M}}c_{13} - 9M^{5/2}e^{-\sqrt{M}}c_{18} - 4M^3e^{-\sqrt{M}}c_{19} + 9M^{5/2}e^{-\sqrt{M}}c_{16} - 12M^3e^{-\sqrt{M}}c_{17}$$
$$+20e^{-5\sqrt{M}}M^3c_{20} - 12M^{5/2}e^{-\sqrt{M}}c_{12}\Big)$$

$$c_{22} = -\frac{1}{96}\frac{1}{M^4\left(e^{\sqrt{M}} - e^{-\sqrt{M}}\right)}\Big(24e^{-5\sqrt{M}}M^{7/2}e^{4\sqrt{M}}c_{14} + 48e^{-5\sqrt{M}}M^{7/2}e^{4\sqrt{M}}c_{13}$$
$$+48e^{-5\sqrt{M}}M^{7/2}e^{6\sqrt{M}}c_{11} + 24e^{-5\sqrt{M}}M^{7/2}e^{6\sqrt{M}}c_{12} - 24e^{-5\sqrt{M}}M^3e^{4\sqrt{M}}c_{14}$$
$$+24e^{-5\sqrt{M}}M^3e^{6\sqrt{M}}c_{12} - 36e^{-5\sqrt{M}}M^3e^{2\sqrt{M}}c_{18} + 36e^{-5\sqrt{M}}M^3e^{8\sqrt{M}}c_{16}$$
$$-15e^{-5\sqrt{M}}M^{5/2}e^{2\sqrt{M}}c_{18} - 12e^{-5\sqrt{M}}M^{5/2}e^{6\sqrt{M}}c_{12} - 12e^{-5\sqrt{M}}M^{5/2}e^{4\sqrt{M}}c_{14}$$
$$-15e^{-5\sqrt{M}}M^{5/2}e^{8\sqrt{M}}c_{16} - 24e^{-5\sqrt{M}}M^3e^{4\sqrt{M}}c_{13} - 24e^{-5\sqrt{M}}M^3e^{6\sqrt{M}}c_{11}$$
$$-36e^{-5\sqrt{M}}M^3e^{2\sqrt{M}}c_{17} + 36e^{-5\sqrt{M}}M^3e^{8\sqrt{M}}c_{15} + 20e^{-5\sqrt{M}}M^3c_{19}e^{10\sqrt{M}}$$
$$-12M^{5/2}c_{12}e^{\sqrt{M}} + 9M^{5/2}c_{16}e^{\sqrt{M}} - 12M^3c_{15}e^{\sqrt{M}} + 12M^{5/2}c_{14}e^{\sqrt{M}} + 24M^3c_{13}e^{\sqrt{M}}$$
$$-4M^3c_{20}e^{\sqrt{M}} - 20e^{-5\sqrt{M}}M^3c_{20} - 4M^3c_{19}e^{\sqrt{M}} - 9M^{5/2}c_{18}e^{\sqrt{M}} + 24M^3c_{11}e^{\sqrt{M}}$$
$$-12M^3c_{17}e^{\sqrt{M}}\Big)$$

$$c_{23} = \frac{1}{2}\frac{c_{13}}{\sqrt{M}} + \frac{1}{4}\frac{c_{14}}{M}$$

$$c_{24} = \frac{1}{4}\frac{c_{14}}{\sqrt{M}}$$

$$c_{25} = \frac{1}{8}\frac{c_{14}}{M^{3/2}} + \frac{1}{4}\frac{c_{13}}{M}$$

$$c_{26} = -\frac{1}{2}\frac{c_{11}}{\sqrt{M}} + \frac{1}{4}\frac{c_{12}}{M}$$

$$c_{27} = \frac{1}{4}\frac{c_{12}}{\sqrt{M}}$$

$$c_{28} = -\frac{1}{8}\frac{c_{12}}{M^{3/2}} + \frac{1}{4}\frac{c_{11}}{M}$$

$$c_{29} = -\frac{3}{32}\frac{c_{18}}{M^{3/2}} + \frac{1}{8}\frac{c_{17}}{M}$$

$$c_{30} = \frac{1}{8}\frac{c_{18}}{M}$$

$$c_{31} = \frac{3}{32}\frac{c_{16}}{M^{3/2}} - \frac{1}{8}\frac{c_{15}}{M}$$

$$c_{32} = -\frac{1}{8}\frac{c_{16}}{M}$$

$$c_{33} = -\frac{1}{24}\frac{c_{20}}{M}$$

$$c_{34} = \frac{1}{24}\frac{c_{19}}{M}$$

$$c_{35} = c_{21} - \frac{1}{8}\frac{c_{12}}{M^{3/2}} + \frac{1}{4}\frac{c_{11}}{M}$$

$$\begin{aligned}
c_{36} = -\frac{1}{96}\frac{1}{M^4\left(e^{\sqrt{M}} + e^{-\sqrt{M}}\right)}\Big(&24e^{-5\sqrt{M}}M^{7/2}e^{4\sqrt{M}}c_{14} + 48e^{-5\sqrt{M}}M^{7/2}e^{4\sqrt{M}}c_{13} \\
&+ 48e^{-5\sqrt{M}}M^{7/2}e^{6\sqrt{M}}c_{11} + 24e^{-5\sqrt{M}}M^{7/2}e^{6\sqrt{M}}c_{12} - 24e^{-5\sqrt{M}}M^{3}e^{4\sqrt{M}}c_{14} \\
&+ 24e^{-5\sqrt{M}}M^{3}e^{6\sqrt{M}}c_{12} - 36e^{-5\sqrt{M}}M^{3}e^{2\sqrt{M}}c_{18} + 36e^{-5\sqrt{M}}M^{3}e^{8\sqrt{M}}c_{16} \\
&- 15e^{-5\sqrt{M}}M^{5/2}e^{2\sqrt{M}}c_{18} - 12e^{-5\sqrt{M}}M^{5/2}e^{6\sqrt{M}}c_{12} - 12e^{-5\sqrt{M}}M^{5/2}e^{4\sqrt{M}}c_{14} \\
&- 15e^{-5\sqrt{M}}M^{5/2}e^{8\sqrt{M}}c_{16} - 24e^{-5\sqrt{M}}M^{3}e^{4\sqrt{M}}c_{13} + 24e^{-5\sqrt{M}}M^{3}e^{6\sqrt{M}}c_{11} \\
&- 36e^{-5\sqrt{M}}M^{3}e^{2\sqrt{M}}c_{17} + 36e^{-5\sqrt{M}}M^{3}e^{8\sqrt{M}}c_{15} + 20e^{-5\sqrt{M}}M^{3}c_{19}e^{10\sqrt{M}} \\
&- 12M^{5/2}c_{12}e^{\sqrt{M}} + 9M^{5/2}c_{16}e^{\sqrt{M}} - 12M^{3}c_{15}e^{\sqrt{M}} + 12M^{5/2}c_{14}e^{\sqrt{M}} \\
&+ 24M^{3}c_{13}e^{\sqrt{M}} - 4M^{3}c_{20}e^{\sqrt{M}} - 20e^{-5\sqrt{M}}M^{3}c_{20} - 4M^{3}c_{19}e^{\sqrt{M}} - 9M^{5/2}c_{18}e^{\sqrt{M}} \\
&+ 24M^{3}c_{11}e^{\sqrt{M}} - 12M^{3}c_{17}e^{\sqrt{M}}\Big) + \frac{1}{8}\frac{c_{14}}{M^{3/2}} + \frac{1}{4}\frac{c_{13}}{M}
\end{aligned}$$

$$c_{37} = c_3 - c_8$$

$$c_{38} = c_4 - c_5$$

$$c_{39} = Br\, c_1 M\, c_2 - 3\, Br\, c_1^2 M^2 c_2^2 - \frac{1}{2} Br\, K$$

$$\begin{aligned}
c_{40} = &-Br\, c_1 c_2^3 e^{-2\sqrt{M}} M + \frac{1}{16} Br\, c_2^4 e^{-4\sqrt{M}} M + \frac{1}{16} Br\, c_1^4 e^{4\sqrt{M}} M - Br\, c_1^3 c_2 e^{2\sqrt{M}} M \\
&+ \frac{1}{4} Br\, c_1^2 e^{2\sqrt{M}} + \frac{1}{4} Br\, c_2^2 e^{-2\sqrt{M}} + Br\, c_1 e^{\sqrt{M}} + Br\, c_2 e^{-\sqrt{M}} - Br\, c_1 M\, c_2 \\
&+ 3\, Br\, c_1^2 M^2 c_2^2 + \frac{1}{2} Br\, K + Br\, c_1 c_2^3 M - Br\, c_2 - \frac{1}{16} Br\, c_2^4 M - \frac{1}{16} Br\, c_1^4 M \\
&+ Br\, c_1^3 c_2 M - \frac{1}{4} Br\, c_1^2 - \frac{1}{4} Br\, c_2^2 - Br\, c_1 + 1
\end{aligned}$$

$$c_{41} = -Br\, c_1 c_2^3 M + Br\, c_2 + \frac{1}{16} Br\, c_2^4 M + \frac{1}{16} Br\, c_1^4 M - Br\, c_1^3 c_2 M + \frac{1}{4} Br\, c_1^2 + \frac{1}{4} Br\, c_2^2 + Br\, c_1$$

$$c_{42} = -Br\, c_1$$

$$c_{43} = -Br\, c_2$$

$$c_{44} = Br\, c_1^3 c_2 M - \frac{1}{4} Br\, c_1^2$$

$$c_{45} = Br\, c_1 c_2^3 M - \frac{1}{4} Br\, c_2^2$$

$$c_{46} = -\frac{1}{16} Br\, c_1^4 M$$

$$c_{47} = -\frac{1}{16} Br\, c_2^4 M$$

$$c_{48} = 2\, Br\, K\, (c_3 - c_8)$$

$$c_{49} = 2\, Br\, K\, (c_4 - c_5)$$

$$c_{50} = 2\, Br\, K\, c_6$$

$$c_{51} = 2\, Br\, K\, c_7$$

$$c_{52} = 2\, Br\, c_1 c_7 + 4\, Br\, c_1 M\, (c_3 - c_8) + 4\, Br\, c_2 M\, c_9$$

