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# ON JORDAN GENERALIZED HIGHER BI-DERIVATIONS ON PRIME GAMMA RINGS

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## Abstract

In this study , we define the concepts of a generalized higher bi-derivation , Jordan generalized higher bi-derivation and Jordan triple generalized higher bi-derivation on Γ-rings and show that a Jordan generalized higher bi-derivation on 2-torsion free prime Γ-ring is a generalized higher bi-derivation .

## 1.Introduction

Let M and Γ be two additive abelian groups . If there exists a mapping $(a, \propto, b) \to a \propto b$ of $M \times \Gamma \times M \to M$ satisfying the following for all a, b, c $\in M$ and $\propto, \beta \in \Gamma$ :

$(i)$ $(a+b) \propto c = a \propto c + b \propto c$ , $a(\propto + \beta)b = a \propto b + a\beta b$ , $a \propto (b+c) = a \propto b + a \propto c$ and

$(ii)$ $(a \propto b)\beta c = a \propto (b\beta c)$ .

Then M is called a $\Gamma - ring$

The notion of a $\Gamma - ring$ was introduced by Nobusawa [9] and generalized by Barnes [2] as defined above . Many properties of $\Gamma - ring$ were obtained by Barnes [2] , kyuno [6] , Luh [7] and others .

let M be a $\Gamma - ring$ . then M is called 2-torsion free if 2a= 0 implies a= 0 for all $a \in M$ . Besides , M is called a prime $\Gamma - ring$ if , for all $a, b \in M$, $a\,\Gamma\, M\,\Gamma\, b = (0)$ implies either a= 0 or b= 0 . and, M is called semiprime if $a\,\Gamma\, M\,\Gamma\, a = (0)$ with $a \in M$ implies a= 0 . Note that every prime $\Gamma - ring$ is obviously semiprime. M is said to be a commutative $\Gamma - ring$ if $a \propto b = b \propto a$ holds for all $a, b \in M$ and $\propto \in \Gamma$ . Let M be a $\Gamma - ring$ . then , for $a, b \in M$ and $\propto \in \Gamma$ , we define $[a, b]_\propto = a \propto b - b \propto a$ , known as the commutator of $a$ $and$ $b$ with respect to $\propto$ .
The notion of derivation and Jordan derivation on a Γ-ring were defined by M. Sapanci and A. Nakajima in [11], as follow

An additive mapping $d: M \to M$ is called a derivation of M if $d(a \propto b) = d(a) \propto b + a \propto d(b)$ for all $a, b \in M$ , $\propto \in \Gamma$ . And , if $d(a \propto a) = d(a) \propto a + a \propto d(a)$ for all $a \in M$ and $\propto \in \Gamma$ , then d is called a Jordan derivation of M .

The concept of Jordan generalized derivation of a Γ-ring has been developed by Y.Ceven and M.A.Ozturk in [3] ,as follow

An additive map $F: M \to M$ is said to be a generalized derivation of M if there exists a derivation $d: M \to M$ such that $F(a \propto b) = F(a) \propto b + a \propto d(b)$ is satisfied for all $a, b \in M$ and $\propto \in \Gamma$ . And , F is said to be a Jordan generalized derivation of M if there

exists a Jordan derivation $d: M \to M$ such that $F(a \propto a) = F(a) \propto a + a \propto d(a)$ holds for all $a \in M$ and $\propto \in \Gamma$ .
A mapping $D: M \times M \to M$ is said to be symmetric if $D(a,b) = D(b,a)$ , for all $a, b \in M$

An bi-additive mapping $d: M \times M \to M$ is called a symmetric bi-derivation on M×M into M if $d(a \propto b\,, c) = d(a,c) \propto b + a \propto d(b,c)$ for all $a, b, c \in M$ , $\propto \in \Gamma$ .

