On Generalized Jordan Isomorphisms of a Gamma- Ring M onto a Gamma- Ring M'

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Abstract:

Let M and M' be two prime Γ-rings .In the present paper we show that under certain conditions of M, every generalized Jordan homomorphism of a Γ-ring M onto a prime Γ–ring M 'is either generalized homomorphism or anti - homomorphism.

Key Words : prime Γ– ring , Isomorphism , Jordan isomorphism .

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1-Introduction:

Let M and Γ be two additive abelian groups, suppose that there is a mapping from M×Γ×M ⟶ M (the image of (a,α,b) being denoted by $a\alpha b$, $a, b \in$ M and $\alpha \in \Gamma$). Satisfying for all $a, b, c \in$ M and $\alpha, \beta \in \Gamma$:

(i) $(a + b)\alpha c = a\alpha c + b\alpha c$
$a(\alpha + \beta)c = a\alpha c + a\beta c$
$a\alpha(b + c) = a\alpha b + a\alpha c$

(ii) $(a\alpha b)\beta c = a\alpha(b\beta c)$

Then M is called a Γ-ring. This definition is due to Barnes [1] .

A Γ-ring M is called a prime if $a\Gamma M\Gamma b = (0)$ implies $a = 0$ or $b = 0$, where $a, b \in$ M.This definition is due to [5].

A Γ-ring M is called semiprime if $a\Gamma M\Gamma a = (0)$ implies $a = 0$, such that $a \in$ M.This definition is due to [5] .

Let M be a 2-torsion free semiprime Γ-ring and suppose that $a, b \in$ M if $a\Gamma m\Gamma b + b\Gamma m\Gamma a = 0$ for all m ∈ M, then $a\Gamma m\Gamma b = b\Gamma m\Gamma a = 0$. This definition is due to [7].

Let M be Γ-ring then M is called 2-torsion free if $2a = 0$ implies $a = 0$, for every $a \in$ M. This definition is due to [6].

An additive mapping θ of a Γ-ring M into a Γ-ring M' is called homomorphism if $\theta(a\alpha b) = \theta(a)\alpha\theta(b)$, for all $a, b \in$ M and $\alpha \in \Gamma$. This definition is due to [1].

An additive mapping θ of Γ-ring M into a Γ-ring M' is called Jordan homomorphism if $\theta(a\alpha b + b\alpha a) = \theta(a)\alpha\theta(b) + \theta(b)\alpha\theta(a)$, for all $a, b \in$ M and $\alpha \in \Gamma$. This definition is due to [4].

Let F be an additive mapping of a Γ-ring M into a Γ-ring M'. F is called a generalized homomorphism if there exists a homomorphism θ from a Γ-ring M into a Γ-ring M', such that

$F(a\alpha b) = F(a)\alpha\theta(b)$, for all $a, b \in M$ and $\alpha \in \Gamma$, where θ is called the relating homomorphism . This definition is due to [4].

And F is called a generalized Jordan homomorphism if there exists a Jordan homomorphism θ from a Γ-ring M into a Γ-ring M', such that
$F(a\alpha b + b\alpha a) = F(a)\alpha\theta(b) + F(b)\alpha\,\theta(a)$, for all $a, b \in M$ and $\alpha \in \Gamma$, where θ is called the relating Jordan homomorphism. This definition is due to [3].

A bijective additive mapping θ from a Γ-ring M onto a Γ-ring M' is called an isomorphism if $\theta(a\alpha b) = \theta(a)\alpha\theta(b)$, for all $a, b \in M$ and $\alpha \in \Gamma$. This definition is due to [2].

A bijective additive mapping θ from a Γ-ring M onto a Γ-ring M' is called an anti - isomorphism if $\theta(a\alpha b) = \theta(b)\alpha\theta(a)$, for all $a, b \in M$ and $\alpha \in \Gamma$.This definition is due to [2].

A bijective additive mapping θ from a Γ-ring M onto a Γ-ring M' is called a Jordan isomorphism if $\theta(a\alpha a) = \theta(a)\alpha\theta(a)$, for all $a \in M$ and $\alpha \in \Gamma$. This definition is due to [2].