$$c_{53} = 4\, Br\, c_1\, M\, c_{10} + 2\, Br\, c_2 \sqrt{M}\, c_6 + 4\, Br\, c_2\, M\, (c_4 - c_5)$$

$$c_{54} = 4\, Br\, c_1 M\, c_7$$

$$c_{55} = 4\, Br\, c_2 M\, c_6$$

$$c_{56} = -2\, Br\, K\, c_9$$

$$c_{57} = -2\, Br\, K\, c_{10}$$

$$c_{58} = -8\, Br\, c_1 M\, c_9$$

$$c_{59} = 8\, Br\, c_2 M\, c_{10}$$

$$c_{60} = 2\, Br\, c_1 \sqrt{M}\, c_6 - 2\, Br\, c_2 \sqrt{M}\, c_7$$

$$c_{61} = \frac{1}{4}\Bigg(1 + \frac{1}{2} c_{60} + \frac{c_{48} e^{\sqrt{M}}}{M} - \frac{2\, c_{51} e^{\sqrt{M}}}{M^{3/2}} + \frac{c_{49}}{M\, e^{\sqrt{M}}} + \frac{2\, c_{50}}{M^{3/2} e^{\sqrt{M}}} + \frac{1}{4}\frac{c_{52}\left(e^{\sqrt{M}}\right)^2}{M} - \frac{1}{4}\frac{c_{54}\left(e^{\sqrt{M}}\right)^2}{M^{3/2}} + \frac{1}{4}\frac{c_{53}}{M\left(e^{\sqrt{M}}\right)^2} + \frac{1}{4}\frac{c_{55}}{M^{3/2}\left(e^{\sqrt{M}}\right)^2} + \frac{1}{9}\frac{c_{56}\left(e^{\sqrt{M}}\right)^3}{M} + \frac{1}{9}\frac{c_{57}}{M\left(e^{\sqrt{M}}\right)^3} + \frac{1}{16}\frac{c_{58}\left(e^{\sqrt{M}}\right)^4}{M} + \frac{1}{16}\frac{c_{59}}{M\left(e^{\sqrt{M}}\right)^4} + \frac{c_{51} e^{\sqrt{M}}}{M} + \frac{c_{50}}{M\, e^{\sqrt{M}}} + \frac{1}{4}\frac{c_{54}\left(e^{\sqrt{M}}\right)^2}{M} + \frac{1}{4}\frac{c_{55}}{M\left(e^{\sqrt{M}}\right)^2} + \frac{1}{4}\frac{c_{54}}{M^{3/2}} - c_{62}\Bigg)$$

$$c_{62} = \frac{c_{48}}{M} - \frac{2\,c_{51}}{M^{3/2}} + \frac{c_{49}}{M} + \frac{2c_{50}}{M^{3/2}} + \frac{1}{4}\frac{c_{52}}{M} - \frac{1}{4}\frac{c_{54}}{M^{3/2}} + \frac{1}{4}\frac{c_{53}}{M} + \frac{1}{4}\frac{c_{55}}{M^{3/2}}$$
$$+\frac{1}{9}\frac{c_{57}}{M} + \frac{1}{9}\frac{c_{56}}{M} + \frac{1}{9}\frac{c_{57}}{M} + \frac{1}{16}\frac{c_{58}}{M} + \frac{1}{16}\frac{c_{59}}{M}$$

$$c_{63} = Br\,c_7 c_6 - \frac{1}{2}Br\,c_{37}c_9\left(e^{\sqrt{M}}\right)^4 - Br\,c_2 c_{31}\left(e^{\sqrt{M}}\right)^2 + \frac{Br\,c_{37}c_{10}}{\left(e^{\sqrt{M}}\right)^2} + \frac{Br\,c_2 c_{36}}{\left(e^{\sqrt{M}}\right)^2}$$
$$+ Br\,c_{38}c_9\left(e^{\sqrt{M}}\right)^2 + Br\,c_1 c_{35}\left(e^{\sqrt{M}}\right)^2 - \frac{Br\,c_1 c_{29}}{\left(e^{\sqrt{M}}\right)^2} + \frac{Br\,c_2 c_{24}}{\sqrt{M}\left(e^{\sqrt{M}}\right)^2}$$
$$+\frac{1}{3}\frac{Br\,c_2 c_{35}}{\left(e^{\sqrt{M}}\right)^6} - \frac{8\,Br\,K\,c_{27}e^{\sqrt{M}}}{M^{3/2}} + \frac{2}{9}\frac{Br\,K\,c_{30}}{M\left(e^{\sqrt{M}}\right)^3} + \frac{2\,Br\,K\,c_{26}e^{\sqrt{M}}}{M} + \frac{2}{3}Br\,c_1\sqrt{M}\,c_{24}$$
$$- Br\,c_1^3 M\,c_6\left(e^{\sqrt{M}}\right)^2 - \frac{2}{3}Br\,c_2\sqrt{M}\,c_{27} + \frac{1}{2}\frac{Br\,c_2 c_{29}}{\left(e^{\sqrt{M}}\right)^4} + \frac{1}{2}\frac{Br\,c_7^2\left(e^{\sqrt{M}}\right)^2}{M}$$

$$c_{64} = 18n\,c_1^2 M\,c_2 - 6n\,c_1^3 M\,e^{2\sqrt{M}} - \frac{6\,n\,c_1 M\,c_2^3}{e^{2\sqrt{M}}}$$

$$c_{65} = 18\,n\,c_2^2\,M\,c_1 - 6\,n\,c_1^2\,M\,c_2\,e^{2\sqrt{M}}$$

$$c_{66} = 3\,n\,c_1^3\,M\,e^{\sqrt{M}} - \frac{3\,n\,c^2 M\,c_1^2}{e^{\sqrt{M}}}$$

$$c_{67} = -3\,n\,c_2^2 c_1\,M\,e^{\sqrt{M}} + \frac{3\,n\,M\,c_2^3}{e^{\sqrt{M}}}$$

$$c_{68} = -\frac{2}{3}n\,c_1^3 M$$

$$c_{69} = -\frac{2}{3}M\,c_2^3\,n$$

$$c_{70} = -\frac{1}{6}c_1\,M^{3/2}e^{\sqrt{M}} + \frac{1}{6}\frac{M^{3/2}c_2}{e^{\sqrt{M}}}$$

$$c_{71} = -\frac{1}{2}M\,c_1 - \frac{1}{2}K\,M\,c_2$$

$$c_{72} = -c_{64}\sqrt{M}\,e^{\sqrt{M}} + c_{65}\sqrt{M}\,e^{-\sqrt{M}} - 2\,c_{66}\sqrt{M}\,e^{2\sqrt{M}} + 2\,c_{67}\sqrt{M}\,e^{-2\sqrt{M}}$$
$$-3\,c_{68}\sqrt{M}\,e^{3\sqrt{M}} + 3\,c_{69}\sqrt{M}\,e^{-3\sqrt{M}} - 3\,c_{70} - 2\,c_{71}$$

$$c_{73} = -18\,n\,c_1^2 M\,c_2 + 6\,n\,c_1^3\,M\,e^{2\sqrt{M}} + \frac{6\,n\,c_1\,M\,c_2^2}{e^{2\sqrt{M}}} - 18\,n\,c_2^2\,M\,c_1 + 6\,n\,c_1^2 M\,c_2\,e^{2\sqrt{M}}$$
$$-3\,n\,c_1^3\,M\,e^{\sqrt{M}} + \frac{3\,n\,c_2\,M\,c_1^2}{e^{\sqrt{M}}} + 3\,n\,c_2^2 c_1\,M\,e^{\sqrt{M}} - \frac{3\,n\,M\,c_2^3}{e^{\sqrt{M}}} + \frac{2}{3}n\,c_1^3\,M$$
$$+\frac{2}{3}M\,c_2^3\,n$$

$$c_{74} = -\frac{3\,n\,c_{67}\sqrt{M}}{\left(e^{\sqrt{M}}\right)^2} - \frac{2\,n\,c_{69}\sqrt{M}}{\left(e^{\sqrt{M}}\right)^3} + 6\,n\,c_{64}\sqrt{M}\,e^{\sqrt{M}} + 2\,n\,c_{68}\left(e^{\sqrt{M}}\right)^3\sqrt{M}$$
$$+2\,n\,M\,c_{71} + \frac{3}{2}n\,M\,c_{70} + 3\,n\,M\,\Big(-c_{64}\sqrt{M}\,e^{\sqrt{M}} + c_{65}\sqrt{M}\,e^{-\sqrt{M}} - 2\,c_{66}\sqrt{M}\,e^{2\sqrt{M}}$$
$$+2\,c_{67}\sqrt{M}\,e^{-2\sqrt{M}} - 3\,c_{68}\sqrt{M}\,e^{3\sqrt{M}} + 3\,c_{69}\sqrt{M}\,e^{-3\sqrt{M}} - 3\,c_{70} - 2\,c_{71}\Big) - \frac{6\,n\,c_{65}\sqrt{M}}{e^{\sqrt{M}}}$$
$$+3\,n\,c_{66}\left(e^{\sqrt{M}}\right)^2\sqrt{M} + \frac{1}{\left(e^{\sqrt{M}}\right)^2}\Big(6\,n\,M\,c_2^2\Big(-c_{64}\sqrt{M}\,e^{\sqrt{M}} + c_{65}\sqrt{M}\,e^{-\sqrt{M}}$$
$$-2\,c_{66}\sqrt{M}\,e^{2\sqrt{M}} + 2\,c_{67}\sqrt{M}\,e^{-2\sqrt{M}} - 3\,c_{68}\sqrt{M}\,e^{3\sqrt{M}} + 3\,c_{69}\sqrt{M}\,e^{-3\sqrt{M}} - 3\,c_{70}$$
$$-2\,c_{71}\Big)\Big) + 12\,n\,M^{3/2}c_1c_2c_{64}e^{\sqrt{M}} - \frac{12\,n\,M^{3/2}c_1c_2c_{65}}{e^{\sqrt{M}}} - 12\,n\,M^{3/2}c_1^2c_{67}$$
$$+12\,n\,M^{3/2}c_1^2c_{65}\,e^{\sqrt{M}} - \frac{12\,n\,M^{3/2}c_2^2\,c_{64}}{e^{\sqrt{M}}} + 6\,n\,M\,c_{73} - 24\,n\,M\,c_2c_1\Big(-c_{64}\sqrt{M}\,e^{\sqrt{M}}$$
$$+c_{65}\sqrt{M}\,e^{-\sqrt{M}} - 2\,c_{66}\sqrt{M}\,e^{2\sqrt{M}} + 2\,c_{67}\sqrt{M}\,e^{-2\sqrt{M}} - 3\,c_{68}\sqrt{M}\,e^{3\sqrt{M}}$$
$$-3\,c_{69}\sqrt{M}\,e^{-3\sqrt{M}} - 3\,c_{70} - 2\,c_{71}\Big) + 12\,n\,M^{3/2}c_2^2\,c_{66} + 6\,n\,M\,c_1^2\Big(-c_{64}\sqrt{M}\,e^{\sqrt{M}}$$
$$+c_{65}\sqrt{M}\,e^{-\sqrt{M}} - 2\,c_{66}\sqrt{M}\,e^{2\sqrt{M}} + 2\,c_{67}\sqrt{M}\,e^{-2\sqrt{M}} - 3\,c_{68}\sqrt{M}\,e^{3\sqrt{M}}$$
$$+3\,c_{69}\sqrt{M}\,e^{-3\sqrt{M}} - 3\,c_{70} - 2\,c_{71}\Big)\,\left(e^{\sqrt{M}}\right)^2$$