And , if $d(a \propto a\,, c) = d(a,c) \propto a + a \propto d(a,c)$ for all $a \in M$ and $\propto \in \Gamma$ , then d is called a Jordan bi-derivation on M×M into M .
The notion of symmetric bi-derivation was introduced by G.Maksa [8] and [5]
An bi-additive map $F: M \times M \to M$ is said to be a generalized symmetric bi-derivation on M×M into M if there exists symmetric bi-derivation $d: M \times M \to M$ such that $F(a \propto b\,, c) = F(a,c) \propto b + a \propto d(b,c)$ is satisfied for all $a, b, c \in M$ and $\propto \in \Gamma$ . And , F is said to be a Jordan generalized bi-derivation on M×M into M if there exists a Jordan bi-derivation $d: M \times M \to M$ such that $F(a \propto a\,, c) = F(a,c) \propto a + a \propto d(a,c)$ holds for all $a, c \in M$ and $\propto \in \Gamma$ .
The notion of generalized symmetric bi-derivations was introduced by Nurcan [1] .

In this paper we show that for our notions of generalized higher bi-derivation and Jordan generalized higher bi-derivation and Jordan triple generalized higher bi-derivation on a Γ-ring . In [10] the authors defined higher bi-derivations and Jordan higher bi-derivations as follows.
Let M be a Γ-ring and $D = (d_i)_{i \in N}$ be a family of biadditive mappings on M×M into M , such that $d_o(a,b) = a$ for all $a, b \in M$ , then D is called a higher bi-derivation on M×M into M if for every $a, b, c, d \in M$ , $\propto \in \Gamma$ and $n \in N$

$$d_n(a \propto b\,, c \propto d) = \sum\nolimits_{i+j=n} d_i(a,c) \propto d_j(b,d)$$

D is said to be a Jordan higher bi-derivation if

$$d_n(a \propto a\,, c \propto c) = \sum\nolimits_{i+j=n} d_i(a,c) \propto d_j(a,c)$$

D is called a Jordan triple higher bi-derivation

$$d_n(a \propto b\beta a\,, c \propto d\beta c) = \sum\nolimits_{i+j+k=n} d_i(a,c) \propto d_j(b,d)\beta d_k(a,c)$$

Note that $d_n(a+b\,, c+d) = d_n(a,c) + d_n(b\,,d)$ for all $a, b, c, d \in M$ and $n \in N$ .
we denote

$$\Psi_n(a,b,c,d)_\propto = d_n(a \propto b\,, c \propto d) - \sum\nolimits_{i+j=n} d_i(a,c) \propto d_j(b,d)$$

for all $a, b, c, d \in M$ , $\propto \in \Gamma$ and $n \in N$
Now , we present the properties of $\Psi_n(a,b,c,d)_\propto$
$\Psi_n(a,b,c,d)_\propto = -\Psi_n(b,a,d,c)_\propto$

A mapping $F: M \to M$ defined by $F(a) = D(a,a)$ , where $D: M \times M \to M$ is a symmetric mapping is called the trace of D it is obvious that in the case $D: M \times M \to M$ is a symmetric mapping which is also biadditive ( i.e. additive in both arguments ) . the trace F of D satisfies the relation $F(a+b) = F(a) + F(b)$ , for all $a, b \in M$ .
In our work we need the following lemma.

lemma 1.1. [4] let M be a 2-torsion free semi prime Γ-ring and suppose that $a, b \in M$ if $a\Gamma m\Gamma b + b\Gamma m\Gamma a = (0)$ for all $m \in M$ , then $a\Gamma m\Gamma b = b\Gamma m\Gamma a = (0)$

**2. Generalized higher bi-derivation on Γ-ring :**

In this section we present the concepts of generalized higher bi-derivation , Jordan generalized higher bi-derivation and Jordan triple generalized higher bi-derivation on Γ-rings and we study the properties of them .

**Definition 2.1**. let M be a Γ-ring and $F = (f_i)_{i\in N}$ be a family of biadditive mappings on M×M into M such that $f_0(a, b) = a$ for all $a, b \in M$ then F is called a generalized higher bi-derivation on M×M into M if there exists a higher bi-derivation $D = (d_i)_{i\in N}$ on M×M into M such that for all $n \in N$ we have .

