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Effect of Heat Transfer on Peristaltic Transport of a Johnson Segalman Fluid Through a Porous Medium in an Inclined Asymmetric Channel

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Abstract

In This paper the influence of heat transfer on peristaltic transport of Johnson Segalman fluid in an inclined asymmetric channel are investigated theoretically and graghically .Aregular perturbation method is used to obtain the analytical solutions for the streame functions, temperature fields, axial pressure gradient, and pressure rise.The effects of the physical parameters of the problem on these distributions are discussed and illustrated graphically through a set of figures.

Keywords: Johnson Segalman fluid; Peristaltic transport; Heat transfer; Porous medium; Inclined asymmetric channel

1. Introduction

The Johnson- Segalman model is a viscoelastic fluid model which was developed to allow for non affine deformation [19]. Recently, this model has been used by a number of researchers [20,24,25] to explain the “spurt” phenomenon. Experimentalists usually associate a spurt with a slip at the wall , and on this issue experiments have been carried out [21,22,26,27,33] . Also Rao and Rajagopal [34] discussed three distinct flows of a Johnson-Segalman fluid . The three flows are cylindrical poiseuill flow . The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, blood pumps in heart lung machine and transport of corrosive fluids where the contact of the fluid with the machinery parts is prohibited. Since the first investigation of Latham [23], anumber of analytical, numerical and experimental [1,2,3,6-10] studies of peristaltic flows of different fluid have been reported under different conditions with reference to physiological and mechanical situations. The peristaltic flows can be divided to Newtonian and non-Newtonian flows that have been reported analytically, numerically, and experimentally by anumber of researchers [29-32].Although most prior studies of peristaltic transport have focused on Newtonian fluids, there are also studies involving non-Newtonian fluid [11-15]. Srinivas and Pushparaj [36] discussed the non-lineare peristaltic transport in an inclined asymmetric channel. Hayat et al.[26] investigated the peristaltic transport of a Johnson-Segalman fluid in an asymmetric channel. Reddy and Roju [35] studied the non-linear peristaltic pumping of Johnson-Segalman fluid in an asymmetric channel under the effect of a magnetic field. Recently, Hayat et. [16] discussed the peristaltic flow in an asymmetric channel for a J-S fluid and previously the examiner Haroun [17] discussed the peristaltic flow in an inclined asymmetric channel for fourth grad fluid. He observed that with the increase in inclined angle the trapped bolus volume increase.

Considering the importance of heat transfer in peristaltic an attempt is made to study the combined effects of heat transfer and inclined angle on the peristaltic transport at a Johnson-Segalman fluid in an asymmetric channel. Nadeem and Noreen Sher Akbar [28] are studied the effect of heat transfer on peristaltic transport of a J-S fluid in an inclined asymmetric channel. El Shehawey and Husseny[4] and El Shehawey et al. [5] studied the peristaltic mechanism of a Newtonian fluid through a porous medium. Hall effects on peristaltic flow of a Maxwell fluid through a porous medium in a channel was studied by Hayat et. [18].

In the present note, a mathematical model is presented to understand the interaction between peristalsis and heat transfer for the motion of a viscous in compressible fluid in a tow-dimentional asymmetric inclined channel. The momentum and energy equations have been linearized under long –wavelength and low Reynold's number assumptions and analytical solutions for the flow variables have been obtained.

2. The mathematical model

The constitutive equations for an incompressible Johnson-Segalman fluid are given by

2.1. Governing equations

Consider an incompressible, Johnson-Segalman fluid confined in a two-dimensional infinit inclined asymmetric channel of width d_1+d_2 (see Fig. 1). We consider an infinite wave train travelling with velocity c along the channel walls. The asymmetry in the channel is induced by assuming the peristaltic wave train on the walls to have different amplitudes and phase. The resulting asymmetric channel walls are defined as

$$Y=\bar{h}_1(\bar{X},\bar{t})=d_1+a_1\sin\left[\frac{2\pi}{\lambda}(\bar{X}-c\bar{t})\right],\qquad \text{upperwall,} \tag{1}$$

$$Y=\bar{h}_2(\bar{X},\bar{t})=-d_2-a_2\sin\left[\frac{2\pi}{\lambda}(\bar{X}-c\bar{t})+\phi\right],\qquad \text{lowerwall.} \tag{2}$$

Here a_1 and a_2 are the amplitude of the waves, λ is the wavelength and $\phi \in [0,\pi]$ is the phase difference. Note that $\phi = 0$ corresponds to an asymmetric channel with waves out of phase and $\phi = \pi$ describes the case where waves are in phase. Moreove a_1, a_2, d_1, d_2 and ϕ satisfy the following inqualit

$$a_1^2+a_2^2+2a_1a_2\cos\phi \le (d_1+d_2)^2 \tag{3}$$

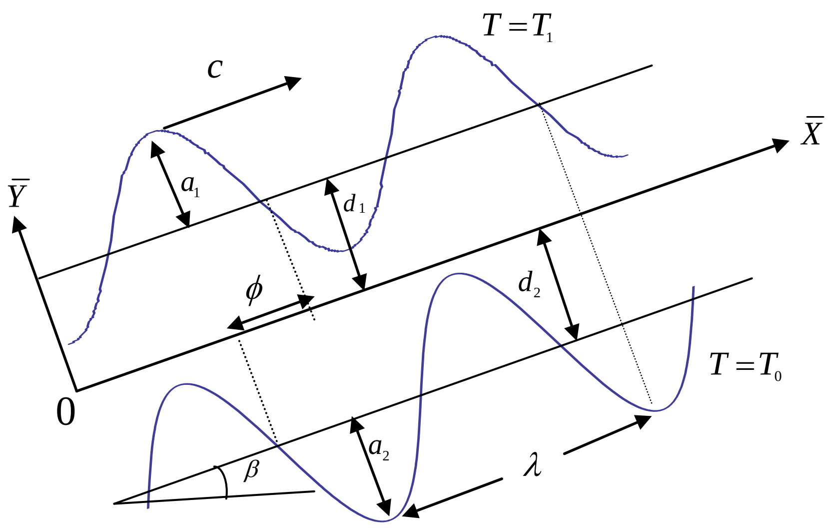

Fig.1. Schematic diagram of the problem

The equations governing the flow of an incompressible fluid are

$$\text{div } \bar{\mathbf{V}}=0, \qquad \text{div } \bar{\boldsymbol{\sigma}} + \rho\bar{\mathbf{f}} = \rho\frac{\mathrm{d}\bar{\mathbf{V}}}{\mathrm{d}\bar{t}}, \tag{4}$$

where $\bar{\mathbf{V}}$ is the velocity, $\bar{\mathbf{f}}$ is the body force per unit mass , ρ is the fluid density, $\mathrm{d}/\mathrm{d}\bar{t}$ is the material derivative and $\bar{\boldsymbol{\sigma}}$ is the cauchy stress tensor given by[19]

$$\bar{\boldsymbol{\sigma}} = -p\bar{\mathbf{I}} + \bar{\mathbf{T}}, \tag{5}$$

$$\bar{\mathbf{T}}=2\mu\bar{\mathbf{D}}+\bar{\mathbf{S}}, \tag{6}$$

$$\bar{\boldsymbol{S}} + m\left[\frac{d\bar{\boldsymbol{S}}}{d\bar{t}} + \bar{\boldsymbol{S}}\left(\bar{\mathbf{W}} - e\bar{\mathbf{D}}\right) + \left(\bar{\mathbf{W}} - e\bar{\mathbf{D}}\right)^T \bar{\mathbf{S}}\right] = 2\eta\bar{\mathbf{D}}, \tag{7}$$

$$\bar{\mathbf{D}} = \frac{1}{2}[\bar{\mathbf{L}} + \bar{\mathbf{L}}^T], \qquad \bar{\mathbf{W}} = \frac{1}{2}[\bar{\mathbf{L}} - \bar{\mathbf{L}}^T], \qquad \bar{\mathbf{L}} = \text{grad } \bar{\mathrm{V}} \tag{8}$$

and bars indicate the quntities in the dimensional coordinates. The equations above include the scalar pressure p , the identity tensor $\bar{\mathbf{I}}$, the extra stress $\bar{\mathbf{S}}$, the dynamic viscosities μ and η, the relaxation time m , the slip parameter e and the respective symmetric and skew symmetric part of the velocity gradient, $\bar{\mathbf{D}}$ and $\bar{\mathbf{W}}$. Note, that model (5) reduces to the Maxwell fluid model for e =1 and $\mu = 0$, and for $m = 0 = \mu$ we receive the classical Navier-Stokes fluid model.