$$c_{75} = \frac{3}{2}n\,c_{66} + 18\,n\,M\,c_2^2\,c_{66} - 21\,n\,c_2^2\,c_{71} - 21\,n\,c_1^2\,c_{71} - \frac{3}{5}n\,M\,c_2^2\,c_{69} - 6\,n\,M\,c_1^2\,c_{69}$$
$$-2\,n\,M\,c_2^2\,c_{65} - n\,M\,c_2^2\,c_{67} - n\,M\,c_1^2\,c_{66} - \frac{3}{5}n\,M\,c_1^2\,c_{68} + \frac{135}{2}\frac{n\,c_1^2\,c_{70}}{\sqrt{M}}$$
$$-\frac{135}{2}\frac{n\,c_2^2\,c_{70}}{\sqrt{M}} - 6\,n\,M\,c_2^2\,c_{68} - 2\,n\,M\,c_1^2\,c_{64} - 9\,n\sqrt{M}\,c_2^2\,c_{72} + 9\,n\sqrt{M}\,c_1^2\,c_{72}$$
$$-3\,n\,M\,c_1\,c_2\,c_{69} + 18\,n\,M\,c_1\,c_2\,c_{65} - 8\,n\,M\,c_1\,c_2\,c_{67} + 18\,n\,M\,c_1\,c_2\,c_{64}$$
$$-3\,n\,M\,c_1c_2\,c_{68} - 8\,n\,M\,c_1\,c_2\,c_{66} + 48\,n\,c_2\,c_1\,c_{71} + 18\,n\,M\,c_1^2\,c_{67} + \frac{3}{2}n\,c_{67}$$
$$+\frac{2}{3}n\,c_{69} + 6\,n\,c_{64} + \frac{2}{3}n\,c_{68} + 6\,n\,c_{65} + 0$$

$$c_{76} = \frac{Br\ c_2}{e^{\sqrt{M}}} - 2\ Br\ \left(e^{\sqrt{M}}\right)^2 \sqrt{M}\ c_1^2 + \frac{9}{4}\frac{Br\ c_2^2}{\left(e^{\sqrt{M}}\right)^2} + \frac{9}{4}\ Br\ \left(e^{\sqrt{M}}\right)^2\ c_1^2 + \frac{2\ Br\ \sqrt{M}\ c_2^2}{\left(e^{\sqrt{M}}\right)^2}$$

$$-\frac{2\ Br\ c_2^2}{e^{\sqrt{M}}} - 2\ Br\ c_1^2\ e^{\sqrt{M}} - Br\ e^{\sqrt{M}} c_1 - 4\ Br\ c_1\ c_2 + \frac{1}{2}\ Br\ M\ c_1^2 + \frac{1}{2} Br\ M\ c_2^2$$

$$-\frac{1}{2} Br\ K + Br\ M\ c_1\ c_2 - \frac{1}{6}\ Br\ M^2\ c_1\ c_2 - 3\ Br\ M^2\ c_1^2\ c_2^2 - c_{77} + 1 - \frac{1}{3}\frac{Br\ M^{3/2} c_2 c_1}{e^{\sqrt{M}}}$$

$$+\frac{1}{3} Br\ e^{\sqrt{M}} M^{3/2} c_1 c_2 + \frac{Br\ M\ c_1 c_2^3}{\left(e^{\sqrt{M}}\right)^2} + Br\ M\ \left(e^{\sqrt{M}}\right)^2 c_2 c_1^3 + \frac{1}{12}\frac{Br\ M^2\ c_2^2}{\left(e^{\sqrt{M}}\right)^2} + \frac{1}{2}\frac{Br\ M\ c_2^2}{\left(e^{\sqrt{M}}\right)^2}$$

$$-\frac{1}{3}\frac{Br\ M^{3/2}\ c_2^2}{e^{\sqrt{M}}} - \frac{1}{16}\ Br\ M\ \left(e^{\sqrt{M}}\right)^4\ c_1^4 - \frac{1}{16}\frac{Br\ c_2^4\ M}{\left(e^{\sqrt{M}}\right)^4} + \frac{1}{3}\ Br\ e^{\sqrt{M}} M^{3/2}\ c_1^2$$

$$+\frac{1}{2} Br\ \left(e^{\sqrt{M}}\right)^2\ M\ c_1^2 - 2\ Br\ c_1\ c_2\ e^{\sqrt{M}} - \frac{2\ Br\ c_1\ c_2}{e^{\sqrt{M}}} + \frac{1}{12} Br\ \left(e^{\sqrt{M}}\right)^2\ M^2\ c_1^2$$

$$c_{77} = Br\ M\ c_1\ c_2^3 + Br\ M\ c_2\ c_1^3 - \frac{2\ Br\ c_1\ c_2}{e^{\sqrt{M}}} - 2\ Br\ c_1\ c_2\ e^{\sqrt{M}} - \frac{1}{16} Br\ M\ c_1^4$$

$$-4\ Br\ c_1\ c_2 + \frac{2\ Br\ c_1\ c_2}{e^{\sqrt{M}}} - \frac{1}{16}\ Br\ c_2^4\ M + 2\ Br\ c_1^2\ e^{\sqrt{M}} - \frac{7}{4}\ Br\ c_2^2 - Br\ c_1$$

$$-Br\ c_2 - \frac{7}{4}\ Br\ c_1^2$$

$$c_{78} = Br\ c_1\ c_{65} - \frac{4\ Br\ c_1\ c_{65}}{M} - 45\ Br\ B\ c_1\ M\ c_2^2\ c_{64} - \frac{1}{6}\ Br\ c_1\ c_{71} - \frac{1}{3} Br\ c_2\ c_{72}$$

$$-\frac{1}{10}\ Br\ c_1\ c_{70} - 6\ Br\ B\ c_2^3\ M^2\ c_{68} + 1 - \frac{1}{3}\ Br\ c_1\ c_{72} + \frac{90\ Br\ B\ c_1\ \sqrt{M}\ c_2^2\ c_{71}}{e^{\sqrt{M}}}$$

$$+90\ Br\ B\ c_1^2\ c_2\ c_{71}\ e^{\sqrt{M}} + \frac{Br\ B\ c_2^3\ M^{3/2} c_{70}}{\left(e^{\sqrt{M}}\right)^3} - \frac{85}{3}\frac{Br\ B\ c_2^3\ c_{70}\sqrt{M}}{\left(e^{\sqrt{M}}\right)^2}$$

$$-2\ Br\ B\ c_1^3\ M^{3/2}\left(e^{\sqrt{M}}\right)^3\ c_{72} - \frac{6\ Br\ B\ c_2^3\ M^2\ c_{64}}{\left(e^{\sqrt{M}}\right)^2} + \frac{90\ Br\ B\ c_1\ c_2^2\ c_{71}}{e^{\sqrt{M}}}$$

$$-90\ Br\ B\ c_1^2\ \sqrt{M}\ c_2\ c_{71}\ e^{\sqrt{M}} - 6\ Br\ B\ c_1^3\ M^2\ c_{65}\left(e^{\sqrt{M}}\right)^2 + \frac{85}{3} Br\ B\ c_1^3\ c_{70}\sqrt{M}\ \left(e^{\sqrt{M}}\right)^3$$

$$+\frac{12\ Br\ B\ c_2^3\ M^2\ c_{66}}{e^{\sqrt{M}}} + \frac{2\ BrB\ c_2^3\ M^{3/2}\ c_{72}}{\left(e^{\sqrt{M}}\right)^3} - \frac{4}{3} Br\ B\ c_1^3\ M^{3/2}\left(e^{\sqrt{M}}\right)^3\ c_{71}$$

$$-\frac{77}{150}\ Br\ B\ c_1^3\ M\ c_{66}\ \left(e^{\sqrt{M}}\right)^5 - \frac{19}{100}\frac{Br\ B\ c_2^3\ M\ c_{69}}{\left(e^{\sqrt{M}}\right)^6} - \frac{19}{100} Br\ B\ c_1^3 M\ c_{68}\left(e^{\sqrt{M}}\right)^6$$

$$-30\ Br\ B\ c_1^3 M\ c_{67}\ e^{\sqrt{M}} + \frac{4}{3}\frac{Br\ B\ c_2^3\ M^{3/2}\ c_{71}}{\left(e^{\sqrt{M}}\right)^3}$$

