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Bounds on Two Parametric New Generalized Fuzzy Entropy

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Abstract

In this paper we define a new two parametric generalized fuzzy average code-word length $L_\alpha^\beta(A)$ for a fuzzy set 'A' and its relationship with two parametric generalized fuzzy entropy $H_\alpha^\beta(A)$ has been discussed. Using $L_\alpha^\beta(A)$, some coding theorems for discrete noiseless channel has been proved. This measure is not only new but some well known measures are the particular cases of our proposed measure that already exist in the literature of fuzzy information theory.

Keywords

Fuzzy set, Membership function, Shannon's entropy, Fuzzy entropy, Code-word length, Kraft inequality, Coding theorem, Holder's inequality and Optimal code length.

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1. Introduction:

Fuzziness and uncertainty are the basic nature of human thinking and many real world objectives. Fuzziness is found in our decision, in our language and in the way of process information. The main objective of information is to remove uncertainty and fuzziness. In fact, we measure information supplied by the amount of probabilistic uncertainty removed in an experiment and the measure of uncertainty removed is also called as a measure of information, while measure of fuzziness is the measure of vagueness and ambiguity of uncertainties. The concept of entropy has been widely used in different areas, e.g. communication theory, statistical mechanics, finance pattern recognition, and neural network etc. Fuzzy set theory developed by Lotfi. A. Zadeh [22] has found wide applications in many areas of science and technology, e.g. clustering, image processing, decision making etc. because of its capability to model non-statistical imprecision or vague concepts. The importance of fuzzy sets comes from the fact that it can deal with imprecise and inexact information, many fuzzy measures have been discussed and derived by Kapur [6], Lowen [10], Nguyen and Walker [14], Parkash [18], Pal and Bezdek [17], Zadeh [22] etc.

Application of fuzzy measures to engineering, fuzzy traffic control, fuzzy aircraft control, medicines, computer science and decision making etc, have already been established. The basic noiseless coding theorems by considering different information measures were investigated by several authors see for instance: Aczel. J [1], Kapur J. N [5], Khan A. B., Autar R. and Ahmad H [8], Van Der Lubbe J.C.A [21], Reyni [19]and obtain the lower bounds for the mean code-word length of a uniquely decipherable code in terms of Shannon's [20] entropy. Kapur [7] has established relationships between probability entropy and coding. But there are situations where probabilistic measures of entropy do not work, to tackle such situations, instead of taking the probability, the idea of fuzziness can be explored.

2. *Preliminaries on fuzzy set theory:*

Let a universe of discourse be $X = \{x_1, x_2, \dots, x_n\}$ then a fuzzy subset of universe X is defined as:

$$A = \{(x_i, \mu_A(x_i)) : x_i \in X, \mu_A(x_i) \in [0,1]\}$$

Where $\mu_A(x_i): X \to [0,1]$ is a membership function and gives the degree of belongingness of the element x_i to the set A and is defined as follows:

$$\mu_A(x_i) = \begin{cases} 0, \text{ if } x_i \notin A \text{ and there is no ambiguity,} \\ 1, \text{ if } x_i \in A \text{ and there is no ambiguity,} \\ 0.5, \text{if } x_i \in A \text{ or } x_i \notin A \text{ and there is maximum ambiguity,} \end{cases}$$

In fact $\mu_A(x_i)$ associates with each $x_i \in X$ gives a grade of membership function in the set A. When $\mu_A(x_i)$ takes values only 0 or 1, there is no uncertainty about it and a set is said to be a crisp (i.e. non-fuzzy) set. Some notions related to fuzzy sets which we shall need in our discussion are as under:

- ***Containment:*** If $A \subset B \Leftrightarrow \mu_A(x_i) \le \mu_B(x_i) \;\forall\; x_i \in X$
- ***Equality:*** If $A = B \Leftrightarrow \mu_A(x_i) = \mu_B(x_i) \;\forall\; x_i \in X$
- ***Compliment:*** If $\bar{A}$ is complement of $A \Leftrightarrow \mu_{\bar{A}}(x_i) = 1 - \mu_A(x_i) \;\forall\; x_i \in X$
- ***Union:*** If $A \cup B$ is union of A & $B \Leftrightarrow \mu_{A\cup B}(x_i) = \text{Max}\{\mu_A(x_i), \mu_B(x_i)\} \;\forall\; x_i \in X$
- ***Intersection:*** If $A \cap B$ is intersection of A & $B \Leftrightarrow \mu_{A\cap B}(x_i) = \text{Min}\{\mu_A(x_i), \mu_B(x_i)\} \;\forall\; x_i \in X$
- ***Product:*** If AB is product of A & $B \Leftrightarrow \mu_{AB}(x_i) = \mu_A(x_i)\mu_B(x_i) \;\forall\; x_i \in X$
- ***Sum:*** If $A + B$ is sum of A & $B \Leftrightarrow \mu_{A+B}(x_i) = \mu_A(x_i) + \mu_B(x_i) - \mu_A(x_i)\mu_B(x_i) \;\forall\; x_i \in X$