The velocity for unsteady tow-dimensional flows is defined as

$$\bar{V}=\left[\bar{U}\left(\bar{X},\bar{Y},\bar{t}\right),\bar{V}\left(\bar{X},\bar{Y},\bar{t}\right),0\right]. \tag{9}$$

From Eqs. (4)-(9) we obtain,

$$\frac{\partial\bar{U}}{\partial\bar{X}}+\frac{\partial\bar{V}}{\partial\bar{Y}}=0 \tag{10}$$

$$\rho\left(\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}}\right)\bar{U}=-\frac{\partial\bar{p}}{\partial\bar{X}}+\mu\left(\frac{\partial^2}{\partial\bar{X}^2}+\frac{\partial^2}{\partial\bar{Y}^2}\right)\bar{U}+\frac{\partial\bar{S}_{\bar{X}\bar{X}}}{\partial\bar{X}}+\frac{\partial\bar{S}_{\bar{X}\bar{Y}}}{\partial\bar{Y}}-\frac{\mu}{\kappa}\bar{U}+\rho g\sin\beta, \tag{11}$$

$$\rho\left(\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}}\right)\bar{V}=-\frac{\partial\bar{p}}{\partial\bar{Y}}+\mu\left(\frac{\partial^2}{\partial\bar{X}^2}+\frac{\partial^2}{\partial\bar{Y}^2}\right)\bar{V}+\frac{\partial^2\bar{S}_{\bar{X}\bar{Y}}}{\partial\bar{X}}+\frac{\partial^2\bar{S}_{\bar{Y}\bar{Y}}}{\partial\bar{Y}}-\frac{\mu}{\kappa}\bar{V}-\rho g\cos\beta, \tag{12}$$

$$2\eta\frac{\partial\bar{U}}{\partial\bar{X}}=\bar{S}_{\bar{X}\bar{X}}+m\left[\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}}\right]\bar{S}_{\bar{X}\bar{X}}-2em\bar{S}_{\bar{X}\bar{X}}\frac{\partial\bar{U}}{\partial\bar{X}}+m\left[(1-e)\frac{\partial\bar{V}}{\partial\bar{X}}-(1+e)\frac{\partial\bar{U}}{\partial\bar{Y}}\right]\bar{S}_{\bar{X}\bar{Y}}, \tag{13}$$

$$\eta\left(\frac{\partial\bar{U}}{\partial\bar{Y}}+\frac{\partial\bar{V}}{\partial\bar{X}}\right)=\bar{S}_{\bar{X}\bar{Y}}+m\left[\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}}\right]\bar{S}_{\bar{X}\bar{Y}}+\frac{m}{2}\left[(1-e)\frac{\partial\bar{U}}{\partial\bar{Y}}-(1+e)\frac{\partial\bar{V}}{\partial\bar{X}}\right]\bar{S}_{\bar{X}\bar{X}}$$

$$+\frac{m}{2}\left[(1-e)\frac{\partial\bar{V}}{\partial\bar{X}}-(1+e)\frac{\partial\bar{U}}{\partial\bar{Y}}\right]\bar{S}_{\bar{Y}\bar{Y}}, \tag{14}$$

$$2\eta\frac{\partial\bar{V}}{\partial\bar{Y}}=\bar{S}_{\bar{Y}\bar{Y}}+m\left[\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}}\right]\bar{S}_{\bar{Y}\bar{Y}}-2em\bar{S}_{\bar{Y}\bar{Y}}\frac{\partial\bar{V}}{\partial\bar{Y}}+m\left[(1-e)\frac{\partial\bar{U}}{\partial\bar{Y}}-(1+e)\frac{\partial\bar{V}}{\partial\bar{X}}\right]\bar{S}_{\bar{X}\bar{Y}}, \tag{15}$$

$$\rho c_p\left(\frac{\partial T}{\partial\bar{t}}+\bar{U}\frac{\partial T}{\partial\bar{X}}+\bar{V}\frac{\partial T}{\partial\bar{Y}}\right)=\bar{S}_{\bar{X}\bar{X}}\frac{\partial\bar{U}}{\partial\bar{X}}+\bar{S}_{\bar{Y}\bar{Y}}\frac{\partial\bar{V}}{\partial\bar{Y}}+\bar{S}_{\bar{X}\bar{Y}}\left(\frac{\partial\bar{U}}{\partial\bar{Y}}+\frac{\partial\bar{V}}{\partial\bar{X}}\right)+k\left(\frac{\partial^2T}{\partial\bar{X}^2}+\frac{\partial^2T}{\partial\bar{Y}^2}\right). \tag{16}$$

In the fixed frame $(\bar{X},\bar{Y})$ the motion is unsteady, while it becomes steady in the wave frame $(\bar{x},\bar{y})$ given by

$$\bar{x}=\bar{X}-ct, \qquad \bar{y}=\bar{Y}, \qquad \bar{u}=\bar{U}-c, \qquad \bar{v}=\bar{V}, \tag{17}$$

when moving away in direction of the wave from the fixed frame $(\bar{X},\bar{Y})$ with speed *c*. Here $\bar{u}$, $\bar{v}$ and $\bar{U}$, $\bar{V}$ are the velocity components in the wave frame and in the fixed frame, respectively.

We put Eq. (17) into Eqs. (11)-(16) and to write contiuuity and motion equations in dimensionless form we introduce the following new quantities

$$x=\frac{\bar{x}}{\lambda},\quad y=\frac{\bar{y}}{d_1},\quad u=\frac{\bar{u}}{c},\quad v=\frac{\bar{v}}{c},\quad h_1=\frac{\bar{h}_1}{d_1},\quad h_2=\frac{\bar{h}_2}{d_1},$$

$$S=\frac{d_1}{\mu c}\bar{S}(\bar{x}),\quad p=\frac{2\pi d_1^{\,2}}{\lambda(\mu+\eta)c}\bar{p}(\bar{x}),\quad \delta=\frac{d_1}{\lambda},\quad d=\frac{d_2}{d_1}, \tag{18}$$

$$a=\frac{a_1}{d_1},\quad b=\frac{a_2}{d_1},\quad R_e=\frac{\rho c d_1}{\mu},\quad W_i=\frac{mc}{d_1},\quad \bar{t}=\frac{ct}{\lambda},$$

$$E_r=\frac{e^2}{c_p(T_0-T_1)},\quad F_r=\frac{c^2}{gd_1},\quad P_r=\frac{\rho c_p v}{k},\quad \theta=\frac{T-T_1}{T_0-T_1},\quad D_r=\frac{k}{d_1^{\,2}},$$

Where ρ is the density, μ is the coefficient of viscosity of the fluid, p is the pressure, β is the inclination of the channel with the horizontal, g is the acceleration due to gravity, T is the temperature, c_p is the specific heat at constant pressure and k is the thermal conductivity.

we have

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{19}$$

$$R_e\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\left(\frac{\mu+\eta}{\mu}\right)\frac{\partial p}{\partial x}+\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}$$

$$-\frac{1}{D_r}(u+1)+\frac{R_e}{F_r}\sin\beta, \tag{20}$$

$$R_e\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\left(\frac{\mu+\eta}{\mu}\right)\frac{\partial p}{\partial y}+\delta^2\left[\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]+\delta^2\frac{\partial S_{xy}}{\partial y}$$

$$+\delta\frac{\partial S_{yy}}{\partial y}-\frac{\delta^2}{D_r}v-\frac{R_e}{F_r}\cos\beta, \tag{21}$$

$$\left(\frac{2\eta\delta}{\mu}\right)\frac{\partial u}{\partial x}=S_{xx}+W_i\delta\left[u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right]S_{xx}-2eW_i\delta S_{xx}\frac{\partial u}{\partial x}$$

$$+W_i\left[\delta^2(1-e)\frac{\partial v}{\partial x}-(1+e)\frac{\partial u}{\partial y}\right]S_{xy}, \tag{22}$$

$$\frac{\eta}{\mu}\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)=S_{xy}+W_i\delta\left[u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right]S_{xy}+\frac{W_i}{2}\left[(1-e)\frac{\partial u}{\partial y}-(1+e)\delta^2\frac{\partial v}{\partial x}\right]S_{xx}$$

$$+\frac{W_i}{2}\left[(1-e)\delta^2\frac{\partial v}{\partial x}-(1+e)\frac{\partial v}{\partial x}\right]S_{yy}, \tag{23}$$

$$\left(\frac{2\eta\delta}{\mu}\right)\frac{\partial v}{\partial y}=S_{yy}+W_i\delta\left[u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right]S_{yy}-2eW_i\delta S_{yy}\frac{\partial v}{\partial y}$$

$$+\left[(1-e)\frac{\partial u}{\partial y}-(1+e)\delta^2\frac{\partial v}{\partial x}\right]S_{xy} \tag{24}$$

$$R_e\delta\left(u\frac{\partial\theta}{\partial x}+\delta v\frac{\partial\theta}{\partial y}\right)=E_r\left[S_{xx}\delta\frac{\partial u}{\partial x}+S_{xy}\delta\frac{\partial v}{\partial x}+S_{xy}\frac{\partial u}{\partial y}+S_{yy}\delta\frac{\partial v}{\partial y}\right]$$

$$+\frac{1}{P_r}\left[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right]. \tag{25}$$

Writing Eqs. (19)-(25) in terms of the stream function $\psi(x,y)$ defined by

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x}, \tag{26}$$

we get

$$\delta R_e\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial y}\right]=-\left(\frac{\mu+\eta}{\mu}\right)\frac{\partial p}{\partial x}+\delta^2\frac{\partial^2\psi}{\partial x\partial y}+\frac{\partial^3\psi}{\partial y^3}+\delta\frac{\partial S_{xx}}{\partial x}$$

$$+\frac{\partial S_{xy}}{\partial y}-\frac{1}{D_r}\left(\frac{\partial\psi}{\partial y}+1\right)+\frac{R_e}{F_r}\sin\beta, \tag{27}$$

$$-\delta^3R_e\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial x}\right]=-\left(\frac{\mu+\eta}{\mu}\right)\frac{\partial p}{\partial y}-\delta^2\left[\delta^2\frac{\partial^3\psi}{\partial x^3}+\frac{\partial^3\psi}{\partial x\partial y^2}\right]+\delta^2\frac{\partial S_{xy}}{\partial x}$$

$$+\delta\frac{\partial S_{yy}}{\partial y}\frac{\delta^2}{D_r}+\frac{\partial\psi}{\partial x}-\delta\frac{R_e}{F_r}\cos\beta, \tag{28}$$

$$\left(\frac{2\eta\delta}{\mu}\right)\frac{\partial^2\psi}{\partial x\partial y}=S_{xx}+W_i\delta\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\right]S_{xx}-2eW_i\delta\frac{\partial^2\psi}{\partial x\partial y}$$

$$-W_i\left[\delta^2(1-e)\frac{\partial^2\psi}{\partial x^2}+(1+e)\frac{\partial^2\psi}{\partial y^2}\right]S_{xy}, \tag{29}$$

$$\frac{\eta}{\mu}\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)-S_{xy}+W_i\delta\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\right]S_{xy}$$

$$+\frac{W_i}{2}\left[(1-e)\frac{\partial^2\psi}{\partial y^2}+\delta^2(1+e)\frac{\partial^2\psi}{\partial x^2}\right]S_{xx}+\frac{W_i}{2}\left[\delta^2(1-e)\frac{\partial^2\psi}{\partial x^2}+(1+e)\frac{\partial^2\psi}{\partial y^2}\right]S_{yy}, \tag{30}$$

$$\left(\frac{2\eta\delta}{\mu}\right)\frac{\partial^2\psi}{\partial x\partial y}=S_{yy}+W_i\delta\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\right]S_{yy}+2eW_i\delta S_{yy}\frac{\partial^2\psi}{\partial x\partial y}$$

$$+W_i\left[(1-e)\frac{\partial^2\psi}{\partial y^2}+\delta^2(1+e)\frac{\partial^2\psi}{\partial x^2}\right]S_{xy}, \tag{31}$$

$$\delta R_e\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\delta\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)\right]=E_r\left[S_{xx}\delta\frac{\partial^2\psi}{\partial x\partial y}-S_{xy}\delta\frac{\partial^2\psi}{\partial x^2}+S_{xy}\frac{\partial^2\psi}{\partial y^2}-S_{yy}\delta\frac{\partial^2\psi}{\partial x\partial y}\right]$$

$$+\frac{1}{P_r}\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right). \tag{32}$$

where δ is wave number, R_e is the Renold's number, W_i is the Weissenberg number, F_r is the Froud number, P_r is the Prandtl number and D_r is the Darcy number. The continuity equation is identically satisfied.