$$c_{79} = \frac{1}{2} Br\, c_1\, c_{64} + \frac{3}{8} Br\, c_2\, c_{69} + \frac{4}{9} Br\, c_1\, c_{66} - Br\, B\, c_2^3\, M\, c_{64} + 27\, Br\, B\, c_1\, M\, c_2^2\, c_{68}$$
$$- \frac{27}{4} Br\, B\, c_1^2\, M\, c_2\, c_{68} - Br\, B\, c_1^3\, M\, c_{65} - 8\, Br\, B\, c_1^2\, M\, c_2\, c_{66} - 9\, Br\, B\, c_1^2\, M\, c_2\, c_{64}$$
$$- 8\, Br\, B\, c_2^2\, M\, c_1\, c_{67} - 9\, Br\, B\, c_2^2\, M\, c_1\, c_{65} + 72\, Br\, B\, c_1^2\, M\, c_2\, c_{67} + 72\, Br\, B\, c_1\, M\, c_2^2\, c_{66}$$
$$+ 144 Br\, B\, c_1\, c_2^2\, c_{71} + 144 Br\, B\, c_1^2\, c_2\, c_{71} + \frac{8}{25} Br\, B\, c_2^3\, M\, c_{67} + \frac{1}{3} Br\, B\, c_1^3\, M\, c_{68}$$
$$+ \frac{1}{3} Br\, B\, c_2^3\, M\, c_{69} + \frac{8}{25} Br\, B\, c_1^3\, M\, c_{66} - 8\, Br\, B\, c_1^3\, M\, c_{67} + \frac{1}{4} Br\, B\, c_1^3\, M\, c_{64}$$
$$+ \frac{1}{4} Br\, B\, c_2^3\, M\, c_{65} - 8\, Br\, B\, c_2^3\, M\, c_{66} - \frac{4}{3}\, Br\, B\, c_1^3\, M\, c_{68} \left(e^{\sqrt{M}}\right)^3 - \frac{36}{25} \frac{Br\, B\, c_2^3\, M\, c_{69}}{e^{\sqrt{M}}}$$
$$+ \frac{4\, Br\, B\, c_2^3\, M\, c_{64}}{\left(e^{\sqrt{M}}\right)^3} - \frac{4}{3} \frac{Br\, B\, c_2^3\, M\, c_{65}}{e^{\sqrt{M}}} - \frac{2\, Br\, B\, c_2^3\, M\, c_{67}}{\left(e^{\sqrt{M}}\right)^3}$$

$$c_{80} = \frac{2\, Br\, K\, c_{30}}{9\, M}$$

$$c_{81} = -\frac{9\, Br\, c_1^2\, M\, c_2\, c_9}{4} + \frac{Br\, c_{37}\, c_9}{2} - \frac{Br\, c_1\, c_{31}}{2} + \frac{Br\, c_2\, c_{34}}{2} + \frac{Br\, c_9\, c_7}{8\sqrt{M}} + \frac{Br\, c_1\, c_{32}}{8\sqrt{M}}$$
$$- \frac{Br\, c_1^3\, M\, c_{37}}{4} - \frac{Br\, c_1^3\, \sqrt{M}\, c_7}{8}$$

$$c_{82} = -\frac{1}{4} Br\, c_1^3\, M\, c_7 - \frac{1}{2} Br\, c_1\, c_{32} + \frac{1}{2}\, Br\, c_9\, c_7$$

$$c_{83} = -\frac{9}{4} Br\, c_1\, M\, c_2^2\, c_{10} - \frac{1}{2} Br\, c_2\, c_{29} + \frac{1}{2} Br\, c_1\, c_{35} + \frac{1}{2} Br\, c_{38}\, c_{10} - \frac{1}{4} Br\, c_2^3\, M\, c_{38}$$
$$- \frac{1}{8\sqrt{M}} Br\, c_2\, c_{30} - \frac{Br\, c_6\, c_{10}}{8\sqrt{M}} + \frac{Br\, c_2^3\, c_6 \sqrt{M}}{8}$$

$$c_{84} = -\frac{Br\, c_2^3\, M\, c_6}{4} + \frac{Br\, c_6\, c_{10}}{2} - \frac{Br\, c_2^3\, M\, c_6}{4} - \frac{Br\, c_2\, c_{30}}{2}$$

$$c_{85} = -\frac{2\, Br\, K\, c_{34}}{25\, M}$$

$$c_{86} = -\frac{2\, Br\, K\, c_{35}}{25\, M}$$

$$c_{87} = -\frac{Br\, c_1\, c_{34}}{3} + \frac{Br\, c_1^3\, M\, c_9}{3} - \frac{5\, Br\, c_9^2}{18}$$

$$c_{88} = -\frac{5\, Br\, c_{10}^2}{18} - \frac{Br\, c_2\, c_{35}}{3} + \frac{Br\, c_2^3\, M\, c_{10}}{3}$$

$$c_{89} = -6\, Br\, c_1\, M^2\, c_2^2\, c_{37} + 6\, Br\, c_1^2\, M^{\frac{3}{2}}\, c_2\, c_6 - 6\, Br\, c_1\, M^{\frac{3}{2}}\, c_2^2\, c_7 - Br\, c_7\, c_6$$
$$- 6\, Br\, c_2^3\, M^2\, c_9 + 8\, Br\, c_9\, M\, c_{10} - Br\sqrt{M}\, c_{37}\, c_6 + Br\, c_7\sqrt{M}\, c_{38}$$
$$- 6\, Br\, c_1^3\, M^2\, c_{10} + Br\, c_2\sqrt{M}\, c_{26} - Br\, c_1\sqrt{M}\, c_{23} - 6\, Br\, c_1^2\, M^2\, c_2\, c_{38}$$

$$c_{90} = -2\, Br\, c_1\, M^2\, c_2^2\, c_7 - 2\, Br\, c_1^2\, M^2\, c_2\, c_6 + \frac{2\, Br\, c_2\sqrt{M}\, c_{27}}{3} - \frac{2\, Br\, c_1\sqrt{M}\, c_{24}}{3}$$

$$c_{91} = -c_1\sqrt{M}\, e^{\sqrt{M}} + c_2\sqrt{M}\, e^{-\sqrt{M}}$$

$$c_{92} = -24\, n\, M\, c_1\, c_2\, c_{64} + \frac{6\, n\, M\, c_2^2\, c_{64}}{\left(e^{\sqrt{M}}\right)^2} + 18\, n\, M\, c_2^2\, c_{68} - \frac{24\, n\, c_2\, c_1\, c_{71}}{e^{\sqrt{M}}}$$
$$- \frac{12\, n\sqrt{M}\, c_2\, c_1\, c_{72}}{e^{\sqrt{M}}} + \frac{72\, n\, c_1\, c_2\, c_{70}}{\sqrt{M}\, e^{\sqrt{M}}} - \frac{24\, n\, M\, c_2^2\, c_{66}}{e^{\sqrt{M}}} + 24\, n\, M\, c_1\, c_2\, c_{66}\, e^{\sqrt{M}}$$
$$+ 6\, n\, M\, c_1^2\, c_{64}\left(e^{\sqrt{M}}\right)^2 - 6\, n\, M\, c_1^2\, c_{65} - 12\, n\sqrt{M}\, c_1^2\, c_{72}\, e^{\sqrt{M}}$$
$$- \frac{72\, n\, c_1^2\, c_{70}\, e^{\sqrt{M}}}{\sqrt{M}} - 6\, n\, c_{64} + 24\, n\, c_1^2\, c_{71}\, e^{\sqrt{M}}$$

$$c_{93} = \frac{24\sqrt{M}\, c_2\, c_1\, c_{71}}{e^{\sqrt{M}}} - 24\, n\sqrt{M}\, c_1^2\, c_{71}\, e^{\sqrt{M}} - \frac{72\, n\, c_1\, c_2\, c_{70}}{e^{\sqrt{M}}} + 72\, n\, c_1^2\, c_{70}\, e^{\sqrt{M}}$$

$$c_{94} = \frac{36\, n\sqrt{M}\, c_1\, c_2\, c_{70}}{e^{\sqrt{M}}} - 36\, n\sqrt{M}\, c_1^2\, c_{70}\, e^{\sqrt{M}}$$

$$c_{95} = -24\, n\sqrt{M}\, c_2\, c_1\, c_{71}\, e^{\sqrt{M}} + \frac{24\, n\sqrt{M}\, c_2^2\, c_{71}}{e^{\sqrt{M}}} - 72\, n\, c_1\, c_2\, c_{70}\, e^{\sqrt{M}}$$
$$+ \frac{72\, n\, c_2^2\, c_{70}}{e^{\sqrt{M}}}$$

$$c_{96} = \frac{36\, n\sqrt{M}\, c_2^2\, c_{70}}{e^{\sqrt{M}}} - 36\, n\sqrt{M}\, c_1\, c_2\, c_{70}\, e^{\sqrt{M}}$$

$$c_{97} = \frac{12n\sqrt{M}\, c_2^2\, c_{72}}{e^{\sqrt{M}}} - 6n\, c_{65} + \frac{6n\, M\, c_2^2\, c_{65}}{\left(e^{\sqrt{M}}\right)^2} + 6n\, M\, c_1^2\, c_{65}\left(e^{\sqrt{M}}\right)^2$$
$$- 24n\, c_2\, c_1\, c_{71}\, e^{\sqrt{M}} + 18n\, M\, c_1^2\, c_{69} + \frac{24n\, c_2^2\, c_{71}}{e^{\sqrt{M}}} - 6n\, M\, c_2^2\, c_{64}$$
$$+ \frac{72n\, c_2^2\, c_{70}}{\sqrt{M}\, e^{\sqrt{M}}} - 24n\, M\, c_1\, c_2\, c_{65} - \frac{72n\, c_1\, c_2\, c_{70}\, e^{\sqrt{M}}}{\sqrt{M}} - 24n\, M\, c_1^2\, c_{67}\, e^{\sqrt{M}}$$
$$- 12n\sqrt{M}\, c_2\, c_1\, c_{72}\, e^{\sqrt{M}} + \frac{24n\, M\, c_1\, c_2\, c_{67}}{e^{\sqrt{M}}}$$