$f_n(a\propto b, c\propto d) = \sum_{i+j=n} f_n(a, c)\alpha d_j(b, d)$ for every a , b , c , d ∈ M and $\alpha \in \Gamma$

F is said to be a Jordan generalized higher bi-derivation on M×M into M if there exists a Jordan higher bi-derivation D= $(d_i)_{i\in N}$ on M×M into M such that for all n∈N we have :

$$f_n(a \propto a, c \propto c) = \sum_{i+j=n} f_i(a, c)\alpha dj(a, c)$$

for every a,c∈M and ∝∈Γ

F is said to be a Jordan triple generalized higher bi-derivation on M×M into M if there exists a Jordan triple generalized higher bi-derivation D= $(d_i)_{i\in N}$ onM×M into M such that for all

n∈N we have :

$$f_n(a \propto b\beta a, c \propto d\beta c) = \sum_{i+j+k=n} f_n(a, c) \propto d_j(b, d)\beta\, d_k(a, c)$$

For every $a, b, c, d \in M$ and $\propto, \beta \in \Gamma$.

Note that $f_n$ (a+b,c+d)= $f_n(a,c)+f_n(b,d)$ for all a,b,c,d∈M and n∈N

**Example 2.2**

Let M= $\{\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix}$: x,y ∈R }, R is real number .

M be a Γ-ring of 2×2 matrices and Γ= { $\begin{pmatrix} r & 0 \\ 0 & 0 \end{pmatrix}$ : r∈R } we use the usual addition and multiplication on matrices of M×Γ×M , we define $f_i$ : M×Γ×M→M , i∈N by

$f_i\left(\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} ka & (1+i)b \\ 0 & 0 \end{pmatrix}$ for all $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix} \in M$

$K = \frac{(i^2 - in + 1) + |i^2 - in + 1|}{2} = \begin{cases} 1 \text{ If } i \in \{0, n\} \\ 0 \text{ If } i \notin \{0, n\} \end{cases} \quad n \in N, 0 \le i \le n$

Then f is generalized higher bi-derivation on Γ – ring because there exists a higher bi-derivation on Γ-ring

$d_i$: M×Γ×M→M , i ∈N defined by

$d_i\left(\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} ma & (m+i)b \\ 0 & 0 \end{pmatrix}$

for all $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix} \in M$

Such that $m = \frac{(1-i)+|1-i|}{2} = \begin{cases} 1 & if \ i = 0 \\ 0 & if \ i \neq 0 \end{cases}$

**Lemma 2.3.** let M be a Γ-ring and $F=(f_i)_{I\in N}$ be a Jordan generalized higher bi- derivation on M×M into M associated with Jordan higher bi- derivation $D=(d_i)_{i\in N}$ of M×M into M . Then for all a,b,c,d,s,t ϵ M , ∝ , βϵΓ and n ϵ N , the following statements hold :

( i ) $f_n(a\propto b + b\propto a , c\propto d + d\propto c) = \sum f_i(a,c) \propto d_j(b,d) + f_i(b,d) \propto d_j(a,c)$

( ii ) $f_n(a\propto b\beta a + a\beta b\propto a , c\propto d\beta c) =$

$$\sum_{i+j+k=n} f_i(a,c) \propto d_j(b,d)\beta d_k(a,c) + f_i(a,c)\beta d_j(b,d) \propto d_k(a,c)$$

Especially , if M is 2-torsion free ,then

$$\text{(iii) } f_n(a \propto b \propto c , c \propto d \propto c) = \sum_{i+j+k=n} f_i(a,c) \propto d_j(b,d) \propto d_j(a,c)$$

(iv) $f_n(a\propto b\propto c + c\propto b\propto a , s\propto d \propto t + t \propto d \propto s) =$

$$\sum_{i+j+k=n} f_i(a,s) \propto d_j(b,d) \propto d_k(c,t) + f_i(c,t) \propto d_j(b,d) \propto d_k(a,s)$$

Proof. (i) is obtained by computing $f_n\big((a+b) \propto (a+b), (c+d) \propto (c+d)\big)$ and (ii) is also obtained by replacing aβb+bβa for b and cβd+dβc for d in (i) , in (ii) . If we replace a+c for a and s+t for c in (iii) , we can get (iv).

