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Fixed Point Result with Compact Metric Space

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Abstract
The goal of this research is to study some fixed point results in compact metric space. We have proved some fixed point theorem for self-mapping satisfying a new contractive condition involving rational expressions in compact metric space.

Introduction–
There are several generalizations of classical contraction mapping theoremofBanach [1].In 1961 Edelstein[3] established the existence of a unique fixed point of a self-map T of a compact metric space satisfying the inequality $d(Tx,Ty) < d(x,y)$ .Which is generalization of Banach.Iseki [4],fisher [5],have proved some results on compact metric space.
Here we are finding some fixed point result involving rational expression in compact metric Space. This paper is the generalization of Bhardwaj et al [2] and Soni [8].

Preliminaries-
Definition 1.1 Fixed point space: Let F be a self-continuous mapping. A space X is calleda fixed point space if every continuous mapping F of X into itself, has a fixed point, inthe sense that$f(x_0) = x_0$.

Definition 1.2 A class {Gi} of open subset of X is said to be an open cover of X, if eachpoint in X belongs to one Gi that is U̇iGi = .X.

A subclass of an open cover which is at least an open cover is called a sub cover.

A compact space is that space in which every open cover has finite sub cover.

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Acompact space is that space in which every open cover has finite sub cover.

Main Result-
Theorem-Let$f$ be a continuous mapping of a compact space X into itself satisfying the Following condition –

$$d(fx,fy) < \alpha_1 \frac{[d(x,fx)+d(y,fx)+d(x,fy)+d(x,y)]}{[1+d(x,fx).d(y,fx).d(y,fy).d(x,y)} + \alpha_2 \frac{[d(x,fx).d(x,fy)+d(y,fy).d(y,fx)}{[d(x,y)+d(x,fx).d(x,fy).d(y,fx).d(x,y)} +$$

$$\alpha_3 \max\{d(x,fx),d(x,fy),d(y,fx),d(y,fy)\} + \alpha_4\, d(x,y)$$

For all $x,y \in X, x \neq y$ and$4\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4 \leq 1$ where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are non-negative andreal, then$f$ has a unique fixed point.

Proof– First we define a function T on X as follows $T(x) = d(x, f(x))$ for all $x \in X$. Since d *and* f are continuous on X.from compactness of X,there exist a point $p \in X$, Such that

$$T(p) = \inf\{T(x): x \in X\} \tag{1.2}$$

If $T(p) \neq 0$, it follows that $p \neq f(p)$, and so $T(f(p)) = d(f(p), f^2(p))$

$$d(f(p), f(f(p)) < \alpha_1 \frac{[d(p,f(p)) + d(f(p),f(p)) + d(p,f^2(p)) + d(p,f(p))]}{[1 + d(p,f(p)).d(f(p),f(p)).d(f(p),f^2(p)).d(p,f(p))}$$
$$+ \alpha_2 \frac{[d(p,f(p)).d(p,f^2(p)) + d(f(p),f^2(p)).d(f(p),f(p))}{[d(p,f(p)) + d(p,f(p)).d(p,f^2(p)).d(f(p),f(p)).d(p,f(p))}$$
$$+ \alpha_3 \max\{d(p,f(p)), d(p,f^2(p)), d(f(p),f(p)), d(f(p),f^2(p))\}$$
$$+ \alpha_4\, d(p,f(p))$$

$$d(f(p),f^2(p)) < \alpha_1[2d(p,f(p)) + d(p,f^2(p))] + \alpha_2.d(p,f^2(p))$$
$$+ \alpha_3\, max\{d(p,f(p)), d(p,f^2(p)), d(f(p),f^2(p))\}$$
$$+ \alpha_4\, d(p,f(p))$$
$$< \alpha_1[2d(p,f(p)) + d(p,f(p) + d(f(p),f^2p)]$$
$$+ \alpha_2[d(p,f(p)) + d(f(p),f^2(p))\,]$$
$$+ \alpha_3\, max\{d(p,f(p)), d(p,f^2(p)), d(f(p),f^2(p))\}$$
$$+ \alpha_4\, d(p,f(p))$$

Case (i) – If

$max\{d(p,f(p)), d(p,f^2(p)), d(f(p),f^2(p))\} = d(f(p),f^2(p))$ $d(f(p),f^2(p)) <$

$$\alpha_1[3d(p,f(p)) + d(f(p),f^2(p))] + \alpha_2[d(p,f(p)) + d(f(p),f^2(p))\,]$$
$$+ \alpha_3\, d(f(p),f^2(p)) + \alpha_4\, d(p,f(p))$$

$$(1-\alpha_1-\alpha_2-\alpha_3)d(f(p),f^2(p)) < (3\alpha_1+\alpha_2+\alpha_4)\, d(p,f(p))$$

$$d(f(p),f^2(p)) = \frac{(3\alpha_1+\alpha_2+\alpha_4)}{(1-\alpha_1-\alpha_2-\alpha_3)}\, d(p,f(p))$$

$$d(f(p),f^2(p)) = \lambda_1\, d(p,f(p))$$

Where $\lambda_1 = \frac{(3\alpha_1+\alpha_2+\alpha_4)}{(1-\alpha_1-\alpha_2-\alpha_3)}$

But $4\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4 \leq 1$, which is a contradiction.