$$c_{98} = -9n\, c_1^2\, c_{70}\;\; 6n\sqrt{M}\, c_1^2\, c_{71}$$

$$c_{99} = 9n\sqrt{M}\, c_1^2\, c_{70}$$

$$c_{100} = 6n\,M\,c_1^2\,c_{66}\left(e^{\sqrt{M}}\right)^2 - 24n\,M\,c_1\,c_2\,c_{66} - \frac{18n\,M\,c_2^2\,c_{68}}{e^{\sqrt{M}}} + \frac{6n\,M\,c_2^2\,c_{66}}{\left(e^{\sqrt{M}}\right)^2}$$
$$+18n\,M\,c_1\,c_2\,c_{68}\,e^{\sqrt{M}} - 3n\,c_1^2\,c_{71} + 3n\sqrt{M}\,c_1^2\,c_{72} - \frac{3}{2}n\,c_{66}\,\frac{9}{2}\,\frac{n\,c_1^2\,c_{70}}{\sqrt{M}}$$
$$+\frac{6n\,M\,c_1\,c_2\,c_{64}}{e^{\sqrt{M}}} - 6n\,M\,c_1^2\,c_{64}\,e^{\sqrt{M}}$$
$$c_{101} = -6n\sqrt{M}\,c_2^2\,c_{71} - 9n\,c_2^2\,c_{70}$$
$$c_{102} = -9n\sqrt{M}\,c_2^2\,c_{70}$$
$$c_{103} = -\frac{3}{2}n\,c_{67} - \frac{6n\,M\,c_2^2\,c_{65}}{e^{\sqrt{M}}} - 18n\,M\,c_1^2\,c_{69}\,e^{\sqrt{M}} - \frac{9}{2}\,\frac{n\,c_2^2\,c_{70}}{\sqrt{M}} - 3n\,c_2^2\,c_{71}$$
$$-24n\,M\,c_1\,c_2\,c_{67} + 6n\,M\,c_1^2\,c_{67}\left(e^{\sqrt{M}}\right)^2 - 3n\sqrt{M}\,c_2^2\,c_{72} + 6n\,M\,c_1\,c_2\,c_{65}\,e^{\sqrt{M}}$$
$$+\frac{18n\,M\,c_1\,c_2\,c_{69}}{e^{\sqrt{M}}} + \frac{6n\,M\,c_2^2\,c_{67}}{\left(e^{\sqrt{M}}\right)^2}$$
$$c_{104} = -\frac{2}{3}n\,c_{68} + 2n\,M\,c_1^2\,c_{64} - 8n\,M\,c_1^2\,c_{66}\,e^{\sqrt{M}} - 24n\,M\,c_1\,c_2\,c_{68} + \frac{6n\,M\,c_2^2\,c_{68}}{\left(e^{\sqrt{M}}\right)^2}$$
$$+\frac{8n\,M\,c_1\,c_2\,c_{66}}{e^{\sqrt{M}}} + 6n\,M\,c_1^2\,c_{68}\left(e^{\sqrt{M}}\right)^2$$
$$c_{105} = -\frac{8n\,M\,c_2^2\,c_{67}}{e^{\sqrt{M}}} + 2n\,M\,c_2^2\,c_{65} - \frac{2}{3}n\,c_{69} + \frac{6n\,M\,c_2^2\,c_{69}}{\left(e^{\sqrt{M}}\right)^2} + 6n\,M\,c_1^2\,c_{69}\left(e^{\sqrt{M}}\right)^2$$
$$+8n\,M\,c_1\,c_2\,c_{67}\,e^{\sqrt{M}} - 24n\,M\,c_1\,c_2\,c_{69}$$
$$c_{106} = \frac{9n\,M\,c_1\,c_2\,c_{68}}{e^{\sqrt{M}}} + 3n\,M\,c_1^2\,c_{66} - 9n\,M\,c_1^2\,c_{68}\,e^{\sqrt{M}}$$
$$c_{107} = -\frac{9n\,M\,c_2^2\,c_{69}}{e^{\sqrt{M}}} + 9n\,M\,c_1\,c_2\,c_{69}\,e^{\sqrt{M}} + 3n\,M\,c_2^2\,c_{67}$$
$$c_{108} = \frac{18}{5}n\,M\,c_1^2\,c_{68}$$
$$c_{109} = \frac{18}{5}n\,M\,c_2^2\,c_{69}$$
$$c_{110} = 6n\,M\,c_1^2\left(e^{\sqrt{M}}\right)^2 c_{70} - 24n\,M\,c_1\,c_2\,c_{70} - n\,M\,c_{72} + \frac{6n\,M\,c_2^2\,c_{70}}{\left(e^{\sqrt{M}}\right)^2}$$
$$c_{111} = \frac{6n\,c_2^2\,M\,c_{71}}{\left(e^{\sqrt{M}}\right)^2} - 24n\,c_1\,M\,c_2\,c_{71} - 3n\,M\,c_{73} + 6n\,c_1^2\,M\left(e^{\sqrt{M}}\right)^2 c_{71}$$
$$c_{112} = -\frac{1}{2}n\,M\,c_{71}$$

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$$c_{113} = -\frac{3}{10} n\, M\, c_{70}$$

$$c_{114} = c_{75}$$

$$c_{115} = -Br\, c_1$$

$$c_{116} = -Br\, c_2$$

$$c_{117} = Br\, c_1^3\, c_2\, M - \frac{1}{4} Br\, c_1^2$$

$$c_{118} = -\frac{1}{4} Br\, c_2^2 + Br\, c_1\, c_2^3\, M$$

$$c_{119} = -\frac{1}{16} Br\, M\, c_1^4$$

$$c_{120} = -\frac{1}{16} Br\, M\, c_2^4$$

$$c_{121} = Br\, c_1^3\, c_2\, M + \frac{1}{16} \frac{Br\, c_2^4\, M}{\left(e^{\sqrt{M}}\right)^4} + \frac{1}{16} Br\, c_1^4 \left(e^{\sqrt{M}}\right)^4 M - Br\, c_1\, M\, c_2 + 3\, Br\, c_1^2\, M^2\, c_2^2$$
$$- \frac{Br\, c_1\, c_2^3\, M}{\left(e^{\sqrt{M}}\right)^2} - Br\, c_1^3\, c_2 \left(e^{\sqrt{M}}\right)^2 M - Br\, c_1 - Br\, c_2 + \frac{1}{2} Br\, K - \frac{1}{4} Br\, c_1^2 - \frac{1}{4} Br\, c_2^2$$
$$- \frac{1}{16} Br\, c_2^4\, M - \frac{1}{16} Br\, c_1^4\, M + \frac{1}{4} Br\, c_1^2 \left(e^{\sqrt{M}}\right)^2 + \frac{1}{4} \frac{Br\, c_2^2}{\left(e^{\sqrt{M}}\right)^2} + Br\, c_1\, e^{\sqrt{M}} + \frac{Br\, c_2}{e^{\sqrt{M}}}$$
$$+ Br\, c_1\, c_2^3\, M$$

$$c_{122} = Br\, c_1\, M\, c_2 - 3\, Br\, c_1^2\, M^2\, c_2^2$$

$$c_{123} = -Br\, c_1\, c_2^3\, M - Br\, c_1^3\, c_2\, M + \frac{1}{16} Br\, c_1^4\, M + \frac{1}{16} Br\, c_2^4\, M + Br\, c_2 + Br\, c_1$$
$$+ \frac{1}{4} Br\, c_1^2 + \frac{1}{4} Br\, c_2^2$$

$$c_{124} = 2\, Br\, e^{\sqrt{M}} \sqrt{M}\, c_1^2 - \frac{2\, Br \sqrt{M}\, c_2\, c_1}{e^{\sqrt{M}}}$$

$$c_{125} = Br\, c_1 + 2\, Br\, c_1^2 - 2\, Br\, e^{\sqrt{M}}\, c_1^2 + 2\, Br\, c_1\, c_2 + \frac{2\, Br\, c_1\, c_2}{e^{\sqrt{M}}}$$

$$c_{126} = 2\, Br\, c_2^2 + 2\, Br\, c_1\, c_2 + 2\, Br\, e^{\sqrt{M}}\, c_1\, c_2 - \frac{2\, Br\, c_2^2}{e^{\sqrt{M}}} + Br\, c_2$$

$$c_{127} = -\frac{2\, Br \sqrt{M}\, c_2^2}{e^{\sqrt{M}}} + 2\, Br\, e^{\sqrt{M}} \sqrt{M}\, c_2\, c_1$$

$$c_{128} = -\frac{1}{4} Br\, c_1^2 - Br\, M\, c_2\, c_1^3$$

$$c_{129} = -\frac{1}{4} Br\, c_2^2 - Br\, M\, c_1\, c_2^3$$

$$c_{130} = -\frac{1}{16}\,Br\,M\,c_1^4$$

$$c_{131} = \frac{1}{16}\,Br\,c_2^4\,M$$

$$c_{132} = -\frac{1}{12}\,Br\left(e^{\sqrt{M}}\right)^2 M^2\,c_1^2 + c_{77} + \frac{1}{6}\,Br\,M^2\,c_1\,c_2 - \frac{1}{12}\,\frac{Br\,M^2\,c_2^2}{\left(e^{\sqrt{M}}\right)^2}$$

$$c_{133} = -\frac{1}{3}\,Br\,e^{\sqrt{M}}\,M^{3/2}\,c_1\,c_2 + \frac{1}{3}\,\frac{Br\,M^{3/2}\,c_2^2}{e^{\sqrt{M}}} - \frac{1}{3}\,Br\,e^{\sqrt{M}}\,M^{3/2}\,c_1^2 + \frac{1}{3}\,\frac{Br\,M^{3/2}\,c_2\,c_1}{e^{\sqrt{M}}}$$

$$c_{134} = +3\,Br\,M^2\,c_1^2\,c_2^2 - \frac{1}{2}\,\frac{Br\,M\,c_2^2}{\left(e^{\sqrt{M}}\right)^2} - \frac{1}{2}\,Br\,M\,c_1^2 - Br\,M\,c_1\,c_2 - \frac{1}{2}\,Br\left(\left(e^{\sqrt{M}}\right)\right)^2 M\,c_1^2 + \frac{1}{2}\,Br\,K - \frac{1}{2}\,Br\,M\,c_2^2$$