**Definition 2.4.** let M be a Γ-ring and $F=(f_i)_{I\in N}$ be a Jordan generalized higher bi- derivation on M×M into M associated with Jordan higher bi- derivation $D=(d_i)_{i\in N}$ of M×M into M . Then for all a,b,c,d,s,t ϵ M , ∝ , βϵΓ and n ϵ N , we define

$$\emptyset_n(a,b,c,d)_\propto = \text{fn}\,(a \propto b, c \propto d) - \sum_{i+j=n} f_i(a,c) \propto d_j(b,d)$$

**Lemma 2.5.** let M be a Γ-ring and $F=(f_i)_{I\in N}$ be a Jordan generalized higher bi- derivation on M×M into M associated with Jordan higher bi- derivation $D=(d_i)_{i\in N}$ of M×M into M . Then for all a,b,c,d,s,t ϵ M , ∝ , βϵΓ and n ϵ N.

$(i)\ \emptyset_n(a,b,c,d)_\propto = -\emptyset_n(b,a,d,c)_\propto$

$(ii)\ \emptyset_n(a+s,b,c,d)_\propto = \emptyset_n(b,a,d,c)_\propto + \emptyset_n(s,b,c,d)_\propto$

$(iii)\ \emptyset_n(a,b+s,c,d)_\propto = \emptyset_n(a,b,c,d)_\propto + \emptyset_n(a,s,c,d)_\propto$

$(iv)\ \emptyset_n(a,b,c+s,d)_\propto = \emptyset_n(a,b,c,d)_\propto + \emptyset_n(a,b,s,d)_\propto$

$(v)\ \emptyset_n(a,b,c,d+s)_\propto = \emptyset_n(a,b,c,d)_\propto + \emptyset_n(a,b,c,s)_\propto$

Proof . These results follow easily by Lemma 2.3 $(i)$ and the definition of $\emptyset_n(a,b,c,d)_\propto$

Note that F is a generalized higher bi-derivation iff $\emptyset_n\ (a,b,c,d)_\propto = 0$ for all a,b,c,d $\in M$, $\propto \in \Gamma$ and $n \in N$.

## 3. The Main Results

In this section we present , the main results of this paper .

**Lemma 3.1.** let M be a 2-torsion free and $f=(f_i)_{I\epsilon N}$ be a Jordan generalized higher bi-derivation on M×M into M associated with Jordan higher bi- derivation $D=(d_i)_{i\epsilon N}$ of M×M into M . then for all a,b,c,d,s,t ϵ M , ∝ , βϵΓ and n ϵ N , if $\emptyset_t\ (a,b,c,d)_\propto=0$ for every t<n and $\Psi_t\ (a,b,c,d)_\propto = 0$ for every t<n then:
$\emptyset_n(a,b,c,d)_\propto \beta m\beta [a,b]_\propto + [a,b]_\propto \beta m\beta\ \Psi_n(a,b,c,d)_\propto = 0$

Proof . let SϵM , since $f_n$ is bi additive mapping then by Lemma 2.3. ( iv ) we obtain :

$$f_n(a \propto b\beta m\beta b \propto a + b \propto a\beta m\beta a \propto b\ , c \propto d\beta s\beta d \propto c + d \propto c\beta s\beta c \propto d)$$

$$= f_n\big((a \propto b)\beta m\beta(b \propto a) + (b \propto a)\beta m\beta(a \propto b)\ , (c \propto d)\beta s\beta(d \propto c) + (d \propto c)\beta s\beta(c \propto d)\big)$$

$$= \sum_{i+j+k=n} f_i(a \propto b, c \propto d)\beta d_j(m,s)\beta d_k(b \propto a, d \propto c) + f_i(b \propto a, d \propto c)\beta d_j(m,s)\beta d_k(a \propto b, c \propto d)$$