Now, the main purpose of this paper is that every generalized Jordan isoomorphism of a Γ-ring M onto a prime Γ-ring M' is either generalized isomorphism or anti isomorphism and every generalized Jordan isomorphism from a Γ-ring M onto a 2-torsion free Γ-ring M' such that $a\alpha b\beta a = a\beta b\alpha a$, for all $a, b \in M$ and $\alpha, \beta \in \Gamma$, $a'\alpha b'\beta a' = a'\beta b'\alpha a'$, for all $a', b' \in M'$.Then F is a generalized Jordan triple isomorphism.

2. Generalized Jordan Isomorphism on Γ- Rings

Definition (2.1):

Let F be a bijective additive mapping of a Γ-ring M onto a Γ-ring M' . F is called a **generalized isomorphism** if there exists an isomorphism θ from a Γ-ring M onto a Γ-ring M' such that

$F(a\alpha b) = F(a)\alpha\theta(b)$, for all $a, b \in M$ and $\alpha \in \Gamma$.

Where θ is called the **relating isomorphism** .

Example(2.2):

Let R be a ring .Let $M = M_{1\times 2}(R)$, $M' = M_{1\times 2}(R)$ and $\Gamma = \left\{ \begin{pmatrix} n \\ 0 \end{pmatrix}, n \in Z \right\}$.Then M and M' are two Γ-rings.

Let F be an additive mapping of a Γ-ring M into a Γ-ring M', such that

$F((a \quad b)) = (-a \quad 0)$, for all $(a \quad b) \in M$.

Then there exists an isomorphism θ from a Γ-ring M onto a Γ-ring M', such that

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$\theta((a \quad b)) = (a \qquad 0)$, for all $(a \quad b) \in M$.

Then F is generalized isomorphism.

Definition (2.3):

Let F be a bijective additive mapping of a Γ-ring M onto a Γ-ring M' . F is called **generalized Jordan isomorphism** if there exists a Jordan isomorphism θ from a Γ- ring M onto a Γ- ring M' such that

$F(a\alpha b + b\alpha a) = F(a)\alpha\theta(b) + F(b)\alpha\theta(a)$, for all $a, b \in M$ and $\alpha \in \Gamma$.

Where θ is called the **relating Jordan isomorphism**.

Definition (2.4):

Let F be an additive mapping of a Γ-ring M onto a Γ-ring M'. F is called **generalized Jordan triple isomorphism** if there exists a Jordan triple isomorphism θ from a Γ-ring M onto a Γ-ring M' such that

$F(a\alpha b\beta a) = F(a)\alpha\theta(b)\beta\theta(a)$, for all $a, b \in M$ and $\alpha, \beta \in \Gamma$.

Where θ is called the **relating Jordan triple isomorphism**.

Definition (2.5):

Let F be an additive mapping of a Γ-ring M onto a Γ-ring M'. F is called **generalized anti - isomorphism** if there exists an anti - isomorphism from a Γ-ring M onto a Γ-ring M' such that

$F(a\alpha b) = F(b)\alpha\theta(a)$, for all $a, b \in M$ and $\alpha \in \Gamma$.

Where θ is called the **relating anti isomorphism**.

Lemma (2.6):

Let F be a generalized Jordan triple isomorphism of a Γ-ring M onto a Γ-ring M'. Then for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$

(i) $F(a\alpha b\beta a + a\beta b\alpha a)=F(a)\alpha\theta(b)\beta\theta(a)+F(a)\beta\theta(b)\alpha\theta(a)$

(ii) $F(a\alpha b\beta c+c\alpha b\beta a)= F(a)\alpha\theta(b)\beta\theta(c) + F(c)\alpha\theta(b)\beta\theta(a)$

(iii) In particular, if M, M' be two commutative Γ-rings and M' is a 2-torsion free Γ-ring , then

$F(a\alpha b\beta c) = F(a)\alpha\theta(b)\beta\theta(c)$

(iv) $F(a\alpha b\alpha c +c\alpha b\alpha a)=F(a)\alpha\theta(b)\alpha\theta(c)+ F(c)\alpha\theta(b)\alpha\theta(a)$