$$c_{135} = 4\,Br\,c_2\,c_{66} + \frac{8\,Br\,c_1\,c_{71}}{M} + 2\,Br\,c_1\,c_{64}\,e^{\sqrt{M}} - \frac{36\,Br\,c_1\,c_{70}}{M^{3/2}} - \frac{2\,Br\,c_1\,c_{72}}{\sqrt{M}} + \frac{4\,Br\,c_1\,c_{72}}{M^{3/2}} + \frac{48\,Br\,c_1\,c_{70}}{M^{5/2}} - \frac{2\,Br\,c_2\,c_{64}}{e^{\sqrt{M}}} - 144\,Br\,B\,c_1^2\,c_2\,c_{71} + 8\,Br\,B\,c_1^3\,M\,c_{67} + \frac{4\,Br\,c_2\,c_{64}}{M\,e^{\sqrt{M}}} + 48\,Br\,B\,c_1^3\,c_{71}\left(e^{\sqrt{M}}\right)^2 - \frac{4\,Br\,c_1\,c_{64}\,e^{\sqrt{M}}}{M} + \frac{2\,Br\,c_2\,c_{64}}{M} - \frac{12\,Br\,c_1\,c_{71}}{M^2} - \frac{2\,Br\,c_1\,c_{73}}{M} - \frac{2\,Br\,c_2\,c_{66}}{M} + \frac{2\,Br\,c_1\,c_{64}}{M} - \frac{12\,Br\,B\,c_2^2\,\sqrt{M}\,c_1\,c_{72}}{\left(e^{\sqrt{M}}\right)^2} + \frac{36\,Br\,B\,c_1\,M\,c_2^2\,c_{64}}{e^{\sqrt{M}}} + \frac{12\,Br\,B\,c_1^2\,M\,c_2\,c_{65}}{e^{\sqrt{M}}} + 24\,Br\,B\,c_1^2\,M\,c_2\,c_{66}\left(e^{\sqrt{M}}\right)^2 - 36\,Br\,B\,c_1^2\,M\,c_2\,c_{64}\,e^{\sqrt{M}} + 36\,Br\,B\,c_1\,M\,c_2^2\,c_{68}\,e^{\sqrt{M}}$$

$$c_{136} = \frac{24\,Br\,c_1\,c_{70}}{M} - \frac{2\,Br\,c_1\,c_{72}}{M} - \frac{36\,Br\,c_1\,c_{70}}{M^2} + \frac{8\,Br\,c_1\,c_{71}}{M^{3/2}} - \frac{4\,Br\,c_1\,c_{71}}{\sqrt{M}} - 24\,Br\,B\,c_1^3\,\sqrt{M}\,c_{71}\left(e^{\sqrt{M}}\right)^2 + \frac{144\,Br\,B\,c_1\,c_2^2\,c_{70}}{\left(e^{\sqrt{M}}\right)^2} + 72\,Br\,B\,c_1^2\,\sqrt{M}\,c_2\,c_{71} - 432\,Br\,B\,c_1^2\,c_2\,c_{70} - \frac{2\,Br\,c_2\,c_{64}}{\sqrt{M}\,e^{\sqrt{M}}} + \frac{2\,Br\,c_1\,c_{64}\,e^{\sqrt{M}}}{\sqrt{M}} + 144\,Br\,B\,c_1^3\,c_{70}\left(e^{\sqrt{M}}\right)^2 - \frac{24\,Br\,B\,c_1\,\sqrt{M}\,c_2^2\,c_{71}}{\left(e^{\sqrt{M}}\right)^2}$$

$$c_{137} = -\frac{2\ Br\ c_1\ c_{71}}{M} - \frac{6\ Br\ c_1\ c_{70}}{\sqrt{M}} + \frac{12\ Br\ c_1\ c_{70}}{M^{3/2}} - 36\ Br\ B\ c_1^3\ c_{70}\ \sqrt{M}\ \left(e^{\sqrt{M}}\right)^2$$
$$+108\ Br\ B\ c_1^2\ c_2\ c_{70}\ \sqrt{M} - \frac{36\ Br\ B\ c_1\ c_2^2\ c_{70}\ \sqrt{M}}{\left(e^{\sqrt{M}}\right)^2}$$

$$c_{138} = -\frac{2\ Br\ c_1\ c_{70}}{M}$$

$$c_{139} = 4\ Br\ c_1\ c_{67} + \frac{2\ Br\ c_2\ c_{65}}{M} + \frac{8\ Br\ c_2\ c_{71}}{M} - \frac{2\ Br\ c_1\ c_{67}}{M} - \frac{4\ Bt\ c_2\ c_{72}}{M^{3/2}} + \frac{36\ Br\ c_2\ c_{70}}{M^{3/2}}$$
$$+\frac{2\ Br\ c_2\ c_{65}}{e^{\sqrt{M}}} - \frac{48\ Br\ c_2\ c_{70}}{M^{5/2}} - 2\ Br\ c_1\ c_{65}\ e^{\sqrt{M}} - 144\ Br\ B\ c_1\ c_2^2\ c_{71}$$
$$+8\ Br\ B\ c_2^3\ M\ c_{66} + \frac{48\ Br\ B\ c_2^3\ c_{71}}{\left(e^{\sqrt{M}}\right)^2} + \frac{4\ Br\ c_1\ c_{65}\ e^{\sqrt{M}}}{M} - \frac{4\ Br\ c_2\ c_{65}}{M\ e^{\sqrt{M}}} - \frac{2\ Br\ c_2\ c_{73}}{M}$$
$$+\frac{2\ Br\ c_1\ c_{65}}{M} - \frac{12\ Br\ c_2\ c_{71}}{M^2} + \frac{24\ Br\ B\ c_2^2\ M\ c_1\ c_{67}}{\left(e^{\sqrt{M}}\right)^2} + \frac{216\ Br\ B\ c_1^2\ c_2\ c_{70}\ \left(e^{\sqrt{M}}\right)^2}{\sqrt{M}}$$
$$+12\ Br\ B\ c_2\ \sqrt{M}\ c_1^2\ c_{72}\ \left(e^{\sqrt{M}}\right)^2 + \frac{36\ Br\ B\ c_1^2\ M\ c_2\ c_{69}}{e^{\sqrt{M}}} + 36\ Br\ B\ c_1^2\ M\ c_2\ c_{65}\ e^{\sqrt{M}}$$
$$-\frac{36\ Br\ B\ c_2^2\ M\ c_1\ c_{65}}{e^{\sqrt{M}}} + 12\ Br\ B\ c_1\ M\ c_2^2\ c_{64}\ e^{\sqrt{M}} - 72\ Br\ B\ c_1^2\ M\ c_2\ c_{67}$$
$$-\frac{648\ Br\ B\ c_1\ c_2^2\ c_{70}}{\sqrt{M}} - 36\ Br\ B\ c_2^2\ \sqrt{M}\ c_1\ c_{72} - 36\ Br\ B\ c_1^3\ M\ c_{69}\ e^{\sqrt{M}}$$
$$+\frac{216\ Br\ B\ c_2^3\ c_{70}}{\sqrt{M}\ \left(e^{\sqrt{M}}\right)^2} + 48\ Br\ B\ c_1^2\ c_2\ c_{71}\ \left(e^{\sqrt{M}}\right)^2 - \frac{12\ Br\ B\ c_2^3\ M\ c_{64}}{e^{\sqrt{M}}}$$
$$+\frac{4\ Br\ B\ c_2^3\ M\ c_{65}}{\left(e^{\sqrt{M}}\right)^3} + 24\ Br\ B\ c_1^3\ M\ c_{67}\left(e^{\sqrt{M}}\right)^2 + \frac{12\ Br\ B\ c_2^3\ \sqrt{M}\ c_{72}}{\left(e^{\sqrt{M}}\right)^2}$$
$$-4\ Br\ B\ c_1^3\ M\ c_{65}\ \left(e^{\sqrt{M}}\right)^3 + \frac{2\ Br\ c_2\ c_{72}}{\sqrt{M}}$$

$$c_{140} = \frac{24\ Br\ c_2\ c_{70}}{M} - \frac{36\ Br\ c_2\ c_{70}}{M^2} - \frac{2\ Br\ c_2\ c_{72}}{M} + \frac{4\ Br\ c_2\ c_{71}}{\sqrt{M}} - \frac{8\ Br\ c_2\ c_{71}}{M^{3/2}}$$
$$-72\ Br\ B\ c_1\ \sqrt{M}\ c_2^2\ c_{71} + 144\ Br\ B\ c_1^2\ c_2\ c_{70}\ \left(e^{\sqrt{M}}\right)^2 + \frac{24\ Br\ B\ c_2^3\ \sqrt{M}\ c_{71}}{\left(e^{\sqrt{M}}\right)^2}$$
$$-432\ Br\ B\ c_1\ c_2^2\ c_{70} - \frac{2\ Br\ c_2\ c_{65}}{\sqrt{M}\ e^{\sqrt{M}}} + \frac{144\ Br\ B\ c_2^3\ c_{70}}{\left(e^{\sqrt{M}}\right)^2} + \frac{2\ Br\ c_1\ c_{65}\ e^{\sqrt{M}}}{\sqrt{M}}$$
$$+\frac{12\ Br\ B\ c_2^3\ \sqrt{M}\ c_{72}}{\left(e^{\sqrt{M}}\right)^2} + 24\ Br\ B\ c_1^2\ \sqrt{M}\ c_2\ c_{71}\ \left(e^{\sqrt{M}}\right)^2$$

$$c_{141}=-\frac{2\,Br\,c_2\,c_{71}}{M}+\frac{6\,Br\,c_2\,c_{70}}{\sqrt{M}}-\frac{12\,Br\,c_2\,c_{70}}{M^{3/2}}-108\,Br\,B\,c_1\,c_2^2\,c_{70}\,\sqrt{M}$$
$$+36\,Br\,B\,c_1^2\,c_2\,c_{70}\,\sqrt{M}\,\left(e^{\sqrt{M}}\right)^2$$

