

An Alternative Solution to Multi Objective Linear Fractional Programming Problem by Using Geometric Programming Technique

Ersoy OZ[1*] Selcuk Alp[2] Nuran Guzel[3]

1. Department of Statistics, Yildiz Technical University, 34220 Esenler-Istanbul, Turkey
2. Department of Industrial Engineering, Yildiz Technical University,34349 Yildiz-Istanbul, Turkey
3. Department of Mathematics, Yildiz Technical University, 34220 Esenler-Istanbul, Turkey

* E-mail of the corresponding author: ersoyoz@yildiz.edu.tr

Abstract

In this study, we have proposed an alternative solution to the multi objective linear fractional programming problems. This method deals with every objective of multi objective linear fractional programming problems gradually by using geometric programming technique to find the pareto optimal solution. The proposed solution procedure has been used in numeric examples and results have been compared with the real solution values.

Keywords: multi objective, fractional programming, geometric programming

1. Introduction

Multi Objective Linear Fractional Programming Problem (MOLFPP), is an optimization problem that includes more than one ratio functions under some linear limitations. These ratio functions are handled as objective functions that measure the efficiency of the system, such as inventory/sales, cost/profit, output/employee, cost/profit and actual cost/standards cost. The solution values that each ratio function takes under the limitations in the problem are different. The optimal solution of MOLFPP can be reached by compensating these different solution values (Guzel 2013).

Many methods and applications have been used for solving the MOLFPP in the literature: Charnes & Cooper (1962) solved the single objective Linear Fractional Programming Problem (LFPP) by converting it to linear programming problem. Choo & Atkins (1982) provided an analysis about the bi-criteria linear fractional programming. Kornbluth & Steuer (1981) and Lai & Hwang (1996) have developed an algorithm for solving the MOLFPP for the all weak-efficient vertices of the feasible region. Mishra & Singh (2013) used the method of LFPP in order to solve Multi Objective Linear Programming Problem. Chakraborty & Gupta (2002) suggested a procedure for solution of Multi Objective Fractional Programming Problem. For this procedure, they reformulated Multi Objective Fractional Programming model and gave computational examples. Youness *et al.* (2014) presented a solution algorithm for bi level Multi Objective Fractional Integer Programming Problem. In this model right side constants have been formed by the blurred numbers. The suggested algorithm uses Taylor series and Kuhn Tucker conditions combinations and integer solutions have been obtained by Gomory Cutting method. Pal *et al.* (2003) showed that blurred MOLFPP can be solved by using objective programming procedure. Two objective functions in the computational examples have been combined by giving blurred values and by converting it to one objective function. At this stage two new limitations have been added. Bhatia & Mehra (1999) developed Lagrange multiplier theorems for the solution of MOLFPP that includes n-set function. Valipour *et al.* 2014) suggested iterative parametric method for the solution of MOLFPP. Roghanian *et al.* (2007) showed that contingent bi level linear multi objective programming problem can be used in cases when the production capacity, resource value and demand on the market are random in supply chain problem. Azar *et al.* (2012) used Multi Objective Programming Problems in planning the production of a metal firm and wood firm. Example was solved by Pal (2003) first with some suggested presuppositions as MOLFPP and then by Dutta *et al.* (1992) with suggested blurred method and they showed that the results are the same. Gomez *et al.* (2006) suggested a solution for the problem of harvesting time for timber in a wood plantation in Cuba by MOLFPP. Fasakhodi *et al.* (2010) have made an optimization by taking the “net return/water consumption” and “labor employment/water consumption” rates into consideration with MOLFPP.

Aggarwal and Patkar (1978) developed a geometric programming approach for Single Objective LFPP. In this study, an alternative model for the solution of MOLFPP is going to be presented. The suggested model is based on developing the solution of Aggarwal & Patkar (1978) by a gradual approach on MOLFPP in cases when there are multiple objectives. Geometric Programming (GP), a special type of nonlinear programming, is a special technique that is developed in order to find the optimum values of posynomial and signomial functions. The solution to the GP problem follows the opposite method with respect to the classic optimization technique and it depends on the technique of first finding the weight values and calculating the optimum value for the objective function, then finding the values of the decision variables. GP started to be modelled and studied as part of nonlinear optimization (Zener 1991; Deffin *et. al* 2000). Some algorithms were used when trying to solve GP. In this study, GP technique is used to solve MOLFPP and the obtained results are compared to the results of the problems given in (Guzel 2013; Dangwal *et al.* 2012; Sen 1983; Sulaiman & Salih 2010).

