

On Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Spaces with Rational Inequality

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Abstract: In this paper, we use the concepts of subcompatibility and subsequential continuity in Intuitionistic Fuzzy Metric Spaces which are respectively weaker than occasionally weak compatibility and reciprocal continuity. With them, we establish a common fixed point theorem for four mapstaking rational inequality.

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1. Introduction

Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [8] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al.[1] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [5]. Further, Alaca et al. [1] proved Intuitionistic fuzzy Banach and Intuitionisticfuzzy Edelstein contraction theorems, with the different definition of Cauchy sequences and completeness than the ones given in [8].Popa ([9]-[10]) introduced the idea of implicit function to prove a common fixed point theorem in metric spaces Singhand Jain [13] further extended the result of Popa ([9]-[10]) in fuzzy metric spaces. In this paper, we use the concepts of subcompatibility and subsequential continuity in Intuitionistic Fuzzy Metric Spaces Using Implicit Relation which are respectively weaker than occasionally weak compatibility and reciprocal continuity. With them, we establish a common fixed point theorem for four maps.

2 Preliminary Notes

Definition 2.1.[11] A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t-norms if $*$ is satisfying conditions:

- (i) $*$ is an commutative and associative;
- (ii) $*$ is continuous;
- (iii) $a * 1 = a$ for all $a \in [0, 1]$;
- (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

Definition 2.2.[11] A binary operation \diamond : $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t -conorm if it satisfies the following conditions:

- (a) \diamond is commutative and associative;
- (b) \diamond is continuous;
- (c) $a \diamond 0 = a$ for all $a \in [0, 1]$
- (d) $a \diamond b = c \diamond d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Definition 2.3.[1] A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space (shortly IFM-Space) if X is an arbitrary set, $*$ is a continuous t-norm, \diamond is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$;

$$(IFM-1) M(x, y, t) + N(x, y, t) \leq 1$$

$$(IFM-2) M(x, y, 0) = 0 \text{ for all } x, y \text{ in } X$$

$$(IFM-3) M(x, y, t) = 1 \text{ for all } x, y \text{ in } X \text{ and } t > 0 \text{ if and only if } x = y$$

$$(IFM-4) M(x, y, t) = M(y, x, t), \text{ for all } x, y \text{ in } X \text{ and } t > 0$$

$$(IFM-5) M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$$

$$(IFM-6) M(x, y, \cdot): [0, \infty) \rightarrow [0, 1] \text{ is left continuous}$$

$$(IFM-7) \lim_{t \rightarrow \infty} M(x, y, t) = 1, \text{ for all } x, y \text{ in } X \text{ and } t > 0,$$

$$(IFM-8) N(x, y, 0) = 1 \text{ for all } x, y \text{ in } X$$

$$(IFM-9) N(x, y, t) = 0 \text{ for all } x, y \text{ in } X \text{ and } t > 0 \text{ if and only if } x = y$$

$$(IFM-10) N(x, y, t) = N(y, x, t), \text{ for all } x, y \text{ in } X \text{ and } t > 0$$

$$(IFM-11) N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$$

$$(IFM-12) N(x, y, \cdot): [0, \infty) \rightarrow [0, 1] \text{ is right continuous}$$

$$(IFM-13) \lim_{t \rightarrow \infty} N(x, y, t) = 0, \text{ for all } x, y \text{ in } X \text{ and } t > 0$$

Then (M, N) is called an intuitionistic fuzzy metric on X . The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t , respectively.

Remark 2.4. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \diamond)$ such that t -norm $*$ and t -conorm \diamond are associated that is,

$$x \diamond y = 1 - ((1 - x) * (1 - y)) \text{ for all } x, y \in X.$$

Example 2.5. Let (X, d) be a metric space. Define t -norm $a * b = \min \{a, b\}$ and t -conorm $a \diamond b = \max \{a, b\}$ and for all $x, y \in X$ and $t > 0$.

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \quad N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Then $(X, M, N, *, \diamond)$ is an IFM-space and the intuitionistic fuzzy metric space (M, N) induced by the metric d is often referred to as the standard intuitionistic fuzzy metric.

Remark 2.6.[1] In intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing for all x, y in X .

Definition 2.7.[1] Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0:$$

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_n, x, t) = 0$$

(c) $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is

Convergent.