2.2 Rate of volume flow and boundary conditions

The dimensional rate of fluid flow in the fixed frame $(\bar{X},\bar{Y})$ is

$$Q=\int_{\bar{h}_1(\bar{X})}^{\bar{h}_2(\bar{X})}\bar{U}(\bar{X},\bar{Y},\bar{t})\,d\bar{Y} \tag{33}$$

In the wave frame $(\bar{x},\bar{y})$ Eq. (33) reduces to

$$q=\int_{\bar{h}_1(\bar{x})}^{\bar{h}_2(\bar{x})}\bar{u}(\bar{x},\bar{y})\,d\bar{y} \tag{34}$$

By Eq.(17), the above rates are related in the following expression

$$Q = q + c\bar{h}_1 - c\bar{h}_2 \tag{35}$$

Applying the averaged flow

$$\bar{Q} = \frac{1}{\tau}\int_0^{\tau} Q d\bar{t} \tag{36}$$

Over a period $(\tau = \lambda \backslash c)$ at afixed position $\bar{X}$, we receive

$$Q = q + c\bar{d}_1 - c\bar{d}_2. \tag{37}$$

With the definition of the dimensionless time averaged flows

$$\Theta \equiv \frac{\bar{Q}}{c\bar{d}_1}, \qquad F \equiv \frac{q}{c\bar{d}_1}, \tag{38}$$

in the fixed and moving frames, respectively, we can write Eq(37) as

$$\Theta = F + 1 + d \tag{39}$$

where

$$F = \int_{h_1(x)}^{h_2(x)} \frac{\partial \psi}{dy} dy = \psi(h_1) - \psi(h_2) \tag{40}$$

and the dimensionless surface of the peristaltic walls are

$$h_1(x) = 1 + a\sin(2\pi x), \qquad h_2(x) = -d - b\sin(2\pi x + \phi), \tag{41}$$

where the inequality

$$a^2 + b^2 + 2ab\cos\phi \le (1+d)^2 \tag{42}$$

holds. The dimensionless boundary conditions in the wave frame are

$$\left.\begin{aligned}
&\psi = \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1 \quad \text{at} \quad y = h_1(x), \\
&\psi = -\frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1 \quad \text{at} \quad y = h_2(x) \\
&\theta = 1 \quad \text{at} \quad y = h_1(x), \qquad \theta = 0 \quad \text{at} \quad y = h_2(x),
\end{aligned}\right\} \tag{43}$$

Here it is pointed out that the conditions on ψ satisfy Eq. (40) and the conditions on $\frac{\partial \psi}{\partial y}$ are no-slip .

2.3 Model equations

under lubrication approach(i.e., neglecting the terms of order δ and Re), from Eqs. (27)-(32) we get

$$\left(\frac{\mu+\eta}{\eta}\right)\frac{\partial p}{\partial x}=\frac{\partial S_{xy}}{\partial y}+\frac{\partial^3\psi}{\partial y^3}-\frac{1}{D_r}\left(\frac{\partial \psi}{\partial y}+1\right)+\frac{R_e}{F_r}\sin\beta, \tag{44}$$

$$\frac{\partial p}{\partial y}=0, \tag{45}$$

$$S_{xx}=W_i\left(1+e\right)\frac{\partial^2\psi}{\partial y^2}S_{xy}, \tag{46}$$

$$\left(\frac{\eta}{\mu}\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)=S_{xy}+\frac{W_i}{2}(1-e)\frac{\partial^2\psi}{\partial y^2}S_{xx}-\frac{W_i}{2}(1+e)\frac{\partial^2\psi}{\partial y^2}S_{yy}, \tag{47}$$

$$S_{yy}=-W_i\left(1-e\right)\frac{\partial^2\psi}{\partial y^2}S_{xy}, \tag{48}$$

$$E_rP_rS_{xy}\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\theta}{\partial y^2}=0, \tag{49}$$

From Eqs. (46)-(48), we write

$$S_{xy}=\frac{\left(\frac{\eta}{\mu}\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)}{1+W_i^2\left(1-e^2\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2}\ , \tag{50}$$

using Eqs.(46),(48) and (50), the Eq.(44) and (49) can be rewritten as

$$\frac{\partial^2}{\partial y^2}\left[\frac{\left(\frac{\eta}{\mu}+1\right)\frac{\partial^2\psi}{\partial y^2}+W_i^2\left(1-e^2\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)^3}{1+W_i^2\left(1-e^2\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2}\right]=0\ , \tag{51}$$

$$\frac{\partial p}{\partial x}=\frac{\partial^3\psi}{\partial y^3}+W_i^2\alpha_1\frac{\partial}{\partial y}\left[\left(\frac{\partial^2\psi}{\partial y^2}\right)^3\right]-\frac{\mu}{(\mu+\eta)D_r}\frac{\partial\psi}{\partial y}+\left(\frac{\mu}{\mu+\eta}\right)\frac{R_e}{F_r}\sin\beta, \tag{52}$$

$$\frac{\partial^2\theta}{\partial y^2}=-E_rP_r\left[\frac{\eta}{\mu}\left(\frac{\partial^2\psi}{\partial y^2}\right)^2-W_i^2\frac{\eta}{\mu}\left(1-e^2\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)^4\right], \tag{53}$$

where $\alpha_1=\dfrac{\left(e^2-1\right)\eta}{(\eta+\mu)}$

3. perturbed systems and perturbation solutions

The Eqs. (51)-(53) are non-lineare and its closed form solution is not possible. thus, we linearize this equations in terms of W_i^2 since W_i is small for the type of flow under consideration. So we expand ψ, p, F and θ as

$$\begin{aligned}\psi&=\psi_0+W_i^2\psi_1+O(W_i^4),\\ p&=p_0+W_i^2p_1+O(W_i^4),\\ F&=F_0+W_i^2F_1+O(W_i^4),\\ \theta&=\theta_0+W_i^2\theta_1+O(W_i^4),\end{aligned} \tag{54}$$

substituting Eq.(54) into Eqs. (51)-(53) and boundary conditions (43), then equating the like powers of W_i^2 we get :

3.1. perturped systems

3.1.1. Zeroth-order system

$$\left.\begin{aligned}&\frac{\partial^4\psi_0}{\partial y^4}=\frac{\mu}{(\mu+\eta)D_r}\frac{\partial^2\psi_0}{\partial y^2},\\ &\frac{\partial p_0}{\partial y}=0,\\ &\frac{\partial p_0}{\partial x}=\frac{\partial^3\psi_0}{\partial y^3}-\frac{\mu}{(\mu+\eta)D_r}\frac{\partial\psi_0}{\partial y}+\left(\frac{\mu}{\mu+\eta}\right)\frac{R_e}{F_r}\sin\beta,\\ &\frac{\partial^2\theta_0}{\partial y^2}=-E_rP_r\left[\frac{\eta}{\mu}\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2\right],\end{aligned}\right\} \tag{55}$$

$$\left.\begin{aligned}
&\psi_0=\frac{F_0}{2}, \qquad \frac{\partial\psi_0}{dy}=-1, \quad \text{at} \quad y=h_1(x),\\
&\psi_0=-\frac{F_0}{2}, \qquad \frac{\partial\psi_0}{dy}=-1, \quad \text{at} \quad y=h_2(x),\\
&\theta_0=1, \quad \text{at} \quad y=h_1(x), \qquad \theta_0=0, \quad \text{at} \quad y=h_2(x),
\end{aligned}\right\} \tag{56}$$

3.1.2. First –order system

$$\left.\begin{aligned}
&\frac{\partial^4\psi_1}{\partial y^4}=\frac{\mu}{(\mu+\eta)D_r}\frac{\partial^2\psi_1}{\partial y^2}-\alpha_1\frac{\partial^2}{\partial y^2}\left[\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^3\right],\\
&\frac{\partial p_1}{\partial y}=0,\\
&\frac{\partial p_1}{\partial x}=\frac{\partial^3\psi_1}{\partial y^3}+\alpha_1\frac{\partial}{\partial y}\left[\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^3\right]-\frac{\mu}{(\mu+\eta)D_r}\frac{\partial\psi_1}{\partial y},\\
&\frac{\partial^2\theta_1}{\partial y^2}=-E_rP_r\left[\frac{2\eta}{\mu}\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2}-\frac{\eta}{\mu}\left(1-e^2\right)\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^4\right],
\end{aligned}\right\} \tag{57}$$

$$\left.\begin{aligned}
&\psi_1=\frac{F_1}{2}, \qquad \frac{\partial\psi_1}{dy}=0, \qquad \text{at} \quad y=h_1(x),\\
&\psi_1=-\frac{F_1}{2}, \qquad \frac{\partial\psi_1}{dy}=0, \qquad \text{at} \quad y=h_2(x),\\
&\theta_1=0, \quad \text{at} \quad y=h_1(x), \qquad \theta_1=0 \quad \text{at} \quad y=h_2(x),
\end{aligned}\right\} \tag{58}$$