$$c_{142}=-\frac{2\,Br\,c_2\,c_{70}}{M}$$

$$c_{143}=-\frac{1}{2}\,Br\,c_1\,c_{64}-\frac{1}{2}\,\frac{Br\,c_2\,c_{68}}{M}+Br\,c_1\,c_{66}\,e^{\sqrt{M}}-\frac{Br\,c_2\,c_{66}}{e^{\sqrt{M}}}+Br\,B\,c_1^3\,M\,c_{65}$$
$$+\frac{1}{2}\,\frac{Br\,c_2\,c_{66}}{M\,e^{\sqrt{M}}}-6\,Br\,B\,c_1^3\,c_{71}\,e^{\sqrt{M}}-\frac{1}{2}\,\frac{Br\,c_1\,c_{66}\,e^{\sqrt{M}}}{M}+\frac{1}{2}\,\frac{Br\,c_2\,c_{66}}{M}-\frac{1}{2}\,\frac{Br\,c_1\,c_{64}}{M}$$
$$+\frac{1}{2}\,\frac{Br\,c_1\,c_{66}}{M}+\frac{18\,Br\,B\,c_1\,M\,c_2^2\,c_{66}}{e^{\sqrt{M}}}-\frac{27}{2}\,\frac{Br\,B\,c_1^2\,c_2\,c_{70}}{\sqrt{M}\,e^{\sqrt{M}}}-\frac{3\,Br\,B\,c_1\,M\,c_2^2\,c_{64}}{\left(e^{\sqrt{M}}\right)^2}$$
$$-\frac{3\,Br\,B\,c_2\,\sqrt{M}\,c_1^2\,c_{72}}{e^{\sqrt{M}}}-18\,Br\,B\,c_1^2\,M\,c_2\,c_{66}\,e^{\sqrt{M}}+9\,Br\,B\,c_1^2\,M\,c_2\,c_{68}\,\left(e^{\sqrt{M}}\right)^2$$
$$-27\,Br\,B\,c_1\,M\,c_2^2\,c_{68}+9\,Br\,B\,c_1^2\,M\,c_2\,c_{64}-\frac{2\,Br\,B\,c_2^3\,M\,c_{66}}{\left(e^{\sqrt{M}}\right)^3}$$
$$+2\,Br\,B\,c_1^3\,M\,c_{66}\,\left(e^{\sqrt{M}}\right)^3+\frac{6\,Br\,B\,c_1^2\,c_2\,c_{71}}{e^{\sqrt{M}}}+3\,Br\,B\,c_1^3\,\sqrt{M}\,c_{72}\,e^{\sqrt{M}}$$
$$-3\,Br\,B\,c_1^3\,M\,c_{64}\left(e^{\sqrt{M}}\right)^2+\frac{27}{2}\,\frac{Br\,B\,c_1^3\,c_{70}\,e^{\sqrt{M}}}{\sqrt{M}}+\frac{9\,Br\,B\,c_2^3\,M\,c_{68}}{\left(e^{\sqrt{M}}\right)^2}$$
$$+\frac{3}{2}\,Br\,c_2\,c_{68}$$

$$c_{144}=\frac{18\,Br\,B\,c_1^2\,c_2\,c_{70}}{e^{\sqrt{M}}}+6\,Br\,B\,c_1^3\,\sqrt{M}\,c_{71}\,e^{\sqrt{M}}-18\,Br\,B\,c_1^3\,c_{70}\,e^{\sqrt{M}}$$
$$-\frac{1}{2}\,\frac{Br\,c_2\,c_{66}}{\sqrt{M}\,e^{\sqrt{M}}}+\frac{1}{2}\,\frac{Br\,c_1\,c_{66}\,e^{\sqrt{M}}}{\sqrt{M}}-\frac{6\,Br\,B\,c_1^2\,\sqrt{M}\,c_2\,c_{71}}{e^{\sqrt{M}}}$$

$$c_{145}=9\,Br\,B\,c_1^3\,c_{70}\,\sqrt{M}\,e^{\sqrt{M}}-\frac{9\,Br\,B\,c_1^2\,c_2\,c_{70}\,\sqrt{M}}{e^{\sqrt{M}}}$$

$$c_{146} = -\frac{1}{2} Br\, c_2\, c_{65} + \frac{3}{2} Br\, c_1\, c_{69} - \frac{1}{2} \frac{Br\, c_2\, c_{65}}{M} + \frac{1}{2} \frac{Br\, c_1\, c_{67}}{M} + \frac{Br\, c_2\, c_{67}}{e^{\sqrt{M}}} - Br\, c_1\, c_{67}\, e^{\sqrt{M}}$$
$$+ Br\, B\, c_2^3\, M\, c_{64} - \frac{6\, Br\, B\, c_2^3\, c_{71}}{e^{\sqrt{M}}} + \frac{1}{2} \frac{Br\, c_1\, c_{67}\, e^{\sqrt{M}}}{M} - \frac{1}{2} \frac{Br\, c_2\, c_{67}}{M\, e^{\sqrt{M}}} + \frac{1}{2} \frac{Br\, c_2\, c_{67}}{M}$$
$$- \frac{1}{2} \frac{Br\, c_1\, c_{69}}{M} - \frac{18\, Br\, B\, c_2^2\, M\, c_1\, c_{67}}{e^{\sqrt{M}}} + \frac{27}{2} \frac{Br\, B\, c_1\, c_2^2\, c_{70}\, e^{\sqrt{M}}}{\sqrt{M}} + \frac{9\, Br\, B\, c_1\, M\, c_2^2\, c_{69}}{\left(e^{\sqrt{M}}\right)^2}$$
$$+ 3\, Br\, B\, c_2^2\, \sqrt{M}\, c_1\, c_{72}\, e^{\sqrt{M}} - 3\, Br\, B\, c_1^2\, M\, c_2\, c_{65} \left(e^{\sqrt{M}}\right)^2 + 18\, Br\, B\, c_1^2\, M\, c_2\, c_{67}\, e^{\sqrt{M}}$$
$$- 27\, Br\, B\, c_1^2\, M\, c_2\, c_{69} + 9\, Br\, B\, c_2^2\, M\, c_1\, c_{65} - \frac{3\, Br\, B\, c_2^3\, \sqrt{M}\, c_{72}}{e^{\sqrt{M}}}$$
$$+ 9\, Br\, B\, c_1^3\, M\, c_{69} \left(e^{\sqrt{M}}\right)^2 - \frac{3\, Br\, B\, c_2^3\, M\, c_{65}}{\left(e^{\sqrt{M}}\right)^2} - \frac{27}{2} \frac{Br\, B\, c_2^3\, c_{70}}{\sqrt{M}\, e^{\sqrt{M}}} + 6\, Br\, B\, c_1\, c_2^2\, c_{71}\, e^{\sqrt{M}}$$
$$- 2\, Br\, B\, c_1^3\, M\, c_{67} \left(e^{\sqrt{M}}\right)^3 + \frac{2\, Br\, B\, c_2^3\, M\, c_{67}}{\left(e^{\sqrt{M}}\right)^3}$$

$$c_{147} = -\frac{6\, Br\, B\, c_2^3\, \sqrt{M}\, c_{71}}{e^{\sqrt{M}}} + 18\, Br\, B\, c_1\, c_2^3\, c_{70}\, e^{\sqrt{M}} - \frac{18\, Br\, B\, c_2^3\, c_{70}}{e^{\sqrt{M}}} - \frac{1}{2} \frac{Br\, c_2\, c_{67}}{\sqrt{M\, e^{\sqrt{M}}}}$$
$$+ \frac{1}{2} \frac{Br\, c_1\, c_{67}\, e^{\sqrt{M}}}{\sqrt{M}} + 6\, Br\, B\, c_1\, \sqrt{M}\, c_2^2\, c_{71}\, e^{\sqrt{M}}$$

$$c_{148} = -\frac{9\, Br\, B\, c_2^3\, c_{70}\, \sqrt{M}}{e^{\sqrt{M}}} + 9\, Br\, B\, c_1\, c_2^2\, c_{70}\, \sqrt{M}\, e^{\sqrt{M}}$$

$$c_{149} = \frac{2}{9} \frac{Br\, c_2\, c_{68}}{M} - \frac{2}{3} \frac{Br\, c_2\, c_{68}}{e^{\sqrt{M}}} + \frac{2}{3} Br\, c_1\, c_{68}\, e^{\sqrt{M}} - \frac{4}{27} \frac{Br\, c_1\, c_{68}\, e^{\sqrt{M}}}{M}$$
$$- \frac{4}{9} Br\, B\, c_1^3\, \sqrt{M}\, c_{72} + \frac{4}{27} \frac{Br\, c_2\, c_{68}}{M\, e^{\sqrt{M}}} - \frac{8}{9} \frac{Br\, B\, c_1^3\, c_{70}}{\sqrt{M}} - \frac{4}{9} Br\, c_1\, c_{66} + \frac{2}{9} \frac{Br\, c_1\, c_{68}}{M}$$
$$+ \frac{16}{27} Br\, B\, c_1^3\, c_{71} - \frac{2}{9} \frac{Br\, c_1\, c_{66}}{M} + \frac{12\, Br\, B\, c_1\, M\, c_2^2\, c_{68}}{e^{\sqrt{M}}} - \frac{4}{3} \frac{Br\, B\, c_1^2\, M\, c_2\, c_{64}}{e^{\sqrt{M}}}$$
$$- \frac{8}{3} \frac{Br\, B\, c_1\, M\, c_2^2\, c_{66}}{\left(e^{\sqrt{M}}\right)^2} - 12\, Br\, B\, c_1^2\, M\, c_2\, c_{68}\, e^{\sqrt{M}} + 8\, Br\, B\, c_1^2\, M\, c_2\, c_{66}$$
$$- \frac{4}{3} \frac{Br\, B\, c_2^3\, M\, c_{68}}{\left(e^{\sqrt{M}}\right)^3} - \frac{8}{3} Br\, B\, c_1^3\, M\, c_{66} \left(e^{\sqrt{M}}\right)^2 + \frac{4}{3} Br\, B\, c_1^3\, M\, c_{68} \left(e^{\sqrt{M}}\right)^3$$
$$+ \frac{4}{3} Br\, B\, c_1^3\, M\, c_{64}\, e^{\sqrt{M}}$$

$$c_{150} = \frac{16}{9} Br\, B\, c_1^3\, c_{70} - \frac{8}{9} Br\, B\, c_1^3\, \sqrt{M}\, c_{71} + \frac{2}{9} \frac{Br\, c_1\, c_{68}\, e^{\sqrt{M}}}{\sqrt{M}} - \frac{2}{9} \frac{Br\, c_2\, c_{68}}{\sqrt{M}\, e^{\sqrt{M}}}$$