In the next section of this study, the MOLFPP and GP will be defined respectively. In the third section, the model that we suggest depending on the GP technique will be clarified.

In the fourth section, the results of the MOLFPP solved with different methods are shown together with the results obtained through our proposed model. In the last section, conclusion and comments will be included.

2. Preliminaries

In this section, the mathematical basis of the methods that will be used in the proposed model will be explained.

2.1 Multiple Objective Linear Fractional Programming Problem

The general format of LFPP may be written as (Chakrabory & Gupta 2002)

$$Z(x) = \frac{c^T x+\alpha}{d^T x+\beta} \tag{1}$$

subject to

$$Ax = b, \tag{2}$$

$$x \geq 0, \tag{3}$$

$$x \in R^n, \tag{4}$$

$$b,\ c^T, d^T \in R^n, \tag{5}$$

$$A \in R^{m\times n}, \tag{6}$$

$$\alpha, \beta \in R. \tag{7}$$

For some values of $x, d^T x+\beta$ may be equal to zero. To avoid such cases, one requires that either $[x \geq 0,\ Ax = b] \Longrightarrow [d^T x+\beta > 0]\ or\ [x \geq 0,\ Ax = b] \Longrightarrow [d^T x+\beta < 0]$.

For convenience, assume that LFP (Eq. 1 - Eq. 7) satisfies the condition that:

$$[x \geq 0,\ Ax = b] \Longrightarrow [d^T x+\beta > 0]. \tag{8}$$

The problem is named as a MOLFPP when there are multiple objective functions. The general format of a MOLFPP may be written as (Chakrabory & Gupta 2002)

$$Z(x) = \{Z_1(x), Z_2(x), \ldots, Z_K(x)\} \tag{9}$$

subject to

$$Ax = b, \tag{10}$$

$$x \geq 0, \tag{11}$$

$$x \in R^n, \tag{12}$$

$$b \in R^n, A \in R^{m\times n} \tag{13}$$

Each $Z_i(x)$ in the objective function Eq. 9 is shown below:

$$Z_i(x) = \frac{c^T x+\alpha}{d^T x+\beta} \tag{14}$$

$$c^T, d^T \in R^n, \tag{15}$$

$$\alpha, \beta \in R. \tag{16}$$

2.2 Geometric Programming Problem

A GP problem is generally defined as follows (Beightler & Phillips 1976):

$$min\ y_0(x) = \sum_{t=1}^{T_0} C_{0t} \prod_{n=1}^{N} x_n^{a_{0tn}} \tag{17}$$

subject to

$$y_m(x) = \sum_{t=1}^{T_m} C_{mt} \prod_{n=1}^{N} x_n^{a_{mtn}} \leq 1, \quad m = 1, 2, \dots, M \tag{18}$$

When all of the C constants are positive, the function is called a posynomial. When at least one of them are negative, it is called a signomial (Boyd *et al.* 2007). To solve the geometric programming problem given in Eq. 17 and Eq. 18, the dual of the GP problem will be used instead of the classic optimization theory. The dual of the GP problem is based on the arithmetic geometric mean inequality (Taha 2010). For the general posynomial case, the dual GP problem is as follows (Beightler & Phillips 1976):

$$max\ \prod_{m=0}^{M} \prod_{t=1}^{T_m} \left(\frac{w_{m0} C_{mt}}{w_{mt}}\right)^{w_{mt}} \tag{19}$$

subject to

$$\sum_{t=1}^{T_0} w_{0t} = 1 \tag{20}$$

$$\sum_{m=1}^{M} \sum_{t=1}^{T_m} a_{mtn} w_{mt} = 0, \quad n = 1, 2, \dots, N \tag{21}$$

$$w_{m0} = \sum_{t=1}^{T_m} w_{mt}, \quad m = 1, 2, \dots, M \tag{22}$$

given the conditions $w_{00} = 1$ and $w_{m0} = \lambda_m$. Eq. 20 is named as the normality condition and Eq. 21 is named as the orthagonality condition.

3. The Proposed Model

3.1 The Solution of Single Objective Linear Fractional Programming Problems

The mathematical form of the method used in solving a Single Objective LFPP is given below (Aggarwal & Patkar 1978):

(P_F) Minimize

$$F(x_1, x_2, \dots, x_m) = \frac{\sum_{i=1}^{m} c_i x_i + c_0}{\sum_{i=1}^{m} d_i x_i + d_0} \tag{23}$$

subject to

$$\sum_{j=1}^{m} a_{ij} x_j - b_i \leq 0, i = 1,2, \dots, n \tag{24}$$

$$x_j \geq 0\ j = 1,2, \dots, m \tag{25}$$

Where $a_{ij}, b_i, c_i\ and\ d_i$ are arbitrary constants and it is assumed that $\sum_{i=1}^{m} d_i x_i + d_0 > 0$ over the feasible region.