Case-(ii)-If $max\{d(p,f(p)), d(p,f^2(p)), d(f(p),f^2(p))\} = d(p,f^2(p))$

Then,

$$d\big(f(p),f^2(p)\big) < \alpha_1[3d(p,f(p)) + d(f(p),f^2(p))] + \alpha_2[d\big(p,f(p)\big) + d\big(f(p),f^2(p)\big)] + \alpha_3\, d\big(p,f^2(p)\big) + \alpha_4\, d(p,f(p))$$

$$< \alpha_1[3d(p,f(p)) + d(f(p),f^2p)] + \alpha_2[d\big(p,f(p)\big) + d\big(f(p),f^2(p)\big)] + \alpha_3[d\big(p,f(p)\big) + d\big(f(p),f^2(p)\big)] + \alpha_4\, d(p,f(p))$$

$$(1-\alpha_1-\alpha_2-\alpha_3)d\big(f(p),f^2(p)\big) < (3\alpha_1+\alpha_2+\alpha_3+\alpha_4)\, d(p,f(p))$$

$$d\big(f(p),f^2(p)\big) = \frac{(3\alpha_1+\alpha_2+\alpha_3+\alpha_4)}{(1-\alpha_1-\alpha_2-\alpha_3)}\, d(p,f(p))$$

$$d\big(f(p),f^2(p)\big) = \lambda_2\, d\big(p,f(p)\big)$$

$$Where\ \lambda_2 = \frac{(3\alpha_1+\alpha_2+\alpha_3+\alpha_4)}{(1-\alpha_1-\alpha_2-\alpha_3)}$$

But $4\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4 \leq 1$, which is a contradiction.

Case-(iii)-*If* $max\{d\big(p,f(p)\big), d\big(p,f^2(p)\big), d\big(f(p),f^2(p)\big)\} = d\big(p,f(p)\big)$

Then,

$$d\big(f(p),f^2(p)\big) < \alpha_1[3d(p,f(p)) + d(f(p),f^2(p))] + \alpha_2[d\big(p,f(p)\big) + d\big(f(p),f^2(p)\big)] + \alpha_3\, d\big(p,f(p)\big) + \alpha_4\, d(p,f(p))$$

$$(1-\alpha_1-\alpha_2)d\big(f(p),f^2(p)\big) < (3\alpha_1+\alpha_2+\alpha_3+\alpha_4)\, d(p,f(p))$$

$$d\big(f(p),f^2(p)\big) = \frac{(3\alpha_1+\alpha_2+\alpha_3+\alpha_4)}{(1-\alpha_1-\alpha_2)}\, d(p,f(p))$$

$$d\big(f(p),f^2(p)\big) = \lambda_2\, d\big(p,f(p)\big)$$

$$Where\ \lambda_2 = \frac{(3\alpha_1+\alpha_2+\alpha_3+\alpha_4)}{(1-\alpha_1-\alpha_2)}$$

But $4\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4 \leq 1$, *which is a contradiction*

$$Here\ \lambda = max\{\lambda_1, \lambda_2\, \lambda_3\}$$

$$\lambda = max\{\frac{(3\alpha_1+\alpha_2+\alpha_4)}{(1-\alpha_1-\alpha_2-\alpha_3)}, \frac{(3\alpha_1+\alpha_2+\alpha_3+\alpha_4)}{(1-\alpha_1-\alpha_2-\alpha_3)}, \frac{(3\alpha_1+\alpha_2+\alpha_3+\alpha_4)}{(1-\alpha_1-\alpha_2)}\}$$

Hence $p = f\ (p)$ *and* p *is a fixed point of f.*

Unique –Now next we prove the uniqueness of p. Let If possible $q \neq p$ be another fixed point
Of f.
Then,

$$d(p,q) = d\big(f(p), f(q)\big)$$
$$< \alpha_1 \frac{[d(p,f(p)) + d(q,f(p)) + d(p,f(q)) + d(p,q)]}{[1 + d(p,f(p)).\,d(q,f(p)).\,d(q,f(q)).\,d(p,q)} + \alpha_2 \frac{[d\big(p,f(p)\big).\,d\big(p,f(q)\big) + d\big(q,f(q)\big).\,d(q,f(p))}{[d(p,q) + d(p,f(p)).\,d(p,f(q)).\,d\big(q,f(p)\big).\,d(p,q)} +$$

$$\alpha_3 \max\{d(p,f(p)), d(p,f(q)), d(q,f(p)), d(q,f(q))\} + \alpha_4\, d(p,q)$$
$$d(p,q) < (3\,\alpha_1 + \alpha_3 + \alpha_4)\, d(p,q)$$

This is contradiction, because $3\,\alpha_1 + \alpha_3 + \alpha_4 < 1$

Hence$p$is unique.

Remark- If we put ,$\alpha_1 = \alpha_2 = \alpha_3 = 0$, $\alpha_4 = 1$, then we get the result of Edelstein [3].

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