3.2 Perturbation solutions

3.2.1. Zeroth-order solution

The solution of Eqs.(55) of zeroth order system, satisfying (56) can be written

$$\psi_0=A_7+A_8y+\frac{(A_5+A_6)\cosh(\sqrt{A_1}\,y)+(A_5-A_6)\sinh(\sqrt{A_1}\,y)}{A_1}, \tag{59}$$

$$\frac{dp_0}{dx}=A_2-A_1A_8, \tag{60}$$

$$\theta_0 = \frac{A_3(\cosh(2\sqrt{A_1}\,\mathrm{y}) - \sinh(2\sqrt{A_1}\,\mathrm{y}))(\mathrm{A}_6{}^2 + \mathrm{A}_5{}^2(\cosh(4\sqrt{A_1}\,\mathrm{y}) + \sinh(4\sqrt{A_1}\,\mathrm{y}))}{4A_1}$$

$$+\frac{4A_1(A_9 + y(A_{10} + A_3 A_5 A_6 y))}{4A_1}. \qquad (61)$$

where

$$A_1 = \frac{\mu}{(\mu+\eta)D_r}, \qquad A_2 = \frac{\mu R_e \sin(\beta)}{(\mu+\eta)F_r}, \qquad A_3 = \frac{-E_r P_r \eta}{\mu},$$

A_5, A_6, A_7, A_8, A_9 and A_{10} are constants can be determinates by using the boundary conditions Eq.(56) .

3.2.2. First-order solution

Substituting the zeroth-order solution (59)-(61) into Eq.(57) and then solving the resulting system with the corresponding boundary conditions Eq.(58),we get

$$\psi_1 = B_3 + B_4 y + \frac{1}{8A_1}(8((B_1 + B_2)\cosh(\sqrt{A_1}\,\mathrm{y}) + (B_1 - B_2)\sinh(\sqrt{A_1}\,\mathrm{y}))$$

$$-6A_5A_6(\mathrm{A}_5(-5 + 2\sqrt{A_1}\,\mathrm{y}) - A_6(5 + 2\sqrt{A_1}\,\mathrm{y}))\cosh(\sqrt{A_1}\,\mathrm{y})$$

$$+(A_5^3 + A_6^3)\cosh(3\sqrt{A_1}y) + (A_5^3 - A_6^3 + 6A_5{}^2 A_6(-5 + 2\sqrt{A_1}y)$$

$$+6A_5A_6{}^2(5 + 2\sqrt{A_1}y) + 2(A_5^3 - A_6^3)\cosh(2\sqrt{A_1}y))\sinh(\sqrt{A_1}y))\alpha_1), \qquad (62)$$

$$\frac{dp_1}{dx} = -A_1 B_4, \qquad (63)$$

$$\theta_1 = B_5 + \frac{1}{4}\Big(\Big(4B_6 y + 4A_3 B_1 B_2 y^2 - 4A_4\cosh(\sqrt{A_1}\,\mathrm{y}) - \sinh(\sqrt{A_1}\,\mathrm{y}\Big)$$

$$\Big(A_6 + A_5\cosh(2\sqrt{A_1}\,\mathrm{y}) + A_5\sinh(2\sqrt{A_1}\,\mathrm{y})\Big)$$

$$+\frac{1}{4A_1}A_3\Big(\cosh(2\sqrt{A_1}\,\mathrm{y}) - \sinh(2\sqrt{A_1}\,\mathrm{y})\Big)$$

$$\Big(B_2{}^2 + B_1{}^2\cosh(4\sqrt{A_1}\,\mathrm{y}) + B_1{}^2\sinh(4\sqrt{A_1}\,\mathrm{y})\Big)\Big)$$

$$+\frac{1}{256A_1}A_3(\cosh(6\sqrt{A_1}\,\mathrm{y})-\sinh(6\sqrt{A_1}\,\mathrm{y}))\alpha_1$$

$$(9A_6{}^6\alpha_1+9A_5{}^6\cosh(12\sqrt{A_1}\,\mathrm{y})\alpha_1+9A_5{}^6\sinh(12\sqrt{A_1}\,\mathrm{y})\alpha_1$$

$$+18A_5{}^3(\cosh(10\sqrt{A_1}\,\mathrm{y})+\sinh(10\sqrt{A_1}\,\mathrm{y}))(-2B_1-3A_5{}^2A_6\alpha_1+3\sqrt{A_1}A_5{}^2A_6y\alpha_1)$$

$$-18A_6{}^3(\cosh(2\sqrt{A_1}\,\mathrm{y})+\sinh(2\sqrt{A_1}\,\mathrm{y}))(2B_2+3A_5A_6{}^2\alpha_1+3\sqrt{A_1}A_5A_6{}^2y\alpha_1)$$

$$+24A_5{}^2(\cosh(8\sqrt{A_1}\,\mathrm{y})+\sinh(8\sqrt{A_1}\,\mathrm{y}))(-6B_2A_5+12B_1A_6-8\sqrt{A_1}B_1A_6y+21A_5{}^2A_6{}^2\alpha_1$$

$$-27\sqrt{A_1}A_5{}^2A_6{}^2\alpha_1+6A_1A_5{}^2A_6{}^2y^2\alpha_1)+24A_6{}^2(\cosh(4\sqrt{A_1}\,\mathrm{y})$$

$$+\sinh(4\sqrt{A_1}\,\mathrm{y}))(12B_2A_5-6B_1A_6+8\sqrt{A_1}B_2A_5y$$

$$+21A_5{}^2A_6{}^2\alpha_1+27\sqrt{A_1}A_5{}^2A_6{}^2\alpha_1+6A_1A_5{}^2A_6{}^2y^2\alpha_1)$$

$$-4A_1A_5A_6y^2(\cosh(6\sqrt{A_1}\,\mathrm{y})+\sinh(6\sqrt{A_1}\,\mathrm{y}))$$

$$\Big(-48B_2A_5-48B_1A_6+32\sqrt{A_1}B_2A_5y-32\sqrt{A_1}B_1A_6y$$

$$-117A_5{}^2A_6{}^2\alpha_1+24A_1A_5{}^2A_6{}^2y^2\alpha_1)\Big) \tag{64}$$

Where

$$\alpha_1=\frac{(n^2-1)\eta}{(\mu+\eta)},\qquad A_4=A_3(1-n^2),$$

B_1, B_2, B_3, B_4, B_5 and B_6 are constants can be determinates by using the boundary conditions Eq. (58) .

Summing up the perturbation results, we find that

$$\psi=A_7+A_8y+\frac{(A_5+A_6)\cosh(\sqrt{A_1}\,\mathrm{y})+(A_5-A_6)\sinh(\sqrt{A_1}\,\mathrm{y})}{A_1}$$

$$W_i\Big(B_3+B_4y+\frac{1}{8A_1}(8((B_1+B_2)\cosh(\sqrt{A_1}\,\mathrm{y})+(B_1-B_2)\sinh(\sqrt{A_1}\,\mathrm{y}))$$

$$-6A_5A_6(A_5(-5+2\sqrt{A_1}\,y)-A_6(5+2\sqrt{A_1}\,y))\cosh(\sqrt{A_1}\,y)$$

$$+(A_5^{\,3}+A_6^{\,3})\cosh(3\sqrt{A_1}y)+(A_5^{\,3}-A_6^{\,3}+6A_5^{\,2}A_6(-5+2\sqrt{A_1}y)$$

$$+6A_5A_6^{\,2}(5+2\sqrt{A_1}y)+2(A_5^{\,3}-A_6^{\,3})\cosh(2\sqrt{A_1}y))\sinh(\sqrt{A_1}y))\alpha_1\Big), \tag{65}$$

$$\frac{dp}{dx}=A_2-A_1A_8-Wi(A_1B_4), \tag{66}$$

$$\theta=\frac{A_3(\cosh(2\sqrt{A_1}\,y)-\sinh(2\sqrt{A_1}\,y))(A_6^{\,2}+A_5^{\,2}(\cosh(4\sqrt{A_1}\,y)+\sinh(4\sqrt{A_1}\,y))}{4A_1}$$

$$+W_i\Big(B_5+\frac{1}{4}\Big(\Big(4B_6y+4A_3B_1B_2y^2-4A_4\cosh(\sqrt{A_1}\,y)-\sinh(\sqrt{A_1}\,y\Big)$$

$$\Big(A_6+A_5\cosh(2\sqrt{A_1}\,y)+A_5\sinh(2\sqrt{A_1}\,y)\Big)$$

$$+\frac{1}{4A_1}A_3\Big(\cosh(2\sqrt{A_1}\,y)-\sinh(2\sqrt{A_1}\,y)\Big)$$

$$\Big(B_2^{\,2}+B_1^{\,2}\cosh(4\sqrt{A_1}\,y)+B_1^{\,2}\sinh(4\sqrt{A_1}\,y)\Big)\Big)$$

$$+\frac{1}{256A_1}A_3(\cosh(6\sqrt{A_1}\,y)-\sinh(6\sqrt{A_1}\,y))\alpha_1$$

$$(9A_6^{\,6}\alpha_1+9A_5^{\,6}\cosh(12\sqrt{A_1}\,y)\alpha_1+9A_5^{\,6}\sinh(12\sqrt{A_1}\,y)\alpha_1$$

$$+18A_5^{\,3}(\cosh(10\sqrt{A_1}\,y)+\sinh(10\sqrt{A_1}\,y))(-2B_1-3A_5^{\,2}A_6\alpha_1+3\sqrt{A_1}A_5^{\,2}A_6y\alpha_1)$$