Making use of transformation, $y_i = t x_i$, $i = 1,2, \dots, m$, we have the following equivalent linear program (Charnes & Cooper 1962),

(P_L) Minimize

$$H(y_1, \dots, y_m, t) = \sum_{i=1}^{m} c_i y_i + c_0 t \tag{26}$$

subject to

$$\sum_{j=1}^{m} d_j y_j + d_0 t - 1 \leq 0 \tag{27}$$

$$-\sum_{j=1}^{m} d_j y_j - d_0 t + 1 \leq 0 \tag{28}$$

$$\sum_{j=1}^{m} a_{ij} y_j - b_i t \leq 0, \quad i = 1,2, \dots, n \tag{29}$$

$$-y_j \leq 0, \quad j = 1,2, \dots, m \tag{30}$$

$$-t \leq 0 \tag{31}$$

Following is one-to-one transformation due to Duffin *et al.* (2000), $H(Y) = \log h, -1 = \log \mu_1, +1 = \log \mu_2, y_j = \log t_j, j = 1,2, \dots, m$ and $t = \log t_{m+1}$. Monotonicity of logarithmic function permits the resulting equivalent program to be expressed as primal geometric program given below:

(P_G) Minimize

$$h(t_1, t_2, \dots, t_{m+1}) = \prod_{j=1}^{m} t_j^{c_j} t_{m+1}^{c_0} \tag{32}$$

subject to

$$\mu_1 \prod_{j=1}^{m} t_j^{d_j} t_{m+1}^{d_0} \leq 1 \tag{33}$$

$$\mu_2 \prod_{j=1}^{m} t_j^{-d_j} t_{m+1}^{-d_0} \leq 1 \quad (34)$$
$$\prod_{j=1}^{m} t_j^{a_{ij}} t_{m+1}^{-b_i} \leq 1, i = 1,2,\dots,n \quad (35)$$
$$0 < t_j^{-1} \leq 1, j = 1,2,\dots,m+1 \quad (36)$$

3.2 The Solution of Multi Objective Linear Fractional Programming Problems

Definition (Goal Optimal solution): Let $\hat{x}$ be a feasible solution to MOLFPP with objective function values

$$Z_i(\hat{x}) = \frac{c^i\hat{x}+c_0^i}{d^i\hat{x}+d_0^i}, i = 1,\dots,k.$$

Then $\hat{x}$ is a goal-optimal solution MOLFPP if for every objective $r = 1,\dots,k.$ The LFPP

$$Max\ Z_r(x) = \frac{c^r x+c_0^r}{d^r x+d_0^r} \quad (37)$$
$$\frac{c^i x+c_0^i}{d^i x+d_0^i} \geq \hat{Z}_i, i \neq r \quad (38)$$
$$Ax \leq b \quad (39)$$
$$x \geq 0 \quad (40)$$

has optimal solution value $Z_r(x^*) = \frac{c^r\hat{x}+c_0^r}{d^r\hat{x}+d_0^r}$.

Definition (Pareto-Optimal solution): Let x^p be a feasible solution to MOLFPP. Then x^p is a Pareto-Optimal solution to MOLFPP if for any other feasible solution $\tilde{x}$, if there is some index i for

$$\frac{c^i\tilde{x}+c_0^i}{d^i\tilde{x}+d_0^i} > \frac{c^i x^p+c_0^i}{d^i x^p+d_0^i},$$

there is another index l for which

$$\frac{c^l\tilde{x}+c_0^l}{d^l\tilde{x}+d_0^l} < \frac{c^l x^p+c_0^l}{d^l x^p+d_0^l}.$$

Theorem: For $\hat{x}$ be a feasible solution to MOLFPP, $\hat{x}$ is pareto-optimal solution if and only if $\hat{x}$ is goal-optimal solution.