$$= f_n(a \propto b, c \propto b)\beta m\beta b \propto a + a \propto b\beta m\beta d_n\ (b \propto a, c \propto d) + f_n\ (b \propto a, d \propto c)\beta m\beta a \propto b + \sum_{\substack{0<i,k<n \\ i+j+k}} f_i(a \propto b, c \propto d)\beta d_j(m,s)\beta d_k(b \propto a, d \propto c) + f_i(b \propto a, d \propto c)\beta d_j(m,s)\beta d_k(a \propto b, c \propto d)$$

$$= f_n(a \propto b, c \propto b)\beta m\beta\ b \propto a + a \propto b\ \beta m\beta\ d_n(b \propto a, d \propto c) + f_n(b \propto a, d \propto c)\ \beta m\beta\ a \propto b + b \propto a\ \beta m\beta\ d_n(a \propto b, c \propto d) + \sum_{\substack{q+t,h+g<n \\ q+t+j+h+g=n}} f_q(a,c) \propto d_t(b,d)\beta d_j(m,s)\beta d_h(b,d) \propto d_g(a,c) + f_q(b,d) \propto d_t(a,c)\beta d_j(m,s)\beta d_h(a,c) \propto d_g(b,d) \quad \dots (1)$$

On the other hand : by lemma 2.3. $(iii)$

$$f_n(a \propto b\ \beta m\beta\ b \propto a + b \propto a\ \beta m\beta\ a \propto b\ , c \propto d\ \beta s\beta\ d \propto c + d \propto c\ \beta s\beta\ c \propto d)$$

$$= f_n(a \propto (b\beta m\beta b) \propto a + b \propto (a\beta m\beta a) \propto b,\ c \propto (d\beta s\beta d) \propto c + d \propto (c\beta s\beta c) \propto d)$$

$$= f_n(a \propto (b\ \beta m\beta\ b) \propto a, c \propto (d\ \beta s\beta\ d) \propto c) + f_n(b \propto (a\beta m\beta a) \propto b, d \propto (c\beta s\beta c) \propto d)$$

$$= \sum_{q+k+g=n} f_q(a,c) \propto d_k( b\, \beta m\beta\, b, d\, \beta s\beta\, d\,) \propto d_g(a,c) + f_q(b,d) \propto d_k(a\beta m\beta a, c\beta s\beta c) \propto d_g\,(b,d)$$

$$= \sum_{q+t+j+h+g=n} f_q(a,c) \propto d_t(b,d)\beta d_j(m,s)\beta d_n(b,d) \propto d_g(a,c) + f_q(b,d) \propto d_t(a,c)\beta d_j(m,s)\beta d_h(a,c) \propto d_g(b,d)$$

$$= \sum_{q+t=n} f_q(a,c) \propto d_t(b,d)\beta m\beta\, b \propto a + a \propto b\, \beta m\beta \sum_{h+g=n} d_h\,(b,d) \propto d_g(a,c) + \sum_{q+t=n} f_q(b,d) \propto d_t(a,c)\beta m\beta\, a \propto b + b \propto a\, \beta m\beta \sum_{h+g} d_h\,(a,c) \propto d_g(b,d)$$

$$+ \sum_{\substack{q+t,h+g<n \\ q+t+j+h+g=n}} f_q(a,c) \propto d_t(b,d)\beta d_j(m,s)\beta d_h(b,d) \propto d_g(a,c) + f_q(b,d) \propto d_t(a,c)\,\beta d_j(m,s)\,\beta d_h(a,c) \propto d_g(b,d) \quad \ldots(2)$$

.

Compare (1) and (2) we get :

$$f_n(a \propto b, c \propto d)\,\beta m\beta b \propto a - \sum\nolimits_{q+t=n} f_q\,(a,c) \propto d_t(b,d)\,\beta m\beta b \propto a \,+\, a \propto b\beta m\beta d_n(b \propto a\,, d \propto c) - a \propto b\,\beta m\beta \sum\nolimits_{h+g=n} d_h(b,d) \propto d_g(a,c) + f_n(b \propto a\,, d \propto c)\,\beta m\beta a \propto b - \sum\nolimits_{q+t=n} f_q\,(b,d) \propto d_t(a,c)\,\beta m\beta\, a \propto b + b \propto a\,\beta m\beta d_n(a \propto b\,, c \propto d) - \, b \propto a\,\beta m\beta \sum\nolimits_{h+g=n} d_h(a,c) \propto d_g(b,d) = 0$$