$$-18A_6^{\,3}(\cosh(2\sqrt{A_1}\,y)+\sinh(2\sqrt{A_1}\,y))(2B_2+3A_5A_6^{\,2}\alpha_1+3\sqrt{A_1}A_5A_6^{\,2}y\alpha_1)$$

$$+24A_5^{\,2}(\cosh(8\sqrt{A_1}\,y)+\sinh(8\sqrt{A_1}\,y))(-6B_2A_5+12B_1A_6-8\sqrt{A_1}B_1A_6y+21A_5^{\,2}A_6^{\,2}\alpha_1$$

$$-27\sqrt{A_1}A_5^{\,2}A_6^{\,2}\alpha_1+6A_1A_5^{\,2}A_6^{\,2}y^2\alpha_1)+24A_6^{\,2}(\cosh(4\sqrt{A_1}\,y)$$

$$+\sinh(4\sqrt{A_1}\,y))(12B_2A_5-6B_1A_6+8\sqrt{A_1}B_2A_5y$$

$$+21A_5^{\,2}A_6^{\,2}\alpha_1+27\sqrt{A_1}A_5^{\,2}A_6^{\,2}\alpha_1+6A_1A_5^{\,2}A_6^{\,2}y^2\alpha_1)$$

$$-4A_1A_5A_6y^2(\cosh(6\sqrt{A_1}\,y)+\sinh(6\sqrt{A_1}\,y))$$

$$\Big(-48B_2A_5-48B_1A_6+32\sqrt{A_1}B_2A_5y-32\sqrt{A_1}B_1A_6y$$

$$-117A_5^{\,2}A_6^{\,2}\alpha_1+24A_1A_5^{\,2}A_6^{\,2}y^2\alpha_1))\Big). \tag{67}$$

The non dimensional pressure rise per wavelength ΔP is defined as

$$\Delta P=\int_0^1\frac{dp}{dx}\,dx, \tag{68}$$

Where $\frac{dp}{dx}$ is defined through Eq.(66).

Not that, if we assume that the Darcy number was closer to infinity,then the results of the problem reduce exactly to the same as that found by Nadeem and Akbar [28].

4. Results and discussions

This section represents the graphical results in order to be able to discuss the quantitative effects of the sundry parameters involved in the analysis .

4.1. Pumping characteristics

We plot the expresion for ΔP in Eq. (68) against Θ for various values of parameters of interest in Figs.(2-9). In Fig.2 the effects of region channel width d on ΔP are seen. Observe that , in the pumping ($\Delta P > 0$) and free pumping ($\Delta P = 0$) for the J-S fluid, an increase in d causes adecreases in pumping. While in the co-pumping ($\Delta P < 0$), the pumping increases with an increase in d. Fig.3 the effects of Weissenberg number on ΔP. Observe that an increase in W_i causes a deacrease in the pumping region($\Delta P > 0$) and behaves oppositely in the free pumping ($\Delta P = 0$) and co-pumping. The observations regarding the effects of upper and lower wave amplitudes b and a (Fig.4 and 5) on ΔP are quite opposite to those in the case of phase difference ϕ Fig.7. Observe that an increase in ϕ causes a decrease in the pumping region ($\Delta P > 0$). It is also noted from Fig.7 that in free pumping and co-pumping ΔP increases with an increase in ϕ. The effects of F_r on pressure rise ΔP are illustrated in Fig.6. It is seen that with the increase in F_r the pressure rise decreases in the all pumping regions ($\Delta P > 0$),($\Delta P = 0$) and ($\Delta P < 0$). The pressure rise increases with increase in β and D_r. But this increasing in ΔP accurs in the co-pumping region which are displayed in (Fig.8 and 9) respectiveiy.

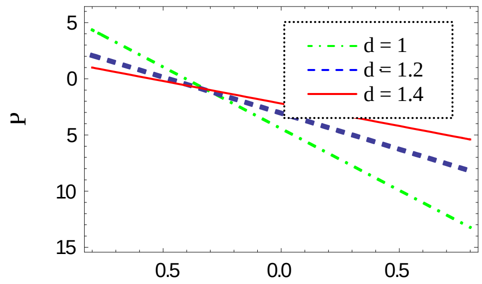

Fig.2. Effect of the width of channel d on variation of Δp versus Θ for : a=0.5, b=0.4, W_i=0.25,ϕ=0.2,μ=0.03,η=0.04, d=0.5, D_r=1, F_r=1, R_e=10, β=0.5, e=0.2.

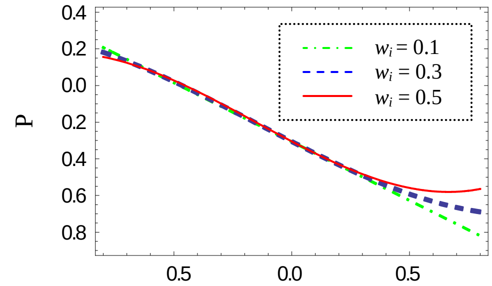

Fig.3. Effect of the Weisenberg number W_i on variation of Δp versus Θ for : a=0.5, ϕ=0.2, μ=0.03, η=0.04, d=0.9, D_r=1, F_r=1, R_e=10, β=0.5, e=0.2.

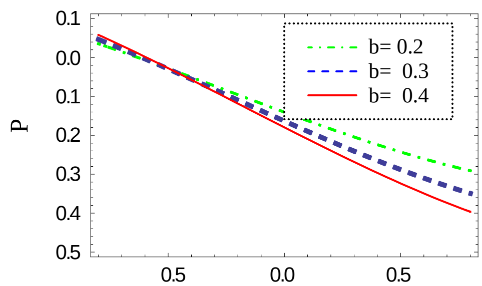

Fig.4. Effect of the lower wave amplitude b on variation of Δp versus Θ for : a=0.5 , ϕ=0.2, μ=0.03, η=0.04, W_i=0.25, *d*=0.5, D_r=1, F_r=1, R_e=10, β=0.5, e=0.2.

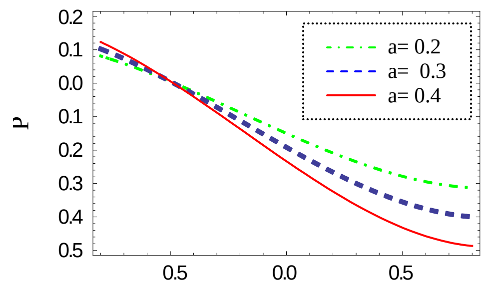

Fig.5. Effect of the upper wave amplitude a on variation of Δp versus Θ for : b=0.5, ϕ=0.5, μ=0.03, η=0.04, W_i=0.01, d=0.1, *Dr*=1, *Fr*=1, *Re*=10, β=0.5, e=0.2.

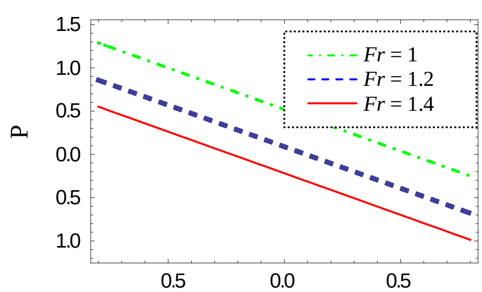

Fig.6. Effect of the Froud number F_r on variation of Δp versus Θ for : b=0.5,a=0.5,μ=0.03,η=0.04,W_i=0.01 ,d=0.1,D_r=1,ϕ=0.2,R_e=10,β=0.5, e=0.2.

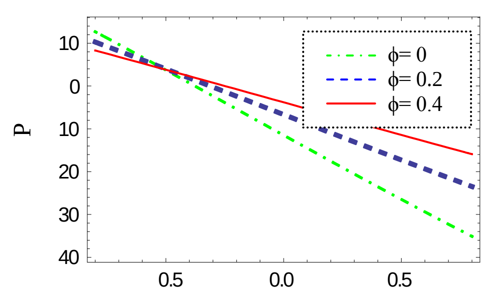

Fig.7. Effect of the phase difference ϕ on variation of Δp versus Θ for : b=0.5,a=0.5,μ=0.03,η=0.04,W_i=0.01 ,d=0.1,D_r=1,F_r=1,R_e=10,e=0.2.

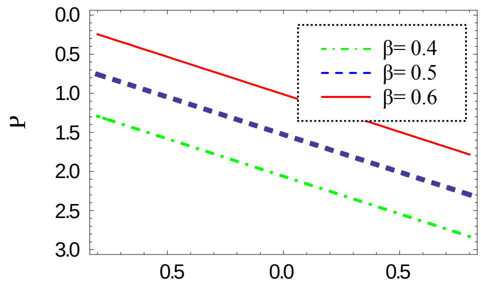

Fig.8. Effect of inclined angle β on variation of Δp versus Θ for : b=0.5,a=0.5,μ=0.03,η=0.04,W_i=0.01, d=3.5,D_r=1,F_r=1,R_e=10,ϕ=0.2,e=0.2.

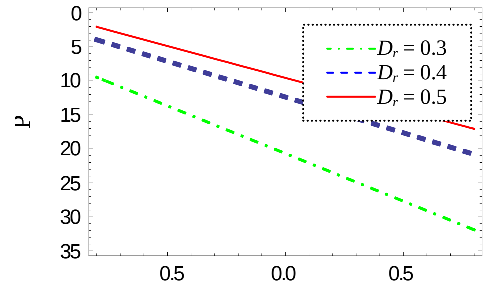

Fig.9. Effect of the Darcy number F_r on variation of Δp versus Θ for : b=0.5,a=0.5,μ=0.03,η=0.04,W_i=0.01, d=0.1,F_r=1,ϕ=0.2,R_e=10,β=0.5,e=0.2.