Proof: $\hat{x}$ is pareto-optimal solution if and only if there does not exist a feasible $\tilde{x}$ satisfying

$$\frac{c^k\tilde{x}+c_0^k}{d^k\tilde{x}+d_0^k} > \frac{c^k\hat{x}+c_0^k}{d^k\hat{x}+d_0^k},$$

for some k, while

$$\frac{c^i\tilde{x}+c_0^i}{d^i\tilde{x}+d_0^i} \geq \frac{c^i\hat{x}+c_0^i}{d^i\hat{x}+d_0^i},$$

for every other i. But the existence of such a $\tilde{x}$ is equivalent to $\hat{x}$ not being optimal to one of the Geometric Programming problems. Thus $\hat{x}$ is pareto-optimal solution to MOLFPP if and only if $\hat{x}$ is goal-optimal solution.

Aggarwal and Patkar (1978) proposed an algorithm depending on the GP solution technique to solve a LFPP. In order to use this algorithm for the MOLFPP solution, objective functions have to be gradually dealt with and solved using the GP technique. Thus the pareto-optimal solution will be obtained.

For the MOLFPP solution, the algorithm consisting of the following steps is used:

Step 1. The objective functions of the problem are arranged as minimizations.

Step 2. The constraints are shown as in Eq. 24.

Step 3. The problem is arranged as a GP problem in the form of Eq. 32 – Eq. 36 considering the first objective and the constraints.

Step 4. The GP problem is solved. The objective function value (s) and the values of the decision variables are found.

Step 5. If the last objective function is solved for, the process is terminated. Otherwise go to Step 6.

Step 6. The last obtained objective function value (s) is injected into the problem as a constrained as shown in Eq. 41.

$$\frac{\sum_{i=1}^{m} c_i x_i + c_0}{\sum_{i=1}^{m} d_i x_i + d_0} \leq s \quad (41)$$

Step 7. The problem is arranged as a GP problem in the form of Eq. 32 – Eq. 36 considering the next objective function and the constraints.

Step 8. Go to Step 4.

4. Numerical Examples

To illustrate the proposed model we consider the following problems:

Problem 1: The problem consisting of two objective functions and three constraints and its solution (Dangwal *et al.* 2012) is given below:

$$Max\left\{Z_1(x) = \frac{-3x_1+2x_2}{x_1+x_2+3},\quad Z_2(x) = \frac{7x_1+x_2}{5x_1+2x_2+1}\right\}$$

subject to

$x_1 - x_2 \geq 1$

$2x_1 + 3x_2 \leq 15$

$x_1 \geq 3$

where $x_1, x_2 \geq 0$

In this problem $Z_1 < 0$, $Z_2 \geq 0$, for each x in the feasible region. The solution of the problem is $Z_1^{max}(3.6, 2.6) = \frac{-14}{23} \cong -0.61, Z_2^{max}(7.5,0) = \frac{15}{11} \cong 1.36$.

Our Solution: The transformations and solutions obtained by applying our proposed model are given below:

Phase 1: The first objective function and the constraints in Problem 1 are arranged according to Eq. 26 – Eq. 31:

$$Z_1^{min} = \frac{3x_1-2x_2}{x_1+x_2+3}$$

subject to

$-x_1 + x_2 + 1 \leq 0$

$2x_1 + 3x_2 - 15 \leq 0$

$-x_1 + 3 \leq 0$

The obtained single objective linear fractional programming problem is transformed into the below format as a geometric programming problem using Eq. 32 – Eq. 36:

$$Z_1^{min} = t_1^3.t_2^{-2}.t_3^0$$

subject to

$0,1.t_1^1.t_2^1.t_3^3 \leq 1$

$10.t_1^{-1}.t_2^{-1}.t_3^{-3} \leq 1$

$t_1^{-1}.t_2^1.t_3^1 \leq 1$

$t_1^2.t_2^3.t_3^{-15} \leq 1$

$t_1^{-1}.t_2^0.t_3^3 \leq 1$

$t_1^{-1} \leq 1$

$t_2^{-1} \leq 1$

$t_3^{-1} \leq 1$

Solving this problem, the result $Z_1^{min}(3.54,2.63) = 0.61$ is obtained.