Proof:

(i) Replace $a\beta b + b\beta a$ for b in Definition (2.3), we get:

$$F(a\alpha(a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a) = F(a)\alpha\theta(a\beta b + b\beta a) + F(a\beta b + b\beta a)\alpha\theta(a)$$

$= F(a)\alpha\theta(a)\beta\theta(b) + F(a)\alpha\theta(b)\beta\theta(a) + F(a)\beta\theta(b)\alpha\theta(a) + F(b)\beta\theta(a)\alpha\theta(a)$ …(1)

On the other hand

$F(a\alpha(a\beta b + b\beta a)+(a\beta b + b\beta a)\alpha a)=F(a\alpha a\beta b + a\alpha b\beta a + a\beta b\alpha a + b\beta a\alpha a)$

$= F(a)\alpha\theta(a)\beta\theta(b)+F(b)\beta\theta(a)\alpha\theta(a)+F(a\alpha b\beta a+a\beta b\alpha a)$ … (2)

Compare (1) and (2), we get:

$F(a\alpha b\beta a+ a\beta b\alpha a)=F(a)\alpha\theta(b)\beta\theta(a) + F(a)\beta\theta(b)\alpha\theta(a)$

(ii) Replace $a + c$ for a in Definition (2.4), we get:

$F((a + c)\alpha b\beta(a + c)) = F(a + c)\alpha\theta(b)\beta\theta(a + c)$

$= F(a)\alpha\theta(b)\beta\theta(a) + F(a)\alpha\theta(b)\beta\theta(c) + F(c)\alpha\theta(b)\beta\theta(a) +F(c)\alpha\theta(b)\beta\theta(c)$ …(1)

On the other hand

$F((a + c)\alpha b\beta(a + c)) = F(a\alpha b\beta a + a\alpha b\beta c + c\alpha b\beta a + c\alpha b\beta c)$

$=F(a)\alpha\theta(b)\beta\theta(a)+F(c)\alpha\theta(b)\beta\theta(c)+F(a\alpha b\beta c+c\alpha b\beta a)$ … (2)

Compare (1) and (2), we get:

$F(a\alpha b\beta c+c\alpha b\beta a) = F(a)\alpha\theta(b)\beta\theta(c) + F(c)\alpha\theta(b)\beta\theta(a)$

(iii) By (ii) and since M, M' be two commutative Γ-rings and M' is a 2-torsion free Γ-ring

$F(a\alpha b\beta c + a\alpha b\beta c) = 2F(a\alpha b\beta c) = F(a)\alpha\theta(b)\beta\theta(c)$

(iv) Replace α for β in (ii), we get:

$F(a\alpha b\alpha c + c\alpha b\alpha a)=F(a)\alpha\theta(b)\alpha\theta(c)+ F(c)\alpha\theta(b)\alpha\theta(a)$

Definition (2.7):

Let F be a generalized Jordan isomorphism of a Γ-ring M onto a Γ-ring M', then for all a, $b \in M$ and $\alpha \in \Gamma$, we define $\delta : M\times\Gamma\times M \longrightarrow M'$ by $\delta(a,b)_\alpha = F(a\alpha b) - F(a)\alpha\theta(b)$.

Lemma (2.8):

If F is a generalized Jordan isomorphism of a Γ-ring M onto a Γ-ring M'.Then all $a, b,c \in M$ and $\alpha, \beta \in \Gamma$

(i) $\delta(a,b)_\alpha = -\delta(b,a)_\alpha$

(ii) $\delta(a + b,c)_\alpha = \delta(a,c)_\alpha + \delta(b,c)_\alpha$

(iii) $\delta(a,b + c)_\alpha = \delta(a,b)_\alpha + \delta(a,c)_\alpha$

(iv) $\delta(a,b)_{\alpha+\beta} = \delta(a,b)_\alpha + \delta(a,b)_\beta$

Proof:

(i) By Definition (2.3):

$F(a\alpha b + b\alpha a) = F(a)\alpha\theta(b) + F(b)\alpha\theta(a)$

$F(a\alpha b) - F(a)\alpha\theta(b) = -(F(b\alpha a) - F(b)\alpha\theta(a))$

$\delta(a,b)_\alpha = -\delta(b,a)_\alpha$

(ii) $\delta(a + b,c)_\alpha = F((a + b)\alpha c) - F((a + b)\alpha\theta(c))$

$= F(a\alpha c + b\alpha c) - F(a)\alpha\theta(c) - F(b)\alpha\theta(c)$

$= F(a\alpha c) - F(a)\alpha\theta(c) + F(b\alpha c) - F(b)\alpha\theta(c)$

$= \delta(a,c)_\alpha + \delta(b,c)_\alpha$

(iii) $\delta(a,b + c)_\alpha = F(a\alpha(b+c)) - F(a)\alpha\theta(b + c)$

$= F(a\alpha b + a\alpha c) - F(a)\alpha\theta(b) - F(a)\alpha\theta(c)$

$= F(a\alpha b) - F(a)\alpha\theta(b) + F(a\alpha c) - F(a)\alpha\theta(c)$

$= \delta(a,b)_\alpha + \delta(a,c)_\alpha$

(iv) $\delta(a,b)_{\alpha+\beta} = F(a(\alpha + \beta)b) - F(a)(\alpha + \beta)\theta(b)$

$= F(a\alpha b) - F(a)\alpha\theta(b) + F(a\beta b) - F(a)\beta\theta(b)$

$= \delta(a,b)_\alpha + \delta(a,b)_\beta$

Remark (2.9):

Note that F is a generalized isomorphism of a Γ-ring M onto a Γ-ring M' if and only if $\delta(a,b)_\alpha = 0$ for all $a, b \in$ M and $\alpha \in \Gamma$.

Lemma (2.10):

Let F be a generalized Jordan isomorphism of a Γ-ring M onto a Γ-ring M'.Then for all a, b, m $\in$ M and $\alpha, \beta \in \Gamma$

(i) $\delta(a,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha+\delta(b,a)_\alpha\beta\theta(m)\beta G(a,b)_\alpha= 0$

(ii) $\delta(a,b)_\alpha\alpha\theta(m)\alpha G(b,a)_\alpha+\delta(b,a)_\alpha\alpha\theta(m)\alpha G(a,b)_\alpha=0$

(iii) $\delta(a,b)_\beta\alpha\theta(m)\alpha G(b,a)_\beta+\delta(b,a)_\beta\alpha\theta(m)\alpha G(a,b)_\beta=0$

Proof:

(i) Let $w = a\alpha b\beta m\beta b\alpha a+b\alpha a\beta m\beta a\alpha b$

since F is a generalized Jordan isomorphism

$F(w) = F(a\alpha(b\beta m\beta b)\alpha a + b\alpha(a\beta m\beta a)\alpha b)$

$= F(a)\alpha\theta(b\beta m\beta b)\alpha\theta(a) + F(b)\alpha\theta(a\beta m\beta a)\alpha\theta(b)$

$= F(a)\alpha\theta(b)\beta\theta(m)\beta\theta(b)\alpha\theta(a) + F(b)\alpha\theta(a)\beta\theta(m)\beta\theta(a)\alpha\theta(b)$ …(1)

On the other hand

$F(w) = F((a\alpha b)\beta m\beta(b\alpha a) + (b\alpha a)\beta m\beta(a\alpha b))$

$= F(a\alpha b)\beta\theta(m)\beta\theta(b\alpha a) + F(b\alpha a)\beta\theta(m)\beta\theta(a\alpha b)$

$F(w) = F(a\alpha b)\beta\theta(m)\beta(\theta(a)\alpha\theta(b) + \theta(b)\alpha\theta(a) - \theta(a\alpha b)) + (- F(a\alpha b) + F(a)\alpha\theta(b) + F(b)\alpha\theta(a))\beta\theta(m)\beta\theta(a\alpha b)$

$= -F(a\alpha b)\beta\theta(m)\beta(\theta(a\alpha b) - \theta(a)\alpha\theta(b)) - F(a\alpha b)\beta\theta(m)\beta(\theta(a\alpha b) - \theta(b)\alpha\theta(a))$

$+ F(a)\alpha\theta(b)\beta\theta(m)\beta\theta(a\alpha b)+F(b)\alpha\theta(a)\beta\theta(m)\beta\theta(a\alpha b)$ …(2)

Compare (1) and (2) , we get :

$0 = -F(a\alpha b)\beta\theta(m)\beta G(a,b)_\alpha - F(a\alpha b)\beta\theta(m)\beta G(b,a)_\alpha +$

$F(a)\alpha\theta(b)\beta\theta(m)\beta\theta(a\alpha b)+F(b)\alpha\theta(a)\beta\theta(m)\beta$

$\theta(a\alpha b) - F(a)\alpha\theta(b)\beta\theta(m)\beta\theta(b)\alpha\theta(a) - F(b)\alpha\theta(a)\beta\theta(m)\beta\theta(a)\alpha\theta(b)$

$0 = -F(a\alpha b)\beta\theta(m)\beta G(a,b)_\alpha - F(a\alpha b)\beta\theta(m)\beta G(b,a)_\alpha +$

$F(a)\alpha\theta(b)\beta\theta(m)\beta(\theta(a\alpha b) - \theta(b)\alpha\theta(a)) +$

$F(b)\alpha\theta(a)\beta\theta(m)\beta(\theta(a\alpha b) - \theta(a)\alpha\theta(b))$

$0 = -F(a\alpha b)\beta\theta(m)\beta G(a,b)_\alpha - F(a\alpha b)\beta\theta(m)\beta G(b,a)_\alpha +$

$F(a)\alpha\theta(b)\beta\theta(m)\beta G(b,a)_\alpha + F(b)\alpha\theta(a)\beta\theta(m)\beta G(a,b)_\alpha$

$0 = -(F(a\alpha b) - F(b)\alpha\theta(a))\beta\theta(m)\beta G(a,b)_\alpha - (F(a\alpha b) - F(a)\alpha\theta(b))\beta\theta(m)\beta G(b,a)_\alpha$

Thus, we have:

$\delta(a,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha+\delta(b,a)_\alpha\beta\theta(m)\beta G(a,b)_\alpha=0$

(ii) Replace α by β in (i), we get (ii).

(iii) Interchanging α and β in (i), we obtain (iii).

Lemma (2.11):

Let F be a generalized Jordan isomorphism of a Γ-ring M onto a 2- torsion free prime Γ-ring M', then for all a, b, m ∈ M and α, β ∈ Γ

(i) $\delta(a,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha=\delta(b,a)_\alpha\beta\theta(m)\beta G(a,b)_\alpha=0$

(ii) $\delta(a,b)_\alpha\alpha\theta(m)\alpha G(b,a)_\alpha=\delta(b,a)_\alpha\alpha\theta(m)\alpha G(a,b)_\alpha=0$

(iii) $\delta(a,b)_\beta\alpha\theta(m)\alpha G(b,a)_\beta=\delta(b,a)_\beta\alpha\theta(m)\alpha G(a,b)_\beta=0$

Proof:

(i) By Lemma (2.10)(i)

$\delta(a,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha+\delta(b,a)_\alpha\beta\theta(m)\beta G(a,b)_\alpha=0$

And since by Lemma (Let M be a 2-torsion free semiprime Γ-ring and suppose that a, b ∈ M if $a\Gamma m\Gamma b + b\Gamma m\Gamma a = 0$ for all m ∈ M, then $a\Gamma m\Gamma b = b\Gamma m\Gamma a = 0$) . Then we get :

$\delta(a,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha=\delta(b,a)_\alpha\beta\theta(m)\beta G(a,b)_\alpha=0$

(ii) Replace α for β in (i), we obtain (ii).

(iii) Interchanging α and β in (i), we get (iii).

Lemma (2.12):

Let F be a generalized Jordan isomorphism of a Γ-ring M onto a prime Γ-ring M', then for all a, b, c, d, m ∈ M and α, β ∈ Γ

(i) $\delta(a,b)_\alpha\beta\theta(m)\beta G(d,c)_\alpha = 0$

(ii) $\delta(a,b)_\alpha\alpha\theta(m)\alpha G(d,c)_\alpha = 0$

(iii) $\delta(a,b)_\alpha\alpha\theta(m)\alpha G(d,c)_\beta = 0$

Proof:

(i) Replacing $a + c$ for a in Lemma (2.11) (i), we get:

$\delta(a + c,b)_\alpha\beta\theta(m)\beta G(b,a + c)_\alpha = 0$

$\delta(a,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha + \delta(a,b)_\alpha\beta\theta(m)\beta G(b,c)_\alpha +$

$\delta(c,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha + \delta(c,b)_\alpha\beta\theta(m)\beta G(b,c)_\alpha = 0$

By lemma (2.11)(i), we get:

$\delta(a,b)_\alpha\beta\theta(m)\beta G(b,c)_\alpha + \delta(c,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha = 0$

Therefore, we get

$\delta(a,b)_\alpha\beta\theta(m)\beta G(b,c)_\alpha\beta\theta(m)\beta\delta(a,b)_\alpha\beta\theta(m)\beta G(b,c)_\alpha = 0$

$= -\,\delta(a,b)_\alpha\beta\theta(m)\beta G(b,c)_\alpha\beta\theta(m)\beta\delta(c,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha = 0$

Since M' is prime Γ-ring and therefore:

$\delta(a,b)_\alpha\beta\theta(m)\beta G(b, c)_\alpha = 0$ …(1)

Now, replacing $b + d$ for b in Lemma (2.11)(i), we get:

$\delta(a,b + d)_\alpha\beta\theta(m)\beta G(b + d,a)_\alpha = 0$

$\delta(a,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha + \delta(a,b)_\alpha\beta\theta(m)\beta G(d,a)_\alpha +$

$\delta(a,d)_\alpha\beta\theta(m)\beta G(b,a)_\alpha + \delta(a,d)_\alpha\beta\theta(m)\beta G(d,a)_\alpha = 0$

By lemma (2.11)(i), we get:

$\delta(a,b)_\alpha\beta\theta(m)\beta G(d,a)_\alpha + \delta(a,d)_\alpha\beta\theta(m)\beta G(b,a)_\alpha = 0$

Therefore, we get:

$\delta(a,b)_\alpha\beta\theta(m)\beta G(d,a)_\alpha\beta\theta(m)\beta\delta(a,b)_\alpha\beta\theta(m)\beta G(d,a)_\alpha = 0$

$= -\,\delta(a,b)_\alpha\beta\theta(m)\beta G(d,a)_\alpha\beta\theta(m)\beta\delta(a,d)_\alpha\beta\theta(m)\beta\, G(b,a)_\alpha = 0$

Since M' is prime Γ-ring and therefore:

$\delta(a,b)_\alpha\beta\theta(m)\beta G(d,a)_\alpha = 0$ …(2)

Now, $\delta(a,b)_\alpha\beta\theta(m)\beta G(b + d,a + c)_\alpha = 0$

$\delta(a,b)_\alpha\beta\theta(m)\beta G(b,a)_\alpha + \delta(a,b)_\alpha\beta\theta(m)\beta G(b,c)_\alpha +$

$\delta(a,b)_\alpha\beta\theta(m)\beta G(d,a)_\alpha + \delta(a,b)_\alpha\beta\theta(m)\beta G(d,c)_\alpha = 0$

Since by lemma (3.2.15) (i) and (1), (2), we get:

$\delta(a,b)_\alpha\beta\theta(m)\beta G(d,c)_\alpha = 0.$

(ii) Replace α for β in (i), we get (ii).

(iii) Replace $\alpha+\beta$ for α in (ii), we get:

$\delta(a,b)_{\alpha+\beta}\alpha\theta(m)\alpha G(d,c)_{\alpha+\beta} = 0$

$\delta(a,b)_\alpha\alpha\theta(m)\alpha G(d,c)_\alpha + \delta(a,b)_\alpha\alpha\theta(m)\alpha G(d,c)_\beta +$

$\delta(a,b)_\beta\alpha\theta(m)\alpha G(d,c)_\alpha + \delta(a,b)_\beta\alpha\theta(m)\alpha G(d,c)_\beta=0$

By (i) and (ii), we get:

$\delta(a,b)_\alpha\alpha\theta(m)\alpha G(d,c)_\beta+\delta(a,b)_\beta\alpha\theta(m)\alpha G(d,c)_\alpha=0$

Therefore, we have:

$\delta(a,b)_\alpha\alpha\theta(m)\alpha G(d,c)_\beta\alpha\theta(m)\alpha\delta(a,b)_\alpha\alpha\theta(m)\alpha G(d,c)_\beta = 0$

$= -\delta(a,b)_\alpha\alpha\theta(m)\alpha G(d,c)_\beta\alpha\theta(m)\alpha\delta(a,b)_\beta\alpha\theta(m)\alpha G(d,c)_\alpha= 0$

Since M' is prime Γ-ring, then:

$\delta(a,b)_\alpha\alpha\theta(m)\alpha G(d,c)_\beta = 0.$

3.2 The main result

Theorem (3.1):

Every generalized Jordan isomorphism of a Γ-ring M onto prime Γ-ring M' is either generalized isomorphism or anti - isomorphism.

Proof:

Let F be a generalized Jordan isomorphism of a Γ-ring M onto prime Γ-ring M'.Then by Lemma (2.12) (i) we get :

$\delta(a,b)_\alpha\beta\theta(m)\beta G(d,c)_\alpha = 0.$

Since M' is prime Γ-ring therefore either $\delta(a,b)_\alpha=0$ or $G(d,c)_\alpha=0$ for all $a, b, c, d \in$ M and $\alpha \in \Gamma$.

If $G(d,c)_\alpha \neq 0$ for all $c, d \in$ M and $\alpha \in \Gamma$ then $\delta(a,b)_\alpha = 0$, hence we get F is generalized isomorphism.

But if $G(d,c)_\alpha = 0$ for all $c, d \in$ M and $\alpha \in \Gamma$, then we get F is anti - isomorphism.

Proposition (3.2):

Let F be a generalized Jordan isomorphism from a Γ-ring M onto a 2-torsion free Γ-ring M', such that $a\alpha b\beta a = a\beta b\alpha a$, for all $a, b \in$ M and $\alpha, \beta \in \Gamma$, $a'\alpha b'\beta a' = a'\beta b'\alpha a'$, for all $a', b' \in$ M'.Then F is a generalized Jordan triple isomorphism.

Proof:

Replace b by $a\beta b + b\beta a$ in Definition (2.3), we get:

$$F(a\alpha(a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a) = F(a)\alpha\theta(a\beta b + b\beta a) + F(a\beta b + b\beta a)\alpha\theta(a)$$

$$= F(a)\alpha\theta(a)\beta\theta(b) + F(a)\alpha\theta(b)\beta\theta(a) + F(a)\beta\theta(b)\alpha\theta(a) + F(b)\beta\theta(a)\alpha\theta(a)$$

Since $a'\alpha b'\beta a' = a'\beta b'\alpha a'$, for all $a', b' \in$ M' and $\alpha, \beta \in \Gamma$, we get:

$$=F(a)\alpha\theta(a)\beta\theta(b)+2F(a)\alpha\theta(b)\beta\theta(a)+F(b)\beta\theta(a)\alpha\theta(a) \quad \ldots(1)$$

On the other hand:

$$F(a\alpha(a\beta b + b\beta a) + (a\beta b + b\beta a)\alpha a) = F(a\alpha a\beta b + a\alpha b\beta a + a\beta b\alpha a + b\beta a\alpha a)$$

Since $a\alpha b\beta a = a\beta b\alpha a$, for all $a, b \in$ M and $\alpha, \beta \in \Gamma$, we get:

$$= F(a)\alpha\theta(a)\beta\theta(b) + F(b)\beta\theta(a)\alpha\theta(a) + 2F(a\alpha b\beta a) \quad \ldots(2)$$

Compare (1) and (2), we get:

$$2F(a\alpha b\beta a) = 2F(a)\alpha\theta(b)\beta\theta(a).$$

Since M' is 2-torsion free Γ-ring .Then F is a generalized Jordan triple isomorphism.

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