$$\emptyset_n(a,b,c,d)_\propto\, \beta m\beta\, b \propto a + a \propto b\, \beta m\beta\, \Psi_n(b,a,d,c)_\propto + \emptyset_n(b,a,d,c)_\propto\, \beta m\beta\, a \propto b + b \propto a\, \beta m\beta\, \Psi_n(a,b,c,d)_\propto = 0$$

$$\emptyset_n(a,b,c,d)_\propto\, \beta m\beta\, b \propto a - a \propto b\, \beta m\beta\, \Psi_n(a,b,c,d)_\propto - \emptyset_n(a,b,c,d)_\propto\, \beta m\beta\, a \propto b + b \propto a\, \beta m\beta\, \Psi_n(a,b,c,d)_\propto = 0$$

$$\emptyset_n(a,b,c,d)_\propto\, \beta m\beta\, [b,a]_\propto + [b,a]_\propto\, \beta m\beta\, \Psi_n(a,b,c,d)_\propto = 0$$

$$\emptyset_n(a,b,c,d)_\propto\, \beta m\beta\, [a,b]_\propto + [a,b]_\propto\, \beta m\beta\, \Psi_n(a,b,c,d)_\propto = 0$$

**Lemma 3.2.** let $M$ be 2-torsion free prime $\Gamma-$ring and $F = (f_i)_{i\in N}$ be a Jordan generalized higher bi-derivation on $M \times M$ into $M$ associated with Jordan higher bi-derivation $D = (d_i)_{i\in N}$ on $M \times M$ into $M$. then for all $a,b,c,d,m \in M$ $,\propto,\beta \in \Gamma$ and $n \in N$

$$\emptyset_n(a,b,c,d)_\propto\, \beta m\beta\, [a,b]_\propto = [a,b]_\propto\, \beta m\beta\, \Psi_n(a,b,c,d)_\propto = 0$$

Proof : By Lemma 3.1. and Lemma 1.1. , we obtain the proof .

**Theorem 3.3.** let M be 2-torsion free prime $\Gamma - \text{ring}$ and $F = (f_i)_{i\in N}$ be a Jordan generalized higher bi-derivation on M×M into M associated with Jordan higher bi-derivation $D = (d_i)_{i\in N}$ on M×M into M , then for all $a, b, c, d, m \in M$ $, \propto, \beta \in \Gamma$ and $n \in N$ $\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [s, t]_\propto = 0$

Proof . Replacing $a + s$ for a in lemma 3.2. we get

$$\emptyset_n(a + s, b, c, d)_\propto \beta m\beta\, [a + s, b]_\propto = 0$$

$$\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [a, b]_\propto + \emptyset_n(a, b, c, d)_\propto \beta m\beta\, [s, b]_\propto + \emptyset_n(s, b, c, d)_\propto \beta m\beta\, [a, b]_\propto + \emptyset_n(s, b, c, d)_\propto \beta m\beta\, [s, b]_\propto = 0$$

By Lemma 3.2. we get $\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [s, b]_\propto + \emptyset_n(s, b, c, d)_\propto \beta m\beta\, [a, b]_\propto = 0$

There fore

$$\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [s, b]_\propto \beta m\beta\, \emptyset_n(a, b, c, d)_\propto \beta m\beta [s, b]_\propto = -\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [s, b]_\propto \beta m\beta\, \emptyset_n(s, b, c, d)_\propto \beta m\beta\, [a, b]_\propto = 0$$

Hence , by the primmess on M :

$$\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [s, b]_\propto = 0 \quad \dots (1)$$

Similarly , by replacing b+t for b in this equality we get :

$$\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [a, t]_\propto = 0 \quad \dots (2)$$

Thus : $\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [a + s , b + t]_\propto = 0$

$$\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [a, b]_\propto + \emptyset_n(a, b, c, d)_\propto \beta m\beta\, [a, t]_\propto + \emptyset_n(a, b, c, d)_\propto \beta m\beta\, [s, b]_\propto + \emptyset_n(a, b, c, d)_\propto \beta m\beta\, [s, t]_\propto = 0$$