4.2. Pressure gradient characteristics

The variation of the axial pressure gradient *dp/dx* with *x* for various values ϕ, *a* ,*b* ,μ ,η, *d*, D_r, F_r , R_e , β, Θ and W_i are shown in Figs.(10-21). Fig.10 studies the effects of phase shift ϕ on the variation of pressure gradient *dp/dx*, and it is noticed that the axial pressure gradient decreases by increasing ϕ . Fig.11 and 12 shows the variation of the axial pressure gradient *dp/dx* with b and a respectively. It is cleare that the axial pressure gradient increases with an increase in amplitudes of the waves. Fig.13 shows the effect of μ on *dp/dx.* It is noted that, the magnitude of *dp/dx* increase with increasing μ and verse to versa in Fig.14 the magnitude of *dp/dx* decreases with increasing η. The effect the Weissenberg number W_i on the variation of pressure gradiant *dp/dx* with *x* is shown in Fig.15. It is observed that, the magnitude of the *dp/dx* increases with increasing W_i . Fig.(16-21) illustrates the variation of the axial pressure gradiant *dp/dx* with x for different values of D_r , β, *d*, R_e , F_r and Θ. It is cleare that the axial pressure gradient *dp/dx* decreases with an increase in D_r Fig.16. Also *dp/dx* increases by increasing β and R_e (Fig.17 and 19). The situation is reversed in Figs.18, 20 and 21, the axial pressure gradiant is decreased with an increase in *d* , F_r and Θ.

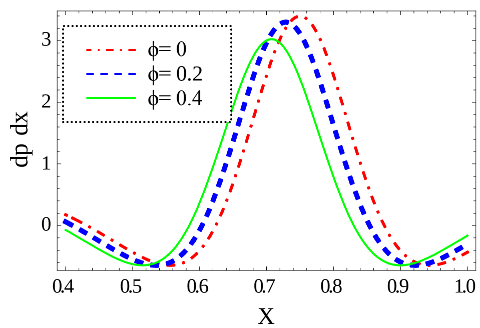

Fig.10. Effect of ϕ on the variation of pressure gradient dp/dx with x: b=0.4, a=0.2, μ=1, η=1,W_i=0.01,D_r=0.5,F_r=1,R_e=10, β=0.5, d=0.3, e=0.8, Θ=0.6.

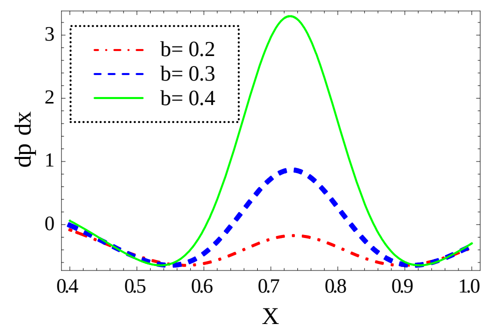

Fig.11. Effect of b on the variation of pressure gradient dp/dx with x: ϕ=0.2, a=0.2, μ=1, η=1,W_i=0.01,D_r=0.5,F_r=1,R_e=10, β=0.5, d=0.3, e=0.8, Θ=0.6.

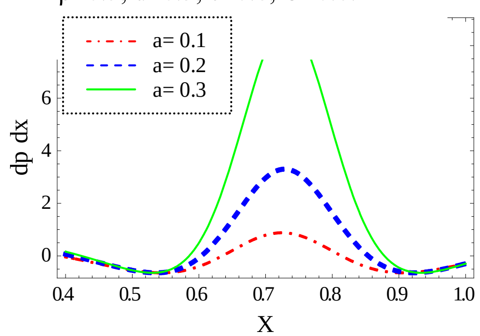

Fig.12. Effect of a on the variation of pressure gradient dp/dx with x: b=0.4, ϕ =0.2, μ=1, η=1,W_i=0.01,D_r=0.5,F_r=1,R_e=10,β =0.5,d=0.3, e=0.8, Θ=0.6.

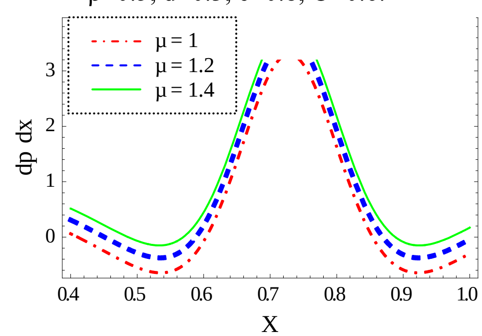

Fig.13. Effect of μ on the variation of pressure gradient dp/dx with x: b=0.4,a=0.2,ϕ =0.2, η=1,W_i=0.01,D_r=0.5,F_r=1,R_e=10,β =0.5,d=0.3, e=0.8, Θ=0.6.

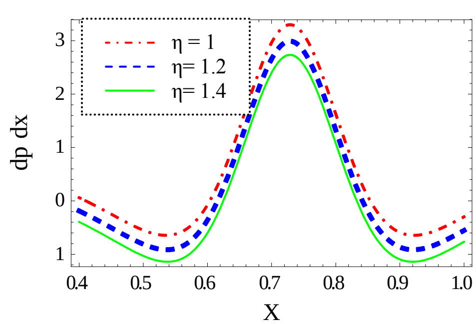

Fig.14. Effect of η on the variation of pressure gradient dp/dx with x: b=0.4, a=0.2, μ=1, ϕ =0.2, W_i=0.01, D_r=0.5,F_r=1, R_e=10,β=0.5,d=0.3, e=0.8, Θ=0.6.

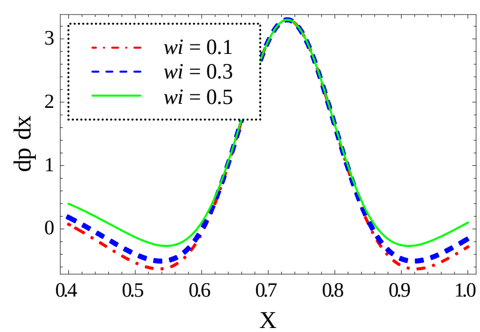

Fig.15. Effect of W_i on the variation of pressure gradient dp/dx with x: b=0.4, a=0.2, μ=1, η=1, ϕ =0.2, D_r=0.5,F_r=1, R_e=10, β=0.5, d=0.3, e=0.8, Θ=0.6.

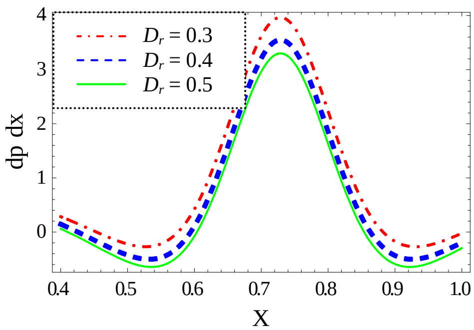

Fig.16. Effect of D_r on the variation of pressure gradient dp/dx with x: b=0.4,a=0.2,ϕ=0.2,μ=1,η=1,W_i=0.01, F_r=1, R_e=10, β=0.5, d=0.3, e=0.8, Θ=0.6.

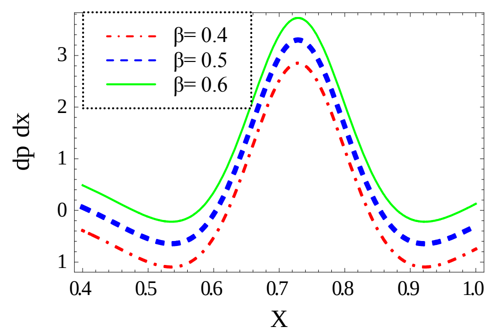

Fig.17. Effect of β on the variation of pressure gradient dp/dx with x: b=0.4,a=0.2,ϕ=0.2,μ=1,η=1,W_i=0.01, D_r=0.5,F_r=1,R_e=10,d=0.3,e=0.8,Θ=06.

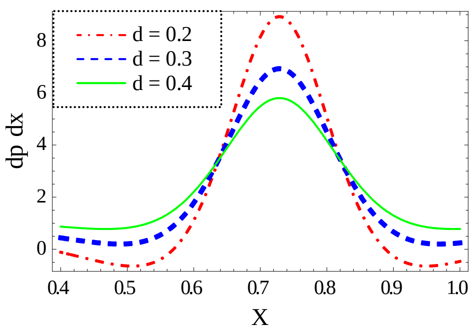

Fig.18. Effect of d on the variation of pressure gradient dp/dx with x: b=0.4,a=0.2,ϕ=0.2,μ=1,η=1,W_i=0.01, D_r=0.5,F_r=1,R_e=10,β=0.5,e=0.8,Θ=0.6.

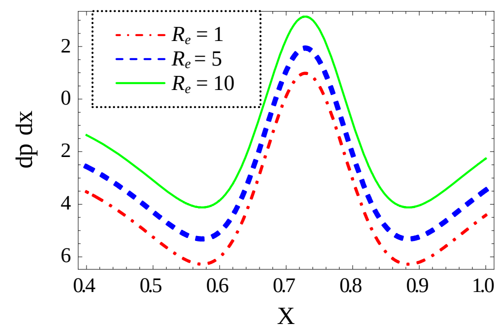

Fig.19. Effect of R_e on the variation of pressure gradient dp/dx with x: b=0.4, a=0.2,ϕ=0.2,μ=1,η=1,W_i=0.01,D_r=0.5, F_r=1,β=0.5,d=0.3, e=0.8, Θ=0.6.

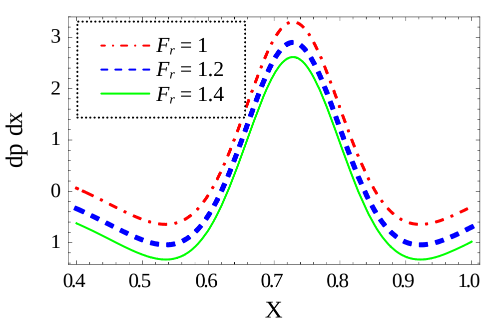

Fig.20. Effect of F_r on the variation of pressure gradient dp/dx with x: b=0.4, a=0.2,ϕ=0.2,μ=1, η=1, W_i=0.01, D_r=0.5, Re=10, β=0.5, d=0.3, e=0.8, Θ=0.6.

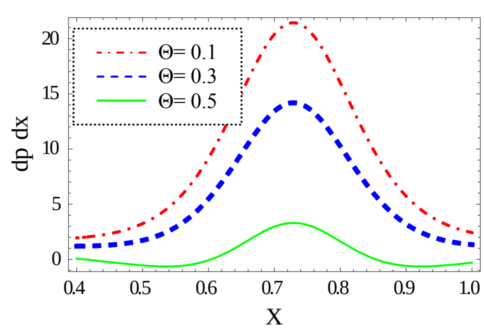

Fig.21. Effect of Θ on the variation of pressure gradient dp/dx with x: b=0.4, a=0.2,ϕ=0.2,μ=1,η=1,W_i=0.01,D_r=0.5, F_r=1,R_e=10,β=0.5, d=0.3, e=0.8.

4.3. Trapping phenomenon

Another interesting phenomenon in peristaltic motion is trapping. In the wave frame, streamelines under certain conditions split to trap a bolus which moves as a whole with the speed of the wave. The effect of Weissenberg number W_i on trapping can be seen through Fig.22 for an inclined asymmetric channel. Furthermore, Fig.22 shows that the bolus is anti-symmetric about the center line and its size decreases with an increase in W_i . It is observed from Figs.23 and 24 that the trapped bolus which are moving as whole increases in size with the increase in b and a. The effects of phase shift ϕ on trapping can be seen from Fig.8. It is depicted that increase in ϕ the trapping bolus which is moving as a whole decreases. The effect of Darcy number D_r on the trapping is illustrated in Fig.26 and it is observed that the size of trapped bolus rapidly increases with increasing D_r . Fig.27 depicts the effects of channel width d on trapping. The trapped bolus exists for small values of d , its size decreases with increasing d.

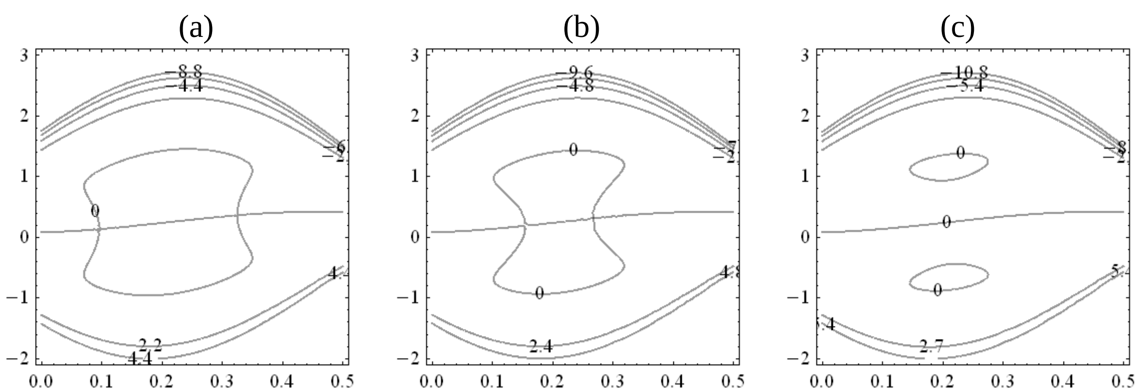

Fig.22. streamelines for b=0.7, ϕ=0.5, a=0.7, μ=0.01, η=0.01, *Dr*=0.7, Θ=1.2, d=0.5, e=0.5, and for different W_i ; **(a)** W_i =1.1, **(b)** W_i =1.2; **(c)** W_i = 1.3.

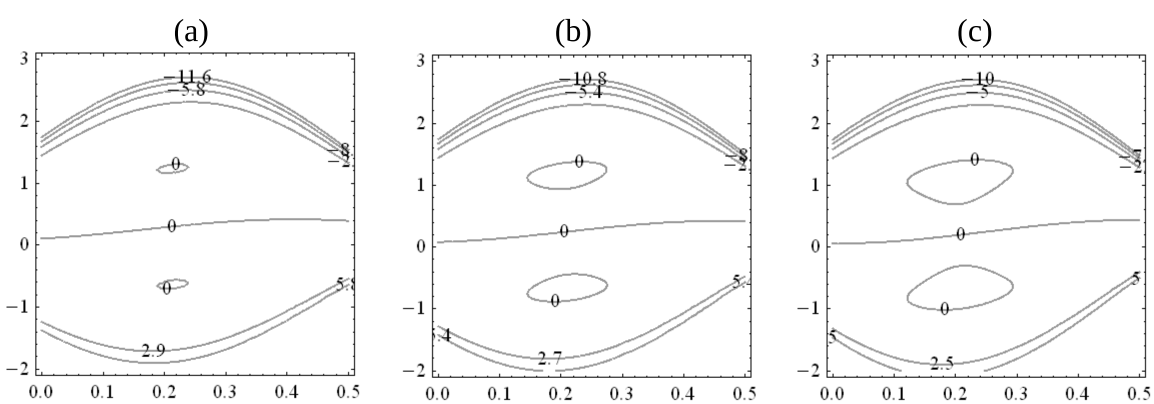

Fig.23. streamelines for W_i=1.3, ϕ=0.5, a=0.7, μ=0.01, η=0.01, *Dr*=0.7, Θ=1.2, d=0.5, e=0.5,and for different b ; **(a)** b= 0.6, **(b)** b=0.7, **(c)** b=0.8.

(a) (b) (c)

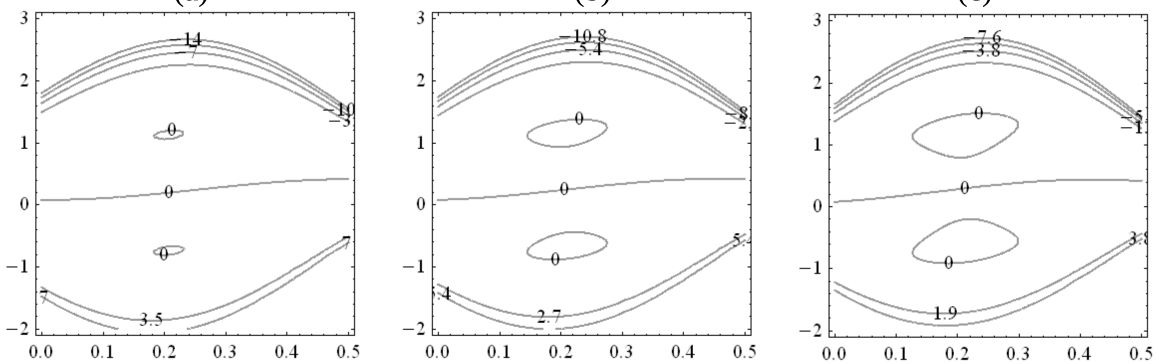

Fig.24. streamelines for W_i=1.3, ϕ=0.5,b=0.7, μ=0.01, η=0.01, *Dr*=0.7, Θ=1.2, d=0.5, e=0.5,and for different a ; **(a)** a= 0.6, **(b)** a=0.7, **(c)** a=0.8.

(a) (b) (c)

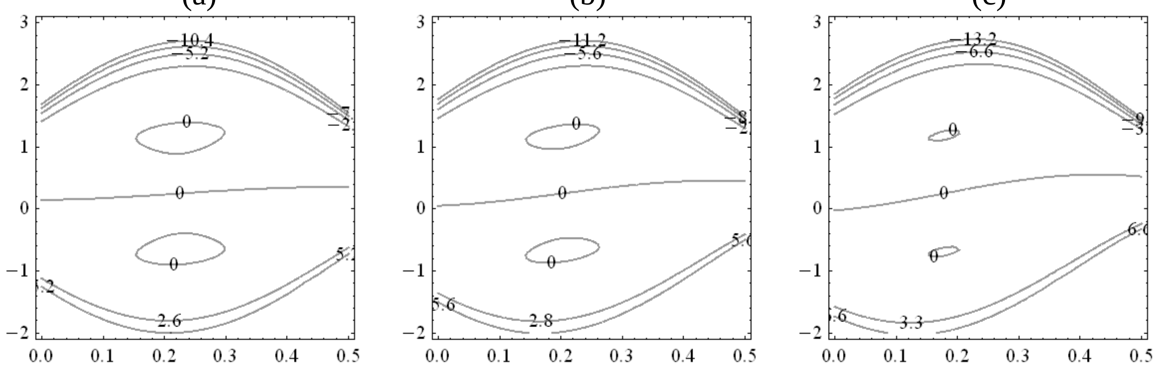

Fig.25. streamelines for W_i=1.3, a=0.7, b=0.7, μ=0.01, η=0.01, *Dr*=0.7, Θ=1.2, d=0.5, e=0.5,and for different ϕ ; **(a)** ϕ= 0.3, **(b)** ϕ= 0.6, **(c)** ϕ= 0.9.

(a) (b) (c)

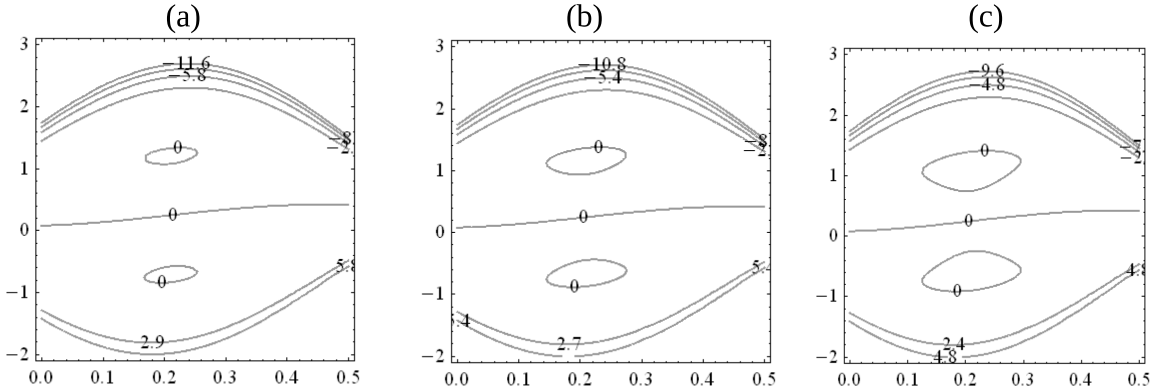

Fig.26. streamelines for W_i=1.3, a=0.7, b=0.7, μ=0.01, η=0.01, ϕ= 0.5, Θ=1.2, d=0.5, e=0.5,and for different D_r ; **(a)** D_r = 0.6, **(b)** D_r = 0.7, **(c)** D_r = 0.9.

(a) (b) (c)

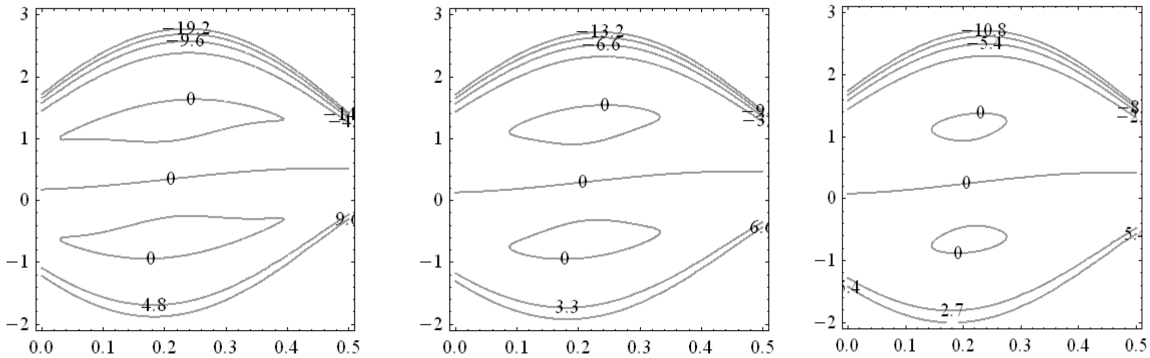

Fig.27. streamelines for W_i=1.3, a=0.7, b=0.7, μ=0.01, η=0.01, ϕ= 0.5, Θ=1.2, D_r = 0.7, e=0.5, and for different d ; **(a)** d=0.3, **(b)** d=0.4, **(c)** d= 0.5.

4.4 Temperature characteristics

The expressions for temperature are given by Eq.(67). To explicity see the effects of various parameters on temperature, Eq.(67) has been numerically evaluated and the results are presented in Fig.28-34. From Fig.28, it can be found that the temperature profiles are almost parabolic and temperature increases with increase of channel width d . Further, it can be noticed that the increase in Weissenberg number W_i causes a decrease in temperature are ploted in Fig.29. Also Fig.30 display the influince of amplitude ratio of the upper wall on the temperature distribution. We note that temperature increases with increasing *a*. Fig.31 displays the influince of amplitude ratio of the lower wall on the temperature distribution. It can be noticed that temperature increases when(1< θ < 0.4) and decreases when (1> θ > 0.4) that accurs by increasing *b*. The effects increasing Darcy number D_r and Eckert number E_r on the temperature are plotted in Figs.32 and 33 ,we note that the increasing in D_r and E_r causes increases in temperature. While with the increase in Prandtl number P_r , the temperature field decreases Fig.34.

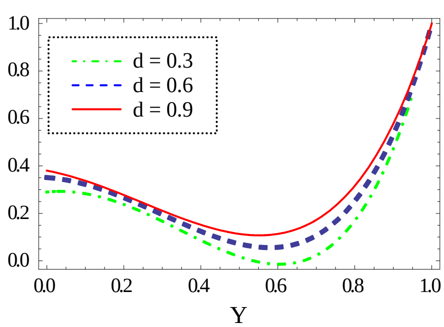

Fig.28. Effect of d on temperature for b= 0.2, ϕ= π/2, a=0.2,μ=0.3,η=0.4,W_i=0.7,D_r=0.1, E_r=-4,P_r=1, e=0.01, x=1, Θ=0.2.

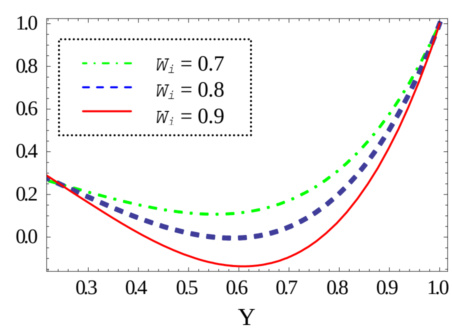

Fig.29. Effect of W_i on temperature for b= 0.2, ϕ= π/2, a=0.2,μ=0.3,η=0.4,d=0.3,D_r=0.1, E_r=-4,P_r=1, e=0.01,x=1,Θ=0.2.

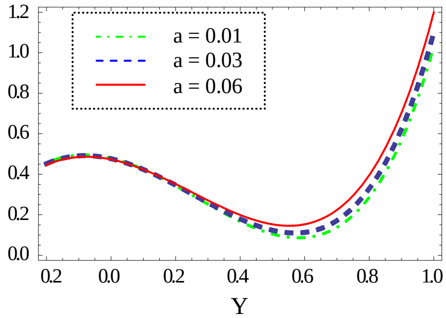

Fig.30. Effect of a on temperature for b=0.2, $\phi=\pi/2$, μ=0.3, η=0.4, d=0.3, D_r=0.1, E_r=-4, P_r=1, W_i=0.8, e=0.01, x=1, Θ=0.2.

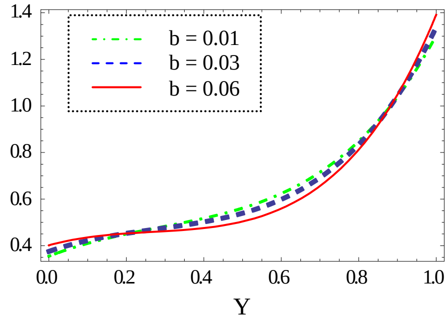

Fig.31. Effect of b on temperature for a=0.2, $\phi=\pi/2$, μ=0.3, η=0.4, d=0.3, D_r=0.1, E_r=-4, P_r=1, W_i=0.8, e=0.01, x=0.9, Θ=0.2.

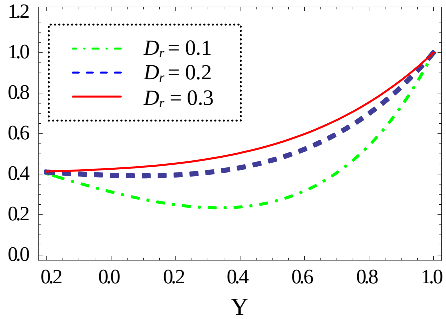

Fig.32. Effect of D_r on temperature for a=0.2,b=0.2, $\phi=\pi/2$, μ=0.3, η=0.4, d=1.3, E_r=-4, P_r=1, W_i=0.8, e=0.01, x=0.9, Θ=0.2.

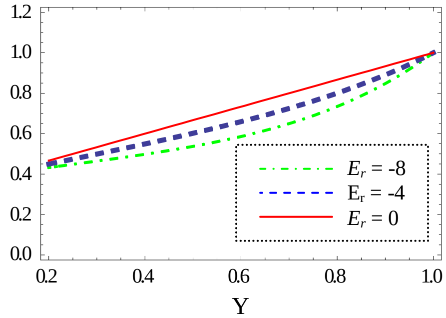

Fig.33. Effect of E_r on temperature for a=0.2,b=0.2, $\phi=\pi/2$, μ=0.3, η=0.4, d=0.3, D_r=0.1, P_r=1, Wi=0.8, e=0.01, x=1, Θ=0.2.

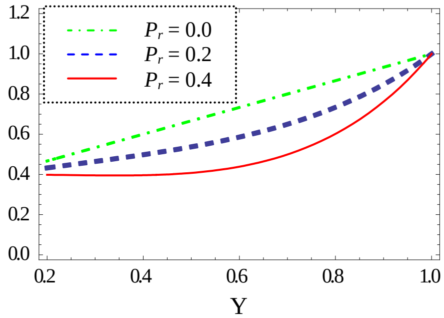

Fig.34. Effect of P_r on temperature for a=0.2, b=0.2, $\phi=\pi/2$, μ=0.3, η=0.4, d=0.3, D_r=0.1, x=1, W_i=0.8, e=0.01, E_r=-4, Θ=0.2.

5. Concluding remarks

In this paper, we investigated the peristaltic transport of a Johnson-Segalman fluid through a porous medium in an inclined asymmetric 2-D channel under the assumptions of long-wavelength and low-Reynolds number. A perturbation solution for small Weissenberg number is obtained for the stream function, axial pressure gradient, pressure rise and temperature field over a wavelength. It is found that :

- The pressure rise over a wavelength ΔP decreases with an increase in d in the pumping ($\Delta P > 0$) and free pumping ($\Delta P = 0$), while the situation is reversed in the co-pumping region ($\Delta P < 0$).
- The pressure rise over a wavelength ΔP decreases with an increase in Wessenberg number W_i and phase difference ϕ in the pumping region ($\Delta P > 0$), while the situation is reversed in the co-pumping region ($\Delta P < 0$) and free pumping ($\Delta P = 0$).
- The pressure rise over a wavelength ΔP decreases by increasing F_r, while increases by increasing D_r and β .
- The axial pressure gradient decreases by increasing d, F_r, ϕ, η and Θ, while increases by increasing a, b, W_i, R_e, β and μ.
- The size of the trapped bolus increases with an increase in a, b, D_r, while decreases with an increase in W_i, ϕ and d .
- The temperature field increases with the increase in d, a, D_r and E_r, while with the increase in W_i, b and P_r the temperature field decreases.

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