Example 2.8. Let $X = \{1/n: n = 1, 2, 3 \dots\} \cup \{0\}$ and let $*$ be the continuous t-norm and \diamond be the continuous t-conorm defined by $a * b = a \cdot b$ and $a \diamond b = \min. \{1, a+b\}$ respectively, for all $a, b \in [0, 1]$. For each $t \in (0, \infty)$ and x, y in X , define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & t > 0 \\ 0, & t = 0 \end{cases} \quad \text{and} \quad N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0 \\ 1, & t = 0 \end{cases}$$

Clearly, $(X, M, N, *, \diamond)$ is complete intuitionistic fuzzy metric space.

The following definition of weakly commuting mappings in intuitionistic fuzzy metric space is given on the lines of Sessa [12].

Definition 2.9[12]. Let A and S be maps from an intuitionistic fuzzy metric space

$(X, M, N, *, \diamond)$ into itself. The maps A and S are said to be weakly commuting if

$M(ASz, SAz, t) \geq M(Az, Sz, t)$, and $N(ASz, SAz, t) \leq N(Az, Sz, t)$ for all $z \in X$ and $t > 0$

Definition 2.10[14]. Let A and S be maps from an IFM-space $(X, M, N, *, \diamond)$ into itself.

The maps A and S are said to be compatible if for all $t > 0$,

$$\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(ASx_n, SAx_n, t) = 0, \text{ whenever } \{x_n\} \text{ is a sequence in } X \text{ such that } \lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z \text{ for some } z \in X.$$

Definition 2.11[6]: Two mappings A and S of a IFM space $(X, M, N, *, \diamond)$ will be called reciprocally continuous if $ASu_n \rightarrow Az$ and $SAu_n \rightarrow Sz$, whenever $\{u_n\}$ is a sequence such that $Au_n, Su_n \rightarrow z$ for some z in X .

If A and S are both continuous, then they are obviously reciprocally continuous but converse is not true. Moreover, in the setting of common fixed point theorems for compatible pair of mappings satisfying contractive conditions, continuity of one of the mappings A and S implies their reciprocal continuity but not conversely.

Definition 2.12. Let $(X, M, N, *, \diamond)$ be a intuitionistic fuzzy metric space. A and S be selfmaps on X . A point x in X is called a coincidence point of A and S iff $Ax = Sx$. In this case, $w = Ax = Sx$ is called a point of coincidence of A and S .

Definition 2.13. A pair of self mappings (A, S) of a IFM space $(X, M, N, *, \diamond)$ is said to be weakly compatible if they commute at the coincidence points i.e., if $Au = Su$ for some u in X , then $ASu = SAu$.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Definition 2.14[2]. Two self mappings A and S of a IFM space $(X, M, N, *, \diamond)$ are said to be occasionally weakly compatible (owc) iff there is a point x in X which is coincidence point of A and S at which A and S commute. In this paper, we weaken the above notion by introducing a new concept called subcompatibility just as defined by H. Bouhadjera[4] in metric space, as follows:

Definition 2.15. Let $(X, M, N, *, \diamond)$ be a intuitionistic fuzzy metric space. Self maps A and S on X are said to be subcompatible iff there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z \text{ for some } z \in X \text{ and satisfy}$$

$$\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(ASx_n, SAx_n, t) = 0.$$

Obviously, two owc maps are subcompatible, however the converse is not true in general. The example below shows that there exist subcompatible maps which are not owc

Example 2.16. Let $X \in [0, \infty)$. For each $t \in (0, \infty)$ and $x, y \in X$, define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & t > 0 \\ 0, & t = 0 \end{cases} \quad \text{and} \quad N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0 \\ 1, & t = 0 \end{cases}$$

Define A and S as follows:

$$A(x) = x^2, S(x) = \begin{cases} x + 2 \text{ if } x \in [0, 4] \cup (9, \infty) \\ x + 12 \text{ if } x \in (4, 9] \end{cases}$$

Let $\{x_n\}$ be a sequence in X defined by $x_n = 2 + \frac{1}{n}$ for $n = 1, 2, 3, \dots$

Then $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = 4, 4 \in X$ and

$$ASx_n \rightarrow 16, SAx_n \rightarrow 16$$

When $n \rightarrow \infty$. Thus, $\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) = 1$ and $\lim_{n \rightarrow \infty} N(ASx_n, SAx_n, t) = 0$.

i.e. A and S are subcompatible. On the other hand, we have

$Ax = Sx$ iff $x = 2$ and $AS(2) \neq SA(2)$, hence A and S are not owc.

Now, our second objective is to introduce subsequential continuity in intuitionistic fuzzymetric space which weakens the concept of reciprocal continuity which was introduced by Pant [8] just as introduced by H. Bouhadjera[4] in metric space, as follows:

Definition 2.17. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Self maps A and S on X are said to be subsequentially continuous iff there exist a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = t \text{ for some } t \in X \text{ and satisfy } \lim_{n \rightarrow \infty} ASx_n = At, \lim_{n \rightarrow \infty} SAx_n = St.$$

Clearly, if A and S are continuous or reciprocally continuous then they are obviously subsequentially continuous. The next example shows that there exist subsequentially continuous pairs of maps which are neither continuous nor reciprocally continuous.

Example 2.18. Let $X \in [0, \infty)$. For each $t \in (0, \infty)$ and $x, y \in X$, define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & t > 0 \\ 0, & t = 0 \end{cases} \quad \text{and} \quad N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0 \\ 1, & t = 0 \end{cases}$$

Define A and S as follows:

$$A(x) = \begin{cases} 1 + x \text{ if } x \in [0, 1] \\ 2x - 1 \text{ if } x \in (1, \infty) \end{cases} \quad S(x) = \begin{cases} 1 - x \text{ if } x \in [0, 1] \\ 3x - 2 \text{ if } x \in [1, \infty) \end{cases}$$

Clearly A and S are discontinuous at $x = 1$.

Let $\{x_n\}$ be a sequence in X defined by $x_n = \frac{1}{n}$ for $n = 1, 2, 3, \dots$

Then, $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = 1, 1 \in X$ and

$ASx_n \rightarrow 2 = A(1), SAx_n \rightarrow 1 = S(1)$ when $n \rightarrow \infty$, therefore, A and S are subsequentially continuous.

Now, let $\{x_n\}$ be a sequence in X defined by $x_n = 1 + \frac{1}{n}$ for $n = 1, 2, 3, \dots$

Then, $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = 1, 1 \in X$ and

$ASx_n \rightarrow 1 \neq 2 = A(1)$ when $n \rightarrow \infty$, so A and S are not reciprocally continuous.

Lemma 2.19. Let $\{u_n\}$ be a sequence in an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$.
 If there exists a constant $k \in (0, 1)$ such that
 $M(u_n, u_{n+1}, kt) \geq M(u_{n-1}, u_n, t)$ and $N(u_n, u_{n+1}, kt) \leq N(u_{n-1}, u_n, t)$ for all $t > 0$ and $n = 1, 2, 3, \dots$.
 Then $\{u_n\}$ is a Cauchy sequence in X .

3. Implicit Relation

Let M_4 be the set of all real continuous functions $\phi, \psi: [0, 1]^3 \rightarrow \mathbb{R}$, non-decreasing in the first argument and satisfying the following conditions:

- A. $\phi(u, u, 1) \geq 0 \Rightarrow u \geq 1$
- B. $\psi(u, u, 0) \leq 0 \Rightarrow u \leq 0$ for all u .

4. Main Result

Now, we prove our main theorem using definition of subcompatible and subsequential continuous maps as follows:

Theorem 4.1. Let A, B, S and T be four self maps of a Intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ with continuous t-norm $*$ and continuous t-conorm \diamond defined by $t*t \geq \text{and}$
 $(1-t)\diamond(1-t) \leq (1-t)$ for all $t \in [0, 1]$. If the pairs (A, S) and (B, T) are subcompatible and subsequentially continuous, then

- (a) A and S have a coincidence point.
- (b) B and T have a coincidence point.
- (c) For some $\phi, \psi \in M_4$ and for all $x, y \in X$ and every $t > 0$.

$$\phi \left\{ \frac{aM(Ax, By, t) + bM(Sx, By, t)}{a + b}, \frac{cM(Sx, Ty, t) + dM(Ty, Ax, t)}{c + d}, \frac{M(Sx, Ax, t) + M(Ty, By, t)}{2} \right\} \geq 0$$

$$\psi \left\{ \frac{aN(Ax, By, t) + bN(Sx, By, t)}{a + b}, \frac{cN(Sx, Ty, t) + dN(Ty, Ax, t)}{c + d}, \frac{N(Sx, Ax, t) + N(Ty, By, t)}{2} \right\} \leq 0$$

Then A, B, S, T have a unique common fixed point.

Where a and b or $(c$ and $d)$ can not be simultaneously zero.

Proof: Since the pair (A, S) and (B, T) are subcompatible and subsequentially continuous, then, there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z, z \in X \text{ and satisfy}$$

$$\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) = M(Az, Sz, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(ASx_n, SAx_n, t) = N(Az, Sz, t) = 0.$$

$$\lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = z', z' \in X \text{ and satisfy}$$

$$\lim_{n \rightarrow \infty} M(BTy_n, TBy_n, t) = M(Bz', Tz', t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(BTy_n, TBy_n, t) = N(Bz', Tz', t) = 0.$$

Therefore, $Az = Sz$ and $Bz' = Tz'$; that is, z is a coincidence point of A and S and z' is a coincidence point of B and T .

Now, we prove $z = z'$.

Put $x = x_n$ and $y = y_n$ in inequality (c), we get

$$\emptyset \left\{ \frac{\frac{aM(Ax_n, By_n, t) + bM(Sx_n, By_n, t)}{a+b}, \frac{cM(Sx_n, Ty_n, t) + dM(Ty_n, Ax_n, t)}{c+d}}{\frac{M(Sx_n, Ax_n, t) + M(Ty_n, By_n, t)}{2}} \right\} \geq 0$$

taking the limit as $n \rightarrow \infty$ yields

$$\emptyset \left\{ \frac{\frac{aM(z, z', t) + bM(z, z', t)}{a+b}, \frac{cM(z, z', t) + dM(z', z, t)}{c+d}}{\frac{M(z, z, t) + M(z', z', t)}{2}} \right\} \geq 0$$

$$\emptyset\{M(z, z', t), M(z, z', t), 1\} \geq 0$$

$$\Psi \left\{ \frac{\frac{aN(Ax_n, By_n, t) + bN(Sx_n, By_n, t)}{a+b}, \frac{cN(Sx_n, Ty_n, t) + dN(Ty_n, Ax_n, t)}{c+d}}{\frac{N(Sx_n, Ax_n, t) + N(Ty_n, By_n, t)}{2}} \right\} \leq 0$$

$$\Psi \left\{ \frac{\frac{aN(z, z', t) + bN(z, z', t)}{a+b}, \frac{cN(z, z', t) + dN(z', z, t)}{c+d}}{\frac{N(z, z, t) + N(z', z', t)}{2}} \right\} \leq 0$$

$$\Psi\{N(z, z', t), N(z, z', t), 0\} \leq 0$$

In view of \emptyset, Ψ we get $z = z'$

Again, we claim that $Az = z$.

Put $x = z$ and $y = y_n$ in inequality (c), we get

$$\emptyset \left\{ \frac{\frac{aM(Az, By_n, t) + bM(Sz, By_n, t)}{a+b}, \frac{cM(Sz, Ty_n, t) + dM(Ty_n, Az, t)}{c+d}}{\frac{M(Sz, Az, t) + M(Ty_n, By_n, t)}{2}} \right\} \geq 0$$

$$\emptyset \left\{ \frac{\frac{aM(Az, z', t) + bM(Az, z', t)}{a+b}, \frac{cM(Az, z', t) + dM(z', Az, t)}{c+d}}{\frac{M(Az, Az, t) + M(z', z', t)}{2}} \right\} \geq 0$$

$$\emptyset\{M(Az, z', t), M(Az, z', t), 1\} \geq 0$$

$$\Psi \left\{ \frac{\frac{aN(Az, By_n, t) + bN(Sz, By_n, t)}{a+b}, \frac{cN(Sz, Ty_n, t) + dN(Ty_n, Az, t)}{c+d}}{\frac{N(Sz, Az, t) + N(Ty_n, By_n, t)}{2}} \right\} \leq 0$$

$$\Psi \left\{ \frac{aN(Az, z', t) + bN(Az, z', t)}{a + b}, \frac{cN(Az, z', t) + dN(z', Az, t)}{c + d}, \frac{N(Az, Az, t) + N(z', z', t)}{2} \right\} \leq 0$$

$$\Psi\{N(Az, z', t), N(Az, z', t), 0\} \leq 0$$

In view of \emptyset, Ψ we get $Az = z' = z$

Again, we claim that $Bz = z$.

Put $x = z$ and $y = z$ in inequality (c), we get

$$\emptyset \left\{ \frac{aM(Az, Bz, t) + bM(Sz, Bz, t)}{a + b}, \frac{cM(Sz, Tz, t) + dM(Tz, Az, t)}{c + d}, \frac{M(Sz, Az, t) + M(Tz, Bz, t)}{2} \right\} \geq 0$$

$$\emptyset \left\{ \frac{aM(z, Bz, t) + bM(z, Bz, t)}{a + b}, \frac{cM(z, Bz, t) + dM(Bz, z, t)}{c + d}, \frac{M(z, z, t) + M(Bz, Bz, t)}{2} \right\} \geq 0$$

$$\emptyset\{M(z, Bz, t), M(z, Bz, t), 1\} \geq 0$$

$$\Psi \left\{ \frac{aN(Az, Bz, t) + bN(Sz, Bz, t)}{a + b}, \frac{cN(Sz, Tz, t) + dN(Tz, Az, t)}{c + d}, \frac{N(Sz, Az, t) + N(Tz, Bz, t)}{2} \right\} \leq 0$$

$$\Psi \left\{ \frac{aN(z, Bz, t) + bN(z, Bz, t)}{a + b}, \frac{cN(z, Bz, t) + dN(Bz, z, t)}{c + d}, \frac{N(z, z, t) + N(Bz, Bz, t)}{2} \right\} \leq 0$$

$$\Psi\{N(z, Bz, t), N(z, Bz, t), 0\} \leq 0$$

In view of \emptyset, Ψ we get $z = Bz = Tz$

Therefore, $z = Az = Bz = Sz = Tz$, that is z is common fixed point of A, B, S and T .

For Uniqueness: Suppose that there exist another fixed point w of A, B, S and T .

By condition (c), take $x = z, y = w$, we have

$$\emptyset \left\{ \frac{aM(Az, Bw, t) + bM(Sz, Bw, t)}{a + b}, \frac{cM(Sz, Tw, t) + dM(Tw, Az, t)}{c + d}, \frac{M(Sz, Az, t) + M(Tw, Bw, t)}{2} \right\} \geq 0$$

$$\emptyset \left\{ \frac{aM(z, w, t) + bM(z, w, t)}{a + b}, \frac{cM(z, w, t) + dM(w, z, t)}{c + d}, \frac{M(z, z, t) + M(w, w, t)}{2} \right\} \geq 0$$

$$\emptyset\{M(z, w, t), M(z, w, t), 1\} \geq 0$$

$$\Psi \left\{ \frac{aN(Az, Bw, t) + bN(Sz, Bw, t)}{a + b}, \frac{cN(Sz, Tw, t) + dN(Tw, Az, t)}{c + d}, \frac{N(Sz, Az, t) + N(Tw, Bw, t)}{2} \right\} \leq 0$$

$$\Psi \left\{ \frac{aN(z, w, t) + bN(z, w, t)}{a + b}, \frac{cN(z, w, t) + dN(w, z, t)}{c + d}, \frac{N(z, z, t) + N(w, w, t)}{2} \right\} \leq 0$$

$$\Psi\{N(z, w, t), N(z, w, t), 0\} \leq 0$$

In view of \emptyset, Ψ we get $z = w$. Therefore, uniqueness follows.

If we put $S = T$, in Theorem 4.1, we get the following result:

Corollary 4.2. Let A, B and S be threeself maps of a Intuitionistic fuzzy metric space

$(X, M, N, *, \diamond)$ with continuous t-norm $*$ and continuous t-conorm \diamond defined by $t*t \geq \text{tand}$

$(1-t)\diamond(1-t) \leq (1-t)$ for all $t \in [0, 1]$. If the pairs (A, S) and (B, S) are subcompatible and

subsequentially continuous, then

(a) A and S have a coincidence point.

(b) B and S have a coincidence point.

(c) For some $\emptyset, \Psi \in M_4$ and for all $x, y \in X$ and every $t > 0$.

$$\emptyset \left\{ \frac{aM(Ax, By, t) + bM(Sx, By, t)}{a + b}, \frac{cM(Sx, Sy, t) + dM(Sy, Ax, t)}{c + d}, \frac{M(Sx, Ax, t) + M(Sy, By, t)}{2} \right\} \geq 0$$

$$\Psi \left\{ \frac{aN(Ax, By, t) + bN(Sx, By, t)}{a + b}, \frac{cN(Sx, Sy, t) + dN(Sy, Ax, t)}{c + d}, \frac{N(Sx, Ax, t) + N(Sy, By, t)}{2} \right\} \leq 0$$

Then A, B, S have a unique common fixed point.

Where a and b or $(c$ and $d)$ can not be simultaneously zero.

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