By using (1) , (2) and Lemma 3.2. we get $\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [s, t]_\propto = 0$

**Theorem 3.4.** let M be 2-torsion free prime $\Gamma - ring$ . Then every Jordan generalized higher bi-derivation on M×M into M is a generalized higher bi-derivation on M×M into M

Proof. Let M be 2-torsion free prime $\Gamma - ring$ and $F = (f_i)_{i\in N}$ be a Jordan generalized higher bi-derivation on M×M into M associated with Jordan higher bi-derivation $D = (d_i)_{i\in N}$ on M×M into M
By Theorem 3.3. $\emptyset_n(a, b, c, d)_\propto \beta m\beta\, [s, t]_\propto = 0$ for all $a, b, c, d, m, s, t \in M , \propto, \beta \in \Gamma$ . and $n \in N$ since M is prime , we get either $\emptyset_n(a, b, c, d)_\propto = 0$ or $[s, t]_\propto = 0$ , for all $a, b, c, d, s, t \in M$ , $\propto \in \Gamma$ , and $n \in N$ if $[a, t]_\propto \neq 0$ for all $s, t \in M$ and $\propto \in \Gamma$ .

Then $\emptyset_n(a, b, c, d)_\propto = 0$ for all $a, b, c, d \in M$ . $\propto \in \Gamma$ and $n \in N$ hence we get , F is a generalized higher bi-derivation on M×M into M.

But , if $[s, t]_\propto = 0$ for all $s, t \in M$ and $\propto \in \Gamma$ , then M is commutative and there fore , we have from lemma 2.3.(i)

 

$$2f_n(a \propto b\,, c \propto d) = 2\sum_{i+j=n} f_i(a,c) \propto d_j(b,d)$$

Since M is 2-torsion free , we obtain that F is a generalized higher bi-derivation on M×M into M .

**Proposition 3.5.** let M be 2-torsion free $\Gamma - ring$ then every Jordan generalized higher be-derivation on M×M into M such that $a \propto b\beta c = a\beta b \propto c$ for all $a, b, c \in M$ and $\propto, \beta \in \Gamma$ is a Jordan triple generalized higher bi-derivation on M×M into M .

Proof. Let M be 2-torsion free $\Gamma - ring$ and F=$(f_i)_{i \in n}$ be a Jordan generalized higher bi-derivation on M×M into M associated with Jordan higher bi-derivation $D = (d_i)_{i \in N}$ on M×M into M

By lemma 2.3. $(ii)$

$$f_n(a \propto b\beta a + a\beta b \propto a\,, c \propto d\beta c + c\beta d \propto c) = \sum_{i+j+k=n} f_i(a,c) \propto d_j(b,d)\beta d_k(a,c) + f_i(a,c)\beta d_j(b,d) \propto d_k(a,c)$$

for all $a, b, c, d \in M.$ $\propto, \beta \in \Gamma$ . and $n \in N$

$$f_n(a \propto b\beta a\,, c \propto d\beta c) + f_n(a\beta b \propto a\,, c\beta d \propto c) = \sum_{i+j+k=n} f_i(a,c) \propto d_j(b,d)\beta d_k(a,c) + \sum_{i+j+k=n} f_i(a,c)\beta d_j(b,d) \propto d_k(a,c)$$

Since $a \propto b\beta c = a\beta b \propto c$ for all $a, b, c \in M$ and $\propto, \beta \in \Gamma$ we get :

$$2f_n(a \propto b\beta a, c \propto d\beta c) = 2\sum_{i+j+k=n} f_i\,(a,c) \propto d_j(b,d)\beta d_k(a,c)$$

Since M is a 2-torsion free we have *:*

$$f_n(a \propto b\beta a, c \propto d\beta c) = \sum_{i+j+k=n} f_i\,(a,c) \propto d_j(b,d)\beta d_k(a,c)$$

i.e F is Jordan triple generalized higher bi-derivation on M×M into M .

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