Phase 2: The single objective linear fractional programming problem below is obtained after arranging the second objective function as minimization, injecting the first objective function into constraints and rewriting all constraints as ≤ 0:

$$Zmin = \frac{-7x_1-x_2}{5x_1+2x_2+1}$$

subject to

$\frac{-3x_1+2x_2}{x_1+x_2+3} \leq 0,61 \rightarrow -3.61x_1 + 1.39x_2 - 1.83 \leq 0$

$-x_1 + x_2 + 1 \leq 0$

$2x_1 + 3x_2 - 15 \leq 0$

$-x_1 + 3 \leq 0$

The obtained single objective fractional programming problem is transformed into the below format as a geometric programming problem:

$Zmin = t_1^{-7}.t_2^{-1}.t_3^0$

subject to

$0{,}1.t_1^5.t_2^2.t_3^1 \leq 1$

$10.t_1^{-5}.t_2^{-2}.t_3^{-1} \leq 1$

$t_1^{-1}.t_2^1.t_3^1 \leq 1$

$t_1^2.t_2^3.t_3^{-15} \leq 1$

$t_1^{-1}.t_2^0.t_3^3 \leq 1$

$t_1^{-3.61}.t_2^{1.39}.t_3^{-1.83} \leq 1$

$t_1^{-1} \leq 1$

$t_2^{-1} \leq 1$

$t_3^{-1} \leq 1$

Solving this problem, the result $Z_2^{min}(7.49{,}0) = -1.36$ is obtained.

Since the objective function is maximization, $Z_1^{max} = -0.61$ and $Z_2^{max} = 1.36$ is obtained.

Problem 2: The problem consisting of three objective functions and four constraints and its solution (Guzel 2013) is given below:

$Max\left\{\frac{-3x_1+2x_2}{x_1+x_2+3}, \frac{7x_1+x_2}{5x_1+2x_2+1}, \frac{x_1+4x_2}{2x_1+3x_2+2}\right\}$

subject to

$x_1 - x_2 \geq 1$

$2x_1 + 3x_2 \leq 15$

$x_1 + 9x_2 \geq 9$

$x_1 \geq 3$

$x_1, x_2 \geq 0$

The values of this problem's solution are $x_1 = 3, x_2 = 2, Z_1 = -0.625,\ Z_2 = 1.15,\ Z_3 = 0.79$.

Our Solution: The transformations and solutions obtained by applying our proposed model are given below:

Phase 1: The first objective function and the constraints in Problem 2 are arranged according to Eq. 26 – Eq. 31:

$Z_1^{min} = \frac{3x_1-2x_2}{x_1+x_2+3}$

subject to

$-x_1 + x_2 + 1 \leq 0$

$2x_1 + 3x_2 - 15 \leq 0$

$-x_1 - 9x_2 + 9 \leq 0$

$-x_1 + 3 \leq 0$

$x_1, x_2 \geq 0$

The obtained single objective linear fractional programming problem is transformed into the below format as a geometric programming problem using Eq. 32 – Eq. 36:

$Z_1^{min} = t_1^3.t_2^{-2}.t_3^0$

subject to

$0.1t_1^1.t_2^1.t_3^3 \leq 1$

$10t_1^{-1}.t_2^{-1}.t_3^{-3} \leq 1$

$t_1^{-1}.t_2^1.t_3^1 \leq 1$

$t_1^2.t_2^3.t_3^{-15} \leq 1$

$t_1^{-1}.t_2^{-9}.t_3^{9} \leq 1$

$t_1^{-1}.t_2^{0}.t_3^{3} \leq 1$

$t_1^{-1} \leq 1$

$t_2^{-1} \leq 1$

$t_3^{-1} \leq 1$

Solving this problem, the result $x_1 = 3.60, x_2 = 2.60, Z_1^{min} = 0.61$ is obtained.

Phase 2: The single objective linear fractional programming problem below is obtained after arranging the second objective function as minimization, injecting the first objective function into constraints and rewriting all constraints as ≤ 0:

$Z_2^{min} = \frac{-7x_1-x_2}{5x_1+2x_2+1}$

subject to

$\frac{3x_1-2x_2}{x_1+x_2+3} \leq 0.61 \rightarrow 2.39x_1 - 2.61x_2 - 1.83 \leq 0$

$-x_1 + x_2 + 1 \leq 0$

$2x_1 + 3x_2 - 15 \leq 0$

$-x_1 - 9x_2 + 9 \leq 0$

$-x_1 + 3 \leq 0$

$x_1, x_2 \geq 0$

The obtained single objective fractional programming problem is transformed into the below format as a geometric programming problem:

$Z_2^{min} = t_1^{-7}.t_2^{-1}.t_3^{0}$

subject to

$0.1\mathrm{t}_1^{5}.\mathrm{t}_2^{2}.\mathrm{t}_3^{1} \leq 1$

$10\mathrm{t}_1^{-5}.\mathrm{t}_2^{-2}.\mathrm{t}_3^{-1} \leq 1$

$\mathrm{t}_1^{-1}.\mathrm{t}_2^{1}.\mathrm{t}_3^{1} \leq 1$

$\mathrm{t}_1^{2}.\mathrm{t}_2^{3}.\mathrm{t}_3^{-15} \leq 1$

$\mathrm{t}_1^{-1}.\mathrm{t}_2^{-9}.\mathrm{t}_3^{9} \leq 1$

$\mathrm{t}_1^{-1}.\mathrm{t}_2^{0}.\mathrm{t}_3^{3} \leq 1$

$\mathrm{t}_1^{2.39}.\mathrm{t}_2^{-2.61}.\mathrm{t}_3^{-1.83} \leq 1$

$\mathrm{t}_1^{-1} \leq 1$

$\mathrm{t}_2^{-1} \leq 1$

$\mathrm{t}_3^{-1} \leq 1$

Solving this problem, the result $x_1 = 3.60, x_2 = 2.60, Z_2^{min} = -1.15$ is obtained.

Phase 3: The single objective linear fractional programming problem below is obtained after arranging the third objective function as minimization, injecting the second objective function into constraints and rewriting all constraints as ≤ 0:

$Z_3^{min} = \frac{-x_1-4x_2}{2x_1+3x_2+2}$

subject to

$\frac{-7x_1-x_2}{5x_1+2x_2+1} \leq -1.15 \rightarrow -1.25x_1 + 1.30x_2 + 1.15 \leq 0$

$2.39x_1 - 2.61x_2 - 1.83 \leq 0$

$-x_1 + x_2 + 1 \leq 0$

$2x_1 + 3x_2 - 15 \leq 0$

$-x_1 - 9x_2 + 9 \leq 0$

$-x_1 + 3 \leq 0$

$x_1, x_2 \geq 0$

The obtained single objective fractional programming problem is transformed into the below format as a geometric programming problem:

$Z_3^{min} = t_1^{-1}.t_2^{-4}.t_3^{0}$

subject to

$0.1\mathrm{t}_1^2.\mathrm{t}_2^3.\mathrm{t}_3^2 \leq 1$

$10\mathrm{t}_1^{-2}.\mathrm{t}_2^{-3}.\mathrm{t}_3^{-2} \leq 1$

$\mathrm{t}_1^{-1}.\mathrm{t}_2^{1}.\mathrm{t}_3^{1} \leq 1$

$\mathrm{t}_1^{2}.\mathrm{t}_2^{3}.\mathrm{t}_3^{-15} \leq 1$

$\mathrm{t}_1^{-1}.\mathrm{t}_2^{-9}.\mathrm{t}_3^{9} \leq 1$

$\mathrm{t}_1^{-1}.\mathrm{t}_2^{0}.\mathrm{t}_3^{3} \leq 1$

$\mathrm{t}_1^{2.39}.\mathrm{t}_2^{-2.61}.\mathrm{t}_3^{-1.83} \leq 1$

$\mathrm{t}_1^{-1.25}.\mathrm{t}_2^{1.30}.\mathrm{t}_3^{1.15} \leq 1$

$\mathrm{t}_1^{-1} \leq 1$

$\mathrm{t}_2^{-1} \leq 1$

$\mathrm{t}_3^{-1} \leq 1$

Solving this problem, the result $x_1 = 3.61, x_2 = 2.59, Z_3^{min} = -0.81$ is obtained.

Since the objective function is maximization, $Z_1^{max} = -0.61, Z_2^{max} = 1.15,\ Z_3^{max} = 0.81$ is obtained.

Problem 3: The problem consisting of four objective functions and five constraints and its solution (Sen 1983; Sulaiman & Salih 2010) is given below:

$Max \left\{\frac{5x_1+3x_2}{x_1+x_2+1}, \frac{9x_1+5x_2}{3x_1+3x_2+3}, \frac{3x_1-4x_2}{x_1+x_2+1}, \frac{3x_1+2x_2}{2x_1+2x_2+2}\right\}$

subject to

$2x_1 + 4x_2 \geq 8$

$x_1 + x_2 \leq 3$

$x_1 + 2x_2 \leq 10$

$2x_1 + x_2 \leq 5$

$x_1 \leq 2$

$x_1, x_2 \geq 0$

The values of this problem's solution are:

$x_1 = 2, x_2 = 1, Z_1 = 3.25$

$x_1 = 2, x_2 = 1, Z_2 = 1.92$

$x_1 = 2, x_2 = 1, Z_3 = 0.5$

$x_1 = 2, x_2 = 1, Z_4 = 1.$

Our Solution: The transformations and solutions obtained by applying our proposed model are given below:

Phase 1: The first objective function and the constraints in Problem 3 are arranged according to Eq. 26 – Eq. 31:

$Z_1^{min} = \frac{-5x_1-3x_2}{x_1+x_2+1}$

subject to

$-2\mathrm{x}_1 - 4\mathrm{x}_2 + 8 \leq 0$

$\mathrm{x}_1 + \mathrm{x}_2 - 3 \leq 0$

$\mathrm{x}_1 + 2\mathrm{x}_2 - 10 \leq 0$

$2\mathrm{x}_1 + \mathrm{x}_2 - 5 \leq 0$

$\mathrm{x}_1 - 2 \leq 0$

$x_1, x_2 \geq 0$

The obtained single objective linear fractional programming problem is transformed into the below format as a geometric programming problem using Eq. 32 – Eq. 36:

$Z_1^{min} = t_1^{-5}.t_2^{-3}.t_3^{0}$

subject to

$0.1t_1^{1}.t_2^{1}.t_3^{1} \leq 1$

$10t_1^{-1}.t_2^{-1}.t_3^{-1} \leq 1$

$t_1^{-2}.t_2^{-4}.t_3^{8} \leq 1$

$t_1^{1}.t_2^{1}.t_3^{-3} \leq 1$

$t_1^{1}.t_2^{2}.t_3^{-10} \leq 1$

$t_1^{2}.t_2^{1}.t_3^{-5} \leq 1$

$t_1^{1}.t_2^{0}.t_3^{-2} \leq 1$

$t_1^{-1} \leq 1$

$t_2^{-1} \leq 1$

$t_3^{-1} \leq 1$

Solving this problem, the result $x_1 = 2, x_2 = 1, Z_1^{min} = -3.25$ is obtained.

Phase 2: The single objective linear fractional programming problem below is obtained after arranging the second objective function as minimization, injecting the first objective function into constraints and rewriting all constraints as ≤ 0:

$Z_2^{min} = \frac{-9x_1-5x_2}{3x_1+3x_2+3}$

subject to

$\frac{-5x_1-3x_2}{x_1+x_2+1} \leq -3.25 \rightarrow -1.75x_1 + 0.25x_2 + 3.25 \leq 0$

$-2x_1 - 4x_2 + 8 \leq 0$

$x_1 + x_2 - 3 \leq 0$

$x_1 + 2x_2 - 10 \leq 0$

$2x_1 + x_2 - 5 \leq 0$

$x_1 - 2 \leq 0$

$x_1, x_2 \geq 0$

The obtained single objective fractional programming problem is transformed into the below format as a geometric programming problem:

$Z_2^{min} = t_1^{-9}.t_2^{-5}.t_3^{0}$

subject to

$0.1t_1^{3}.t_2^{3}.t_3^{3} \leq 1$

$10t_1^{-3}.t_2^{-3}.t_3^{-3} \leq 1$

$t_1^{-2}.t_2^{-4}.t_3^{8} \leq 1$

$t_1^{1}.t_2^{1}.t_3^{-3} \leq 1$

$t_1^{1}.t_2^{2}.t_3^{-10} \leq 1$

$t_1^{2}.t_2^{1}.t_3^{-5} \leq 1$

$t_1^{1}.t_2^{0}.t_3^{-2} \leq 1$

$t_1^{-1.75}.t_2^{0.25}.t_3^{3.25} \leq 1$

$t_1^{-1} \leq 1$

$t_2^{-1} \leq 1$

$t_3^{-1} \leq 1$

Solving this problem, the result $x_1 = 2, x_2 = 1, Z_2^{min} = -1.92$ is obtained.

Phase 3: The single objective linear fractional programming problem below is obtained after arranging the third objective function as minimization, injecting the second objective function into constraints and rewriting all constraints as ≤ 0:

$$Z_3^{min} = \frac{-3x_1+4x_2}{x_1+x_2+1}$$

subject to

$$\frac{-9x_1-5x_2}{3x_1+3x_2+3} \leq -1.92 \rightarrow -3.24x_1 + 0.76x_2 + 5.76 \leq 0$$
$$-1.75x_1 + 0.25x_2 + 3.25 \leq 0$$
$$-2x_1 - 4x_2 + 8 \leq 0$$
$$x_1 + x_2 - 3 \leq 0$$
$$x_1 + 2x_2 - 10 \leq 0$$
$$2x_1 + x_2 - 5 \leq 0$$
$$x_1 - 2 \leq 0$$
$$x_1, x_2 \geq 0$$