$$c_{151} = -\frac{4}{3}\, Br\, B\, c_1^3\, c_{70}\, \sqrt{M}$$

$$c_{152} = -\frac{4}{9}\, Br\, c_2\, c_{67} + \frac{2}{3}\, \frac{Br\, c_2\, c_{69}}{e^{\sqrt{M}}} - \frac{2}{3}\, Br\, c_1\, c_{69}\, e^{\sqrt{M}} - \frac{4}{27}\, \frac{Br\, c_2\, c_{69}}{M\, e^{\sqrt{M}}} + \frac{8}{9}\, \frac{Br\, B\, c_2^3\, c_{70}}{\sqrt{M}}$$
$$+ \frac{4}{9}\, Br\, B\, c_2^3\, \sqrt{M}\, c_{72} + \frac{4}{27}\, \frac{Br\, c_1\, c_{69}\, e^{\sqrt{M}}}{M} - \frac{2}{9}\, \frac{Br\, c_2\, c_{67}}{M} + \frac{16}{27}\, Br\, B\, c_2^3\, c_{71} + \frac{2}{9}\, \frac{Br\, c_2\, c_{69}}{M}$$
$$+ \frac{2}{9}\, \frac{Br\, c_1\, c_{69}}{M} - \frac{4}{3}\, Br\, B\, c_2^2\, M\, c_1\, c_{65}\, e^{\sqrt{M}} - \frac{8}{3}\, Br\, B\, c_1^2\, M\, c_2\, c_{67}\, \left(e^{\sqrt{M}}\right)^2$$
$$- \frac{12\, Br\, B\, c_1\, M\, c_2^2\, c_{69}}{e^{\sqrt{M}}} + 12\, Br\, B\, c_1^2\, M\, c_2\, c_{69}\, e^{\sqrt{M}} + 8\, Br\, B\, c_2^2\, M\, c_1\, c_{67}$$
$$+ \frac{4}{3}\, \frac{Br\, B\, c_2^3\, M\, c_{65}}{e^{\sqrt{M}}} - \frac{4}{3}\, Br\, B\, c_1^3\, M\, c_{69}\, \left(e^{\sqrt{M}}\right)^3 + \frac{4}{3}\, \frac{Br\, B\, c_2^3\, M\, c_{69}}{\left(e^{\sqrt{M}}\right)^3} - \frac{8}{3}\, \frac{Br\, B\, c_2^3\, M\, c_{67}}{\left(e^{\sqrt{M}}\right)^2}$$

$$c_{153} = \frac{16}{9}\, Br\, B\, c_2^3\, c_{70} + \frac{2}{9}\, \frac{Br\, c_1\, c_{69}\, e^{\sqrt{M}}}{\sqrt{M}} - \frac{2}{9}\, \frac{Br\, c_2\, c_{69}}{\sqrt{M}\, e^{\sqrt{M}}}$$

$$c_{154} = \frac{4}{3}\, Br\, B\, c_2^3\, c_{70}\, \sqrt{M}$$

$$c_{155} = -\frac{9}{4}\, \frac{Br\, B\, c_1\, M\, c_2^2\, c_{68}}{\left(e^{\sqrt{M}}\right)^2} - \frac{3}{8}\, Br\, c_1\, c_{68} - \frac{1}{4}\, Br\, B\, c_1^3\, M\, c_{64}$$
$$- \frac{9}{4}\, Br\, B\, c_1^3\, M\, c_{68} \left(e^{\sqrt{M}}\right)^2 + \frac{27}{4}\, Br\, B\, c_1^2\, M\, c_2\, c_{68} - \frac{1}{8}\, \frac{Br\, c_1\, c_{68}}{M}$$
$$+ \frac{3}{2}\, Br\, B\, c_1^3\, M\, c_{66}\, e^{\sqrt{M}} - \frac{3}{2}\, \frac{Br\, B\, c_1^2\, M\, c_2\, c_{66}}{e^{\sqrt{M}}}$$

$$c_{156} = -\frac{1}{4}\, Br\, B\, c_2^3\, M\, c_{65} + \frac{3}{2}\, \frac{Br\, B\, c_2^3\, M\, c_{67}}{e^{\sqrt{M}}} - \frac{3}{8}\, Br\, c_2\, c_{69} - \frac{9}{4}\, Br\, B\, c_1^2\, M\, c_2\, c_{69} \left(e^{\sqrt{M}}\right)^2$$
$$- \frac{1}{8}\, \frac{Br\, c_2\, c_{69}}{M} + \frac{27}{4}\, Br\, B\, c_1\, M\, c_2^2\, c_{69} - \frac{9}{4}\, \frac{Br\, B\, c_2^3\, M\, c_{69}}{\left(e^{\sqrt{M}}\right)} - \frac{3}{2}\, Br\, B\, c_2^2\, M\, c_1\, c_{67}\, e^{\sqrt{M}}$$

$$c_{157} = -\frac{36}{25}\, \frac{Br\, B\, c_1^2\, M\, c_2\, c_{68}}{e^{\sqrt{M}}} + \frac{36}{25}\, Br\, B\, c_1^3\, M\, c_{68}\, e^{\sqrt{M}} - \frac{8}{25}\, Br\, B\, c_1^3\, M\, c_{66}$$

$$c_{158} = \frac{36}{25}\, \frac{Br\, B\, c_2^3\, M\, c_{69}}{e^{\sqrt{M}}} - \frac{8}{25}\, Br\, B\, c_2^3\, M\, c_{67} - \frac{36}{25}\, Br\, B\, c_1\, M\, c_2^2\, c_{69}\, e^{\sqrt{M}}$$

$$c_{159} = -\frac{1}{3}\, Br\, B\, c_1^3\, M\, c_{68}$$

$$c_{160} = -\frac{1}{3}\, Br\, B\, c_2^3\, M\, c_{69}$$

$$c_{161} = -18\,Br\,B\,c_1\,M^2\,c_2^2\,c_{64} + 6\,Br\,B\,c_2^3\,M^2\,c_{68} + 6\,Br\,B\,c_1^3\,M^2\,c_{69}$$
$$+ c_{79} + 6\,Br\,B\,c_2\,M^2\,c_1^2\,c_{64}\left(e^{\sqrt{M}}\right)^2 - Br\,c_1\,c_{65} + Br\,c_1\,c_{73}$$
$$+12\,Br\,B\,c_2^2\,M^2\,c_1\,e^{\sqrt{M}}\,c_{66} - 18\,Br\,B\,c_1^2\,M^{3/2}\,c_2\,e^{\sqrt{M}}\,c_{72}$$
$$+\frac{6\,Br\,B\,c_1\,M^2\,c_2^2\,c_{65}}{\left(e^{\sqrt{M}}\right)^2} + \frac{12\,Br\,B\,c_1^2\,M^2\,c_2\,c_{67}}{e^{\sqrt{M}}} + \frac{18\,Br\,B\,c_1\,M^{3/2}\,c_2^2\,c_{72}}{e^{\sqrt{M}}}$$
$$+ Br\,c_2\,c_{73} - Br\,c_2\,c_{64} - 18\,Br\,B\,c_1^2\,M^2\,c_2\,c_{65} + Br\,c_1\,M\,c_{65} + \frac{6\,Br\,B\,c_2^3\,M^2\,c_{64}}{\left(e^{\sqrt{M}}\right)^2}$$
$$+6\,Br\,B\,c_1^3\,M^2\,c_{65}\left(e^{\sqrt{M}}\right)^2 + 2\,Br\,B\,c_1^3\,M^{3/2}\left(e^{\sqrt{M}}\right)^3 c_{72} - \frac{12\,Br\,B\,c_2^3\,M^2\,c_{66}}{e^{\sqrt{M}}}$$
$$-12\,Br\,B\,c_1^3\,M^2\,e^{\sqrt{M}}\,c_{67} - \frac{2\,Br\,B\,c_2^3\,M^{3/2}\,c_{72}}{\left(e^{\sqrt{M}}\right)^3} + Br\,c_1\sqrt{M}\,e^{\sqrt{M}}\,c_{72}$$
$$-\frac{Br\,c_2\sqrt{M}\,c_{72}}{e^{\sqrt{M}}} + Br\,c_2\,M\,c_{64}$$

$$c_{162} = \frac{1}{3}Br\,c_1\,c_{72} + \frac{12\,Br\,B\,c_1\,M^{3/2}\,c_2^2\,c_{71}}{e^{\sqrt{M}}} - 12\,Br\,B\,c_1^2\,M^{3/2}\,c_2\,e^{\sqrt{M}}\,c_{71}$$
$$+\frac{1}{3}Br\,c_2\,c_{72} + \frac{4}{3}Br\,B\,c_1^3\,M^{3/2}\left(e^{\sqrt{M}}\right)^3 c_{71} - \frac{4}{3}\frac{Br\,B\,c_2^3\,M^{3/2}\,c_{71}}{\left(e^{\sqrt{M}}\right)^3}$$
$$-\frac{1}{3}\frac{Br\,c_2\sqrt{M}\,c_{73}}{e^{\sqrt{M}}} + \frac{1}{3}Br\,c_1\sqrt{M}\,e^{\sqrt{M}}\,c_{73} - \frac{2}{3}\frac{Br\,c_2\sqrt{M}\,c_{71}}{e^{\sqrt{M}}}$$
$$+\frac{2}{3}Br\,c_1\sqrt{M}\,e^{\sqrt{M}}\,c_{71}$$

$$c_{163} = \frac{1}{6}Br\,c_2\,c_{71} + \frac{1}{6}Br\,c_1\,c_{71} + \frac{9\,Br\,B\,c_1\,M^{3/2}\,c_2^2\,c_{70}}{e^{\sqrt{M}}} - 9\,Br\,B\,c_1^2\,M^{3/2}\,c_2\,e^{\sqrt{M}}\,c_{70}$$
$$+Br\,B\,c_1^3\,M^{3/2}\left(e^{\sqrt{M}}\right)^3 c_{70} - \frac{Br\,B\,c_2^3\,M^{3/2}\,c_{70}}{\left(e^{\sqrt{M}}\right)^3} - \frac{1}{2}\frac{Br\,c_2\sqrt{M}\,c_{70}}{e^{\sqrt{M}}}$$
$$+\frac{1}{2}Br\,c_1\sqrt{M}\,e^{\sqrt{M}}\,c_{70} + \frac{1}{6}Br\,c_1\sqrt{M}\,e^{\sqrt{M}}\,c_{72} - \frac{1}{6}\frac{Br\,c_2\sqrt{M}\,c_{72}}{e^{\sqrt{M}}}$$

$$c_{164} = \frac{1}{10}Br\,c_1\,c_{70} - \frac{1}{10}\frac{Br\,c_2\sqrt{M}\,c_{71}}{e^{\sqrt{M}}} + \frac{1}{10}Br\,c_1\sqrt{M}\,e^{\sqrt{M}}\,c_{71}$$

$$c_{165} = \frac{1}{15}Br\,c_1\sqrt{M}\,e^{\sqrt{M}}\,c_{70}$$