3. *Basic concepts:*

Let X is a discrete random variable taking values $x_1, x_2, \dots, x_n$ with respective probabilities $P = (p_1, p_2, \dots, p_n), p_i \ge 0 \;\forall\; i = 1, 2, \dots, n$ and $\sum_{i=1}^{n} p_i = 1$. Shannon [20] gives the following measure of information and call it as entropy.

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i \tag{1.1}$$

The measure (1.1) serves as a suitable measure of entropy. Let $p_1, p_2, p_{3,\dots,} p_n$ be the probabilities of n codewords to be transmitted and let their lengths l_1, l_2,…,l_n satisfy Kraft [9] inequality,

$$\sum_{i=1}^{n} D^{-l_i} \leq 1 \tag{1.2}$$

For uniquely decipherable codes, Shannon [20] showed that for all codes satisfying (1.2), the lower bound of the mean codeword length,

$$L = \sum_{i=1}^{n} p_i l_i \tag{1.3}$$

lies between $H(P)$ and $H(P) + 1$.Where D is the size of code alphabet.

If $x_1, x_2, \dots, x_n$ are members of the universe of discourse, with respective membership functions $\mu_A(x_1)$, $\mu_A(x_2)$, $\mu_A(x_3)$,…, $\mu_A(x_n)$ then all $\mu_A(x_1), \mu_A(x_2), \mu_A(x_3), \dots, \mu_A(x_n)$ lies between 0 and 1 but these are not probabilities because their sum is not unity. $\mu_A(x_i)$ gives the element x_i the degree of belongingness to the set "A". The function $\mu_A(x_i)$ associates with each $x_i \in R^n$ a grade of membership to the set "A" and is known as membership function.

Denote

$$F.S = \begin{bmatrix} x_1 & x_2 & \dots & x_n \\ \mu_A(x_1) & \mu_A(x_2) & \dots & \mu_A(x_n) \end{bmatrix}, 0 \leq \mu_A(x_i) \leq 1 \;\; \forall \;\; x_i \tag{1.4}$$

We call the scheme (1.4) as a finite fuzzy information scheme. Every finite scheme describes a state of uncertainty. Since $\mu_A(x_i)$ and $1 - \mu_A(x_i)$ gives the same degree of fuzziness, therefore corresponding to entropy due to Shannon [20], De-Luca and Termini [4] suggested the following measure of fuzzy entropy as

$$H(A) = -\sum_{i=1}^{n} \left[\mu_A(x_i) \log \mu_A(x_i) + \left(1 - \mu_A(x_i)\right) \log\left(1 - \mu_A(x_i)\right)\right] \tag{1.5}$$

De-Luca and Termini [4] introduced a set of four properties and these properties are widely accepted as for defining new fuzzy entropy. In fuzzy set theory, the entropy is a measure of fuzziness which expresses the amount of average ambiguity in making a decision whether an element belongs to a set or not. So, a measure of average fuzziness $H(A)$ in a fuzzy set A should have the following properties to be valid fuzzy entropy measure:

I. (Sharpness): $H(A)$ is minimum if and only if A is a crisp set,

i.e $\mu_A(x_i) = 0$ or 1; for all x_i, $i = 1, 2, \dots, n$.

II. (Maximality): $H(A)$ is maximum if and only if A is most fuzzy set,

i.e $\mu_A(x_i) = \frac{1}{2}$; for all x_i, $i = 1, 2, \dots, n$.

III. (Resolution): $H(A^*) \leq H(A)$, where A^* is sharpened version of A.

IV. (Symmetry): $H(A) = H(A^c)$, where A^c is the complement of A.

i.e $\mu_{A^c}(x_i) = 1 - \mu_A(x_i)$; for all x_i i = 1, 2, …, n

Generalized fuzzy coding theorems by considering different fuzzy information measures were investigated by several authors see for instance the papers: M.A.K. Baig and Mohd Javid Dar [11], [12] & [13], Ashiq Hussain and M.A.K Baig [2], Parkash and P. K. Sharma [15] & [16], Bhandari and Pal [3], Kapur [6].

In this particular paper two parametric new generalized fuzzy code-word mean length is considered and bounds have been obtained in terms of two parametric new generalized fuