

FUZZY INVENTORY MODEL FOR ITEMS WITH WEIBULL DISTRIBUTION DETERIORATION, POWER DEMAND, LINEAR HOLDING COST, SALVAGE COST AND PARTIAL BACKLOGGING

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Abstract

The objective of this research article is to develop an inventory model which incorporates power pattern demand, Weibull distribution deterioration, shortages and partial backlogging of orders. Holding cost is taken as time dependent and deteriorated items are assumed to have a salvage value. The cost parameters are fuzzified and the total cost is defuzzified using Graded mean representation, signed distance and centroid methods. The values obtained by these methods are compared with the help of numerical examples. The convexity of the cost function is depicted graphically. Sensitivity analysis is performed to study the effect of change in some parameters.

Keywords: Inventory, Power demand, Partial backlogging, Deterioration, Triangular Fuzzy Number, Defuzzification, Graded mean represented method, Signed Distance Method, centroid method.

1. Introduction

Several research articles have been developed to determine the optimal policies for effective inventory management. Earlier many researchers regarded demand rate to be constant which is a feature of static environment. There are very few such instances. Due to the technological revolution, consumers become quickly aware of the new products and their availability. Consequently there is a sudden change in demand.

Though Naddor predicted such a situation in 1966, it was not much in focus for research till recently. He emphasized that demand is one of the significant components of inventory management. Goel and Aggarwal(1981) proposed an order-level inventory model with power demand for deteriorating items. Datta andPal(1988) developed an EOQ model with power demand and variable deterioration. Lee and Wu (2002) included shortages in their EOQ model with power demand. Dye(2004) extended the model by considering time-proportional backlogging. Later Singh etal.(2009) developed a model for perishable items with power demand pattern and partial backlogging. Rajeswari and Vanjikkodi(2011) established an inventory model for time dependent deteriorating item with partially backlogged and power pattern demand. Rajeswari and Vanjikkodi (2012) analyzed an EOQ model for Weibull deteriorating items with power demand. Mishra and Singh(2013) proposed an EOQ model for perishable items with power demand in which shortages are allowed and partially backlogged. Sicilia(2013) et al proposed an enhanced model with power demand pattern.

Also the goods kept in stock are often subject to damage, decay or obsolescence, which accounts for deterioration. The deterioration of goods is a realistic phenomenon in many inventory systems and controlling them is an important issue. Due to deterioration the problem of shortages occur and hence a fraction of demand of the customers in a given period of time. Researchers have continuously modified the deteriorating inventory models so as to become more practicable and realistic.

Some examples of items in which appreciable deterioration can take place during the storage period are food, electronic components, chemicals, etc.,Whitin (1957) was the first to consider the deterioration of the fashion goods. Covert and Philip (1973) developed an EOQ model for items with Weibull distribution deterioration. Chakrabarty, et., al., (1998) extended Covert and Philip's model to a EOQ model for items with

Weibull distribution deterioration, shortages, and trended demand. Wu and Lee (2003) proposed an EOQ inventory model for items with Weibull distributed deterioration, shortages and time –varying demand. Skouri et.al., (2009) developed inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. Amutha and Chandrasekaran (2012) considered an inventory model for deteriorating products with Weibull distribution deterioration, time-varying demand and partial backlogging. Srichandan Mishra et., al. (2014), develop an inventory model for deteriorating items and the salvage value is incorporated to the deteriorated units.

Uncertainties and imprecision are inherent in estimating the various parameters associated with the inventory. Earlier they were classically modeled using the probability theory. However, there are uncertainties that cannot be appropriately treated by usual probability. The advent of the fuzzy theory was used to deal with such situations. The fuzzy set theory was first introduced by Zadeh (1965) and has now been applied in different inventory control systems to model their behaviour more realistically. In 1983 Zimmerman developed using fuzzy sets in operations research. Park (1987) provided a fuzzy set theoretic interpretation of economic order quantity. Vujosevic, Petrovic and Petrovic (1996) developed an EOQ formula by considering inventory cost as a fuzzy number. Li et al., (2002) proposed Fuzzy models for single-period inventory problem. Chen et al,. (2006) developed Fuzzy inventory model for deteriorating items with permissible delay in payments. Syed and Aziz (2007) used the signed distance method for their fuzzy inventory model. De and Rawat (2011) analyzed a fuzzy inventory model using triangular fuzzy number. Sumana saha and Tripti Chakrabarty (2012) formulated a fuzzy EOQ model for time dependent deteriorating items and time dependent demand with shortages. Sushil Kumar and Rajput (2015) developed a fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging.

In this paper a fuzzy inventory model involving items deteriorating in a Weibull pattern with power demand, shortages, and partially backlogging is developed and analysed. Holding cost is taken as a linear function of time and the salvage value of the deteriorated items is considered. Instability in cost is considered in the form of triangular fuzzy number. The average total fuzzy inventory cost is determined and minimized. The fuzzy model is then defuzzified using graded mean , signed distance and centroid methods. To illustrate the model a numerical example is used. Convexity of the cost function is displayed graphically. Sensitivity analysis is also carried out to study the effect of some model parameters of the system.

2. Preliminaries

Definition 2.1 If $\tilde{A}$ = (a, b, c) is a triangular fuzzy number then the graded mean integration representation of $\tilde{A}$ is defined as

$$P(A) = \frac{\int_0^{w_A} h\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right)dh}{\int_0^{w_A} hdh} = \frac{a+4b+c}{6}$$

with $0 < h \leq w_A$ and $0 < w_A \leq 1$.

Definition 2.2 If $\tilde{A}$ = (a, b, c) is a triangular fuzzy number then the signed distance of $\tilde{A}$ is defined as

$$d\left(\tilde{A},\tilde{0}\right) = \int_0^1 d\left(\left[A_L(\alpha)_\alpha, A_R(\alpha)_\alpha\right],\tilde{0}\right) = \frac{1}{4}(a+2b+c)$$

Definition 2.3 The Centroid of a triangular fuzzy number $\tilde{A}$ = (*a*, *b*, *c*) is defined as $C(\tilde{A}) = \frac{a+b+c}{3}$

3. Notations and Assumptions

3.1 Notations

A – The ordering cost per order.

C – The purchase cost per unit.

h(t) – (a+bt)The inventory holding cost per unit per time unit.

π_b – The backordered cost per unit short per time unit.

π_l – The cost of lost sales per unit.

π_d – The deteriorating cost per unit.

θ – The deterioration rate.

λ – The salvage rate per deteriorating unit.

t_1 – The time at which the inventory level reaches zero, $t_1 \geq 0$.

t_2 – Portion of the cycle time during which back orders are received, $t_2 \geq 0$.

T – $(=t_1+t_2)$ The length of cycle time.

I_{MI} – The maximum inventory level during [0, T].

I_{MB} – The maximum backordered units during stock out period.

Q – $(=I_{MI}+I_{MB})$ the order quantity during a cycle of length T.

$I_1(t)$ – The level of positive inventory at time t, $0 \leq t \leq t_1$.

$I_2(t)$ – The level of demanded units at time t, $t_1 \leq t \leq T$.

TCUT – The total cost per unit time .

$\tilde{C}$ – Fuzzy purchase cost per unit.

$\tilde{h}(t)$ – $(\tilde{a}+bt)$ fuzzy holding cost per unit per time unit.

$\tilde{\pi}_b$ – Fuzzy backordered cost per unit short per time unit.

$\tilde{\pi}_l$ – Fuzzy cost of lost sales per unit.

$\tilde{\pi}_d$ – Fuzzy deteriorating cost per unit.

$TCU\tilde{T}$ – Fuzzy total cost per time unit

3.2 Assumptions

- The inventory system deals with single item.
- The demand rate is $\frac{dt^{(1-n)/n}}{nT^{1/n}}$ at any time t, where d is a positive constant, n may be any positive number, T is the cycle time.
- The variable deterioration rate is $\theta(t) = \alpha\beta t^{\beta-1}$, $0 < \theta << 1$, $t > 0$.
- The lead-time is zero or negligible.
- The planning horizon is infinite.
- During the stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. The proportion of the customers who would like to accept the backlogging at time "t" is with the waiting time $(T-t)$ for the next replenishment. That is, for the shortage inventory, the backlogging rate is $B(t)=\frac{1}{1+\delta(T-t)}$; $\delta > 0$ denotes the backlogging parameter and $t_1 \leq t \leq T$.
- Holding cost is linearly time dependent. That is h(t)=a+bt
- Holding cost, deteriorating cost, back order cost, last sale cost and the purchase cost are fuzzified.

4. Mathematical Model

4.1 Crisp model

Inventory level before the inventory runs out of stock:

During the period $[0, t_1]$, the change in the inventory level takes place due to deterioration and demand. The change in the inventory level I(t) during this period is modelled using the differential equation:

$$\frac{dI_1(t)}{dt}+\alpha\beta t^{\beta-1}I_1(t)=-\frac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}, \qquad 0 \leq t \leq t_1 \tag{1}$$

with the condition $I_1(t_1) = 0$ at $t = t_1$.

Thus the level of the inventory at any time t during the interval $[0,t_1]$ is

$$I_1(t)=\frac{d}{T^{\frac{1}{n}}}\left[\left(1-\alpha t^{\beta}\right)\left(t_1^{\frac{1}{n}}-t^{\frac{1}{n}}\right)+\frac{\alpha}{1+n\beta}\left(t_1^{\frac{1+n\beta}{n}}-t^{\frac{1+n\beta}{n}}\right)\right], \qquad 0 \leq t \leq t_1 \tag{2}$$

The maximum inventory level in $[0,t_1]$ is $I_{MI}=I_1(0)=\frac{d}{T^{\frac{1}{n}}}\left[t_1^{\frac{1}{n}}+\frac{\alpha t_1^{\frac{1+n\beta}{n}}}{1+n\beta}\right]$ (3)

Inventory level during shortage period

During the interval $[t_1, T]$ the inventory runs out of items and back orders pile up due to demand. In this period only a fraction of demand is backlogged. The state of inventory during $[t_1, T]$ is represented by the differential equation:

$$\frac{dI_2(t)}{dt} = -\frac{\left(\dfrac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}\right)}{1+\delta(T-t)}, \qquad t_1 \le t \le T \tag{4}$$

with the condition $I_2(t_1) = 0$ at $t = t_1$.

Hence the backlogged demand during the period $[t_1, T]$ is:

$$I_2(t) = -\frac{d}{T^{\frac{1}{n}}}\left[(1-\delta T)\left(t^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \frac{\delta}{1+n}\left(t^{\frac{1+n}{n}} - t_1^{\frac{1+n}{n}}\right)\right], \quad t_1 \le t \le T \tag{5}$$

The maximum backordered units are $I_{MB} = -I_2(T) = \dfrac{d}{T^{\frac{1}{n}}}\left[(1-\delta T)\left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \dfrac{\delta}{1+n}\left(T^{\frac{1+n}{n}} - t_1^{\frac{1+n}{n}}\right)\right]$ (6)

The order size for a cycle is $Q = I_{MI} + I_{MB}$.

$$Q = \frac{d}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1+2n}{n}}}{2(1+2n)} + \delta T t_1^{\frac{1}{n}} - \frac{\delta}{1+n}\left(nT^{\frac{1+n}{n}} + t_1^{\frac{1+n}{n}}\right)\right] \tag{7}$$

Costs involved in the inventory maintenence:

The total cost per replenishment cycle consists of the following cost components.

Ordering cost per cycle: $I_{OC} = A$ (8)

Holding cost: $I_{HC} = \int_0^{t_1} h(t)I_1(t)dt = \dfrac{d}{T^{\frac{1}{n}}}\left[\dfrac{at_1^{\frac{1+n}{n}}}{(1+n)} + \dfrac{a\alpha\beta t_1^{\frac{1+n+n\beta}{n}}}{(1+\beta)(1+n+n\beta)} + \dfrac{bt_1^{\frac{1+2n}{n}}}{2(1+2n)} + \dfrac{b\alpha\beta t_1^{\frac{1+2n+n\beta}{n}}}{2(2+\beta)(1+2n+n\beta)}\right]$ (9)

Cost due to backorder: $I_{BC} = \dfrac{d\pi_b}{T^{\frac{1}{n}}}\left[\dfrac{(1-2\delta T)t_1^{\frac{1+n}{n}}}{1+n} + (\delta T^2 - T)t_1^{\frac{1}{n}} + \dfrac{nT^{\frac{1+n}{n}}}{1+n} - \dfrac{2n^2\delta T^{\frac{1+2n}{n}}}{(1+n)(1+2n)} + \dfrac{\delta t_1^{\frac{1+2n}{n}}}{(1+2n)}\right]$ (10)

Cost due to lost sales: $I_{LS} = \pi_l \int_{t_1}^{T}\left[1 - \dfrac{1}{1+\delta(T-t)}\right]\left(\dfrac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}\right)dt = \dfrac{\pi_l d\delta}{T^{\frac{1}{n}}}\left[\dfrac{nT^{\frac{1+n}{n}}}{1+n} - Tt_1^{\frac{1}{n}} + \dfrac{t_1^{\frac{1+n}{n}}}{1+n}\right]$ (11)

Cost due to deterioration: $I_{DC} = \pi_d\left\{Q - \int_0^{t_1}\left(\dfrac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}}\right)dt - \int_{t_1}^{T}\dfrac{1}{1+\delta(T-t)}\left(\dfrac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}\right)dt\right\} = \pi_d \dfrac{d\alpha t_1^{\frac{1+\beta n}{n}}}{(1+\beta n)T^{\frac{1}{n}}}$ (12)

Purchase cost: $I_{PC} = C \times Q_2 = \dfrac{Cd}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} + \dfrac{\alpha t_1^{\frac{1+n\beta}{n}}}{1+n\beta} + \delta T t_1^{\frac{1}{n}} - \dfrac{\delta}{1+n}\left(nT^{\frac{1+n}{n}} + t_1^{\frac{1+n}{n}}\right)\right]$ (13)

Salvage cost: $I_{SC} = \lambda C\left\{Q - \int_0^{t_1}\left(\dfrac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}}\right)dt - \int_{t_1}^{T}\dfrac{1}{1+\delta(T-t)}\left(\dfrac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}\right)dt\right\} = \dfrac{\lambda C d\alpha t_1^{\frac{1+\beta n}{n}}}{(1+\beta n)T^{\frac{1}{n}}}$ (14)

The total inventory cost per unit time is: $TCUT = \dfrac{1}{T}\left[I_{OC} + I_{HC} + I_{BC} + I_{LS} + I_{PC} + I_{DC} - I_{SC}\right]$

$$TCUT=\frac{1}{T}\left\{A+\frac{d}{T^{\frac{1}{n}}}\left[\frac{at_1^{\frac{1+n}{n}}}{(1+n)}+\frac{a\alpha\beta t_1^{\frac{1+n+n\beta}{n}}}{(1+\beta)(1+n+n\beta)}+\frac{bt_1^{\frac{1+2n}{n}}}{2(1+2n)}+\frac{b\alpha\beta t_1^{\frac{1+2n+n\beta}{n}}}{2(2+\beta)(1+2n+n\beta)}\right]+\frac{\pi_1 d\delta}{T^{\frac{1}{n}}}\left(\frac{nT^{\frac{1+n}{n}}}{1+n}-Tt_1^{\frac{1}{n}}+\frac{t_1^{\frac{1+n}{n}}}{1+n}\right)+\frac{(\pi_d-\lambda C)d\alpha t_1^{\frac{1+\beta n}{n}}}{(1+\beta n)T^{\frac{1}{n}}}\right.$$

$$\left.+\frac{d\pi_b}{T^{\frac{1}{n}}}\left[\frac{(1-2\delta T)t_1^{\frac{1+n}{n}}}{1+n}+(\delta T^2-T)t_1^{\frac{1}{n}}+\frac{nT^{\frac{1+n}{n}}}{1+n}-\frac{2n^2\delta T^{\frac{1+2n}{n}}}{(1+n)(1+2n)}+\frac{\delta t_1^{\frac{1+2n}{n}}}{(1+2n)}\right]+\frac{Cd}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}}+\frac{\alpha t_1^{\frac{1+n\beta}{n}}}{1+n\beta}+\delta Tt_1^{\frac{1}{n}}-\frac{\delta}{1+n}\left(nT^{\frac{1+n}{n}}+t_1^{\frac{1+n}{n}}\right)\right]\right\} \quad (15)$$

The total cost is highly nonlinear. Using the necessary and sufficiency conditions $dTCUT/dt_1=0$ and $d^2TCUT/dt_1^2>0$, the optimal value of t_1 and the minimum total cost per unit time are obtained. (16)

4.2 Fuzzy Model

To account for situations closer to reality it may be assumed that some of these parameters namely $\tilde{C},\tilde{a},\tilde{\pi}_b,\tilde{\pi}_l,\tilde{\pi}_d$ may change with in some limits.

Let $\tilde{\pi}_l=(\pi_{l1},\pi_{l2},\pi_{l3})$, $\tilde{\pi}_b=(\pi_{b1},\pi_{b2},\pi_{b3})$, $\tilde{C}=(C_1,C_2,C_3)$, $\tilde{a}=(a_1,a_2,a_3)$, $\tilde{\pi}_d=(\pi_{d1},\pi_{d2},\pi_{d3})$ are triangular fuzzy numbers.

Total cost of the system per unit time in fuzzy sense is given by

$$T\tilde{CUT}=\frac{1}{T}\left\{A+\frac{d}{T^{\frac{1}{n}}}\left[\frac{\tilde{a}t_1^{\frac{1+n}{n}}}{(1+n)}+\frac{\tilde{a}\alpha\beta t_1^{\frac{1+n+n\beta}{n}}}{(1+\beta)(1+n+n\beta)}+\frac{bt_1^{\frac{1+2n}{n}}}{2(1+2n)}+\frac{b\alpha\beta t_1^{\frac{1+2n+n\beta}{n}}}{2(2+\beta)(1+2n+n\beta)}\right]+\frac{\tilde{\pi}_l d\delta}{T^{\frac{1}{n}}}\left(\frac{nT^{\frac{1+n}{n}}}{1+n}-Tt_1^{\frac{1}{n}}+\frac{t_1^{\frac{1+n}{n}}}{1+n}\right)+\frac{(\tilde{\pi}_d-\lambda\tilde{C})d\alpha t_1^{\frac{1+\beta n}{n}}}{(1+\beta n)T^{\frac{1}{n}}}\right.$$

$$\left.+\frac{d\tilde{\pi}_b}{T^{\frac{1}{n}}}\left[\frac{(1-2\delta T)t_1^{\frac{1+n}{n}}}{1+n}+\frac{nT^{\frac{1+n}{n}}}{1+n}+(\delta T^2-T)t_1^{\frac{1}{n}}-\frac{2n^2\delta T^{\frac{1+2n}{n}}}{(1+n)(1+2n)}+\frac{\delta t_1^{\frac{1+2n}{n}}}{(1+2n)}\right]+\frac{\tilde{C}d}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}}+\frac{\alpha t_1^{\frac{1+n\beta}{n}}}{1+n\beta}+\delta Tt_1^{\frac{1}{n}}-\frac{\delta}{1+n}\left(nT^{\frac{1+n}{n}}+t_1^{\frac{1+n}{n}}\right)\right]\right\} \quad (17)$$

The fuzzy total cost $T\tilde{CUT}$, is defuzzified by graded mean representation, signed distance and centroid methods.

4.2.1 Total cost - Graded Mean Method

The fuzzy total cost is expressed as

$$TCUT_{dG}=\left[TCUT_{dG_1},TCUT_{dG_2},TCUT_{dG_3}\right]$$

where

$$TCUT_{dG_i}=\frac{1}{T}\left\{A+\frac{d}{T^{\frac{1}{n}}}\left[\frac{a_i t_1^{\frac{1+n}{n}}}{(1+n)}+\frac{a_i\alpha\beta t_1^{\frac{1+n+n\beta}{n}}}{(1+\beta)(1+n+n\beta)}+\frac{bt_1^{\frac{1+2n}{n}}}{2(1+2n)}+\frac{b\alpha\beta t_1^{\frac{1+2n+n\beta}{n}}}{2(2+\beta)(1+2n+n\beta)}\right]+\frac{\pi_{li}d\delta}{T^{\frac{1}{n}}}\left(\frac{nT^{\frac{1+n}{n}}}{1+n}-Tt_1^{\frac{1}{n}}+\frac{t_1^{\frac{1+n}{n}}}{1+n}\right)+\frac{(\pi_{di}-\lambda C_j)d\alpha t_1^{\frac{1+\beta n}{n}}}{(1+\beta n)T^{\frac{1}{n}}}\right.$$

$$\left.+\frac{d\pi_{bi}}{T^{\frac{1}{n}}}\left[\frac{(1-2\delta T)t_1^{\frac{1+n}{n}}}{1+n}+(\delta T^2-T)t_1^{\frac{1}{n}}+\frac{nT^{\frac{1+n}{n}}}{1+n}-\frac{2n^2\delta T^{\frac{1+2n}{n}}}{(1+n)(1+2n)}+\frac{\delta t_1^{\frac{1+2n}{n}}}{(1+2n)}\right]+\frac{C_i d}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}}+\frac{\alpha t_1^{\frac{1+n\beta}{n}}}{1+n\beta}+\delta Tt_1^{\frac{1}{n}}-\frac{\delta}{1+n}\left(nT^{\frac{1+n}{n}}+t_1^{\frac{1+n}{n}}\right)\right]\right\}$$

where i=1,2,3 and j=3,2,1 (18)

$$TCUT_{dG}=\frac{1}{6}\left[TCUT_{dG_1}+4TCUT_{dG_2}+TCUT_{dG_3}\right] \quad (19)$$

With the help of the necessary and sufficient conditions $dTCUT_{dG}/dt_1=0$ and $d^2TCUT_{dG}/dt_1^2>0$ (20) the optimal value of t_1 can be obtained and thereby the minimum the total cost per unit time and the order size Q are determined.

4.2.2 Total cost -Signed Distance Method,

Total cost TCUT is expressed as:

$TCUT_{dS}=\left[TCUT_{dS_1},TCUT_{dS_2},TCUT_{dS_3}\right]$ where $TCUT_{dS_i}$ are defined by (18).

$$TCUT_{dS}=\frac{1}{4}\left[TCUT_{dS_1}+2TCUT_{dS_2}+TCUT_{dS_3}\right] \quad (21)$$

The total cost function $TCUT_{dS}$ is highly nonlinear and is minimized following the same process as has been stated above.

4.2.3 Total cost -Centroid Method

$TCUT_{dC} = \left[TCUT_{dC_1}, TCUT_{dC_2}, TCUT_{dC_3}\right]$ where $TCUT_{dC_i}$ are defined by (18).

$$TCUT_{dC} = \frac{1}{3}\left[TCUT_{dC_1} + TCUT_{dC_2} + TCUT_{dC_3}\right] \qquad (22)$$

The total cost function $TCUT_{dC}$ is highly nonlinear and is minimized following the same process as has been stated above.

Numerical example and Sensitivity analysis:

Consider an inventory system with following parametric values:

5.1 Crisp Model,

d=100 units, n=2, T=1, A = \$250/order, C =\$ 10/unit, a=\$0.50/unit/year, b=\$0.05/unit/year, π_b=\$2/unit/year, π_d=\$11/unit, π_l=\$9/unit, δ=0.5, α=0.04, β=4, λ=0.1.

The solution of crisp model is

TCUT =\$1264.5≅ \$ 1265, t_1^* = 0.6132 years, Q* =97.8664units ≅ 98 units.

5.2 Fuzzy Model,

$\tilde{C}$ = (7.5, 8, 8.8), $\tilde{a}$ = (0.4, 0.5, 0.65), $\tilde{\pi}_b$ = (11, 12, 14), $\tilde{\pi}_l$ = (13.5, 15, 15.5), $\tilde{\pi}_d$ = (10.5, 11, 11.7)

The optimal values of the fuzzy model determined by the three methods are tabulated below.

Table 1 Computations with fuzzy parameters

Method	**Fuzzy Perameters**	**t_1^* (years)**	**$TCUT_{dG}^*$(\$)**	**Q^*(units)**
Grade Mean Representation Method	$\tilde{C}, \tilde{a}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.5993	1281.1	97.6859
	$\tilde{a}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.6163	1264.8	97.9056
	$\tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.6188	1264.7	97.9370
	$\tilde{\pi}_l, \tilde{\pi}_d$	0.6212	1264.7	97.9668
	$\tilde{\pi}_d$	0.6135	1264.5	97.8702
Signed Distance method	$\tilde{C}, \tilde{a}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.5919	1289.4	97.5865
	$\tilde{a}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.6177	1264.9	97.9232
	$\tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.6215	1264.7	97.9706
	$\tilde{\pi}_l, \tilde{\pi}_d$	0.6251	1264.8	98.0149
	$\tilde{\pi}_d$	0.6136	1264.5	97.8715
Centroid Method	$\tilde{C}, \tilde{a}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.5841	1297.6	97.4793
	$\tilde{a}, \tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.6191	1265.0	97.9407
	$\tilde{\pi}_b, \tilde{\pi}_l, \tilde{\pi}_d$	0.6241	1264.8	98.0026
	$\tilde{\pi}_l, \tilde{\pi}_d$	0.6288	1264.9	98.0599
	$\tilde{\pi}_d$	0.6137	1264.5	97.8728

Sufficient condition: $\frac{d^2TCUT}{dt_1^2} = 128.1232 > 0$, $\frac{d^2TCUT_{dG}}{dt_1^2} = 121.8845 > 0$, $\frac{d^2TCUT_{dS}}{dt_1^2} = 118.6601 > 0$, $\frac{d^2TCUT_{dC}}{dt_1^2} = 115.3485 > 0$

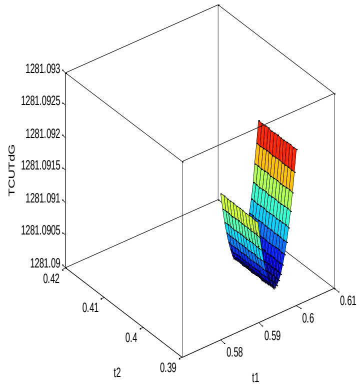

Figure 1. Grade Mean Method

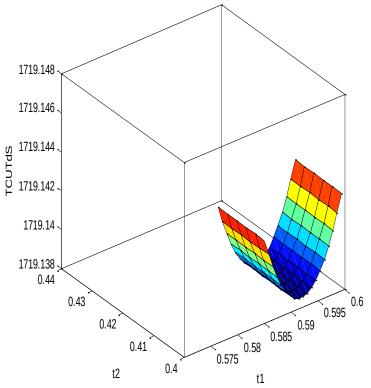

Figure 2. Signed Distance method

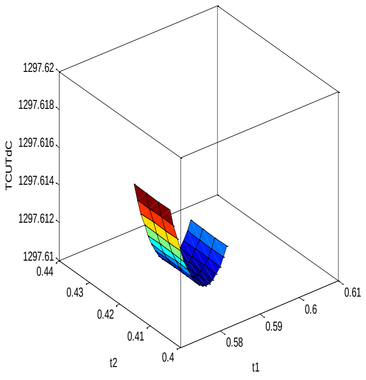

Figure 3. Centroid Method

Fig.1, Fig.2 and Fig.3 represent the convexity of the defuzzified total cost function.

Table 2.Sensitivity analysis for the parameters δ, α, β

Parameters	Values of the parameters	t_1(years)	$TCUT_{dG}$(\$)	Q(units)
Δ	0.40	0.6328	1281.8	98.4977
	0.45	0.6175	1281.5	98.1337
	0.50	0.5993	1281.1	97.6859
	0.55	0.5770	1280.7	97.1137
	0.60	0.5482	1280.2	96.3362
A	0.02	0.6321	1280.6	98.0714
	0.03	0.6143	1280.9	97.8680
	0.04	0.5993	1281.1	97.6859
	0.05	0.5863	1281.3	97.5199
	0.06	0.5749	1281.5	97.3679
B	2	0.5056	1284.1	96.3730
	3	0.5639	1282.0	97.2327
	4	0.5993	1281.1	97.6859
	5	0.6229	1280.6	97.9620
	6	0.6394	1280.3	98.1434

The following observations have been made on the basis of table 2 by changing the backlogging parameter, scale parameter and shape parameters:

- Increase in δ results in decrease in inventory period, decrease in order quantity and total cost per time unit.
- Increase in α results in decrease in inventory period, decrease in order quantity and increase in total cost per time unit.
- Increase in β results in increase in inventory period, increase in order quantity and decrease in total cost per time unit.

5. CONCLUSION

Optimal inventory policies have been derived for the proposed inventory models(crisp and fuzzy). When compared to other types of deterioration The total cost is minimum for Weibull pattern deterioration

(Rajeswari and Vanjikkodi(2015)). When the cost parameters are triangular fuzzy numbers, the total cost obtained by graded mean, signed distance and centroid methods indicate that the grade mean method yields the best solution.

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