The obtained single objective fractional programming problem is transformed into the below format as a geometric programming problem:

$$Z_3^{min} = t_1^{-3}.t_2^{4}.t_3^{0}$$

subject to

$$0.1t_1^1.t_2^1.t_3^1 \leq 1$$
$$10t_1^{-1}.t_2^{-1}.t_3^{-1} \leq 1$$
$$t_1^{-2}.t_2^{-4}.t_3^{8} \leq 1$$
$$t_1^1.t_2^1.t_3^{-3} \leq 1$$
$$t_1^1.t_2^2.t_3^{-10} \leq 1$$
$$t_1^2.t_2^1.t_3^{-5} \leq 1$$
$$t_1^1.t_2^0.t_3^{-2} \leq 1$$
$$t_1^{-1.75}.t_2^{0.25}.t_3^{3.25} \leq 1$$
$$t_1^{-3.24}.t_2^{0.76}.t_3^{5.76} \leq 1$$
$$t_1^{-1} \leq 1$$
$$t_2^{-1} \leq 1$$
$$t_3^{-1} \leq 1$$

Solving this problem, the result $x_1 = 2, x_2 = 1, Z_3^{min} = -0.5$ is obtained.

Phase 4: The single objective linear fractional programming problem below is obtained after arranging the fourth objective function as minimization, injecting the third objective function into constraints and rewriting all constraints as ≤ 0:

$$Z_4^{min} = \frac{-3x_1-2x_2}{2x_1+2x_2+2}$$

subject to

$$\frac{-3x_1+4x_2}{x_1+x_2+1} \leq -0.5 \rightarrow -2.5x_1 + 4.5x_2 + 0.5 \leq 0$$
$$-3.24x_1 + 0.76x_2 + 5.76 \leq 0$$
$$-1.75x_1 + 0.25x_2 + 3.25 \leq 0$$
$$-2x_1 - 4x_2 + 8 \leq 0$$
$$x_1 + x_2 - 3 \leq 0$$
$$x_1 + 2x_2 - 10 \leq 0$$
$$2x_1 + x_2 - 5 \leq 0$$

$x_1 - 2 \leq 0$

$x_1, x_2 \geq 0$

The obtained single objective fractional programming problem is transformed into the below format as a geometric programming problem:

$Z_4^{\min} = t_1^{-3}.t_2^{-2}.t_3^{0}$

subject to

$0.1t_1^2.t_2^2.t_3^2 \leq 1$

$10t_1^{-2}.t_2^{-2}.t_3^{-2} \leq 1$

$t_1^{-2}.t_2^{-4}.t_3^{8} \leq 1$

$t_1^{1}.t_2^{1}.t_3^{-3} \leq 1$

$t_1^{1}.t_2^{2}.t_3^{-10} \leq 1$

$t_1^{2}.t_2^{1}.t_3^{-5} \leq 1$

$t_1^{1}.t_2^{0}.t_3^{-2} \leq 1$

$t_1^{-1.75}.t_2^{0.25}.t_3^{3.25} \leq 1$

$t_1^{-3.24}.t_2^{0.76}.t_3^{5.76} \leq 1$

$t_1^{-2.5}.t_2^{4.5}.t_3^{0.5} \leq 1$

$t_1^{-1} \leq 1$

$t_2^{-1} \leq 1$

$t_3^{-1} \leq 1$

Solving this problem, the result $x_1 = 2, x_2 = 1, Z_4^{min} = -1$ is obtained.

Since the objective function is maximization, $Z_1^{max} = 3.25, Z_2^{max} = 1.92,\ Z_3^{max} = 0.5, Z_4^{max} = 1$ is obtained.

5. Conclusion

In this study, we developed an alternative solution to the MOLFPP by using GP technique as a useful and effective method. In this method, depending on the number of the objectives in the objective function, maximization or minimization of the objectives and the number of limitations, different conversions are not done, only one conversion is done. In this way the complications of conversion in the other methods is minimized. It is observed that in the literature conversion methods and methods based on Fuzzy theory are used in solving the MOLFPP problems. The use of GP in the solution of MOLFPP is an interesting approach. It is also observed that same or close results to the real solution values are obtained in all examples. Three numerical examples have been solved to illustrate the proposed model. Same results with the results in resources were obtained in the first and the third problem. In the second problem, on the other hand, close results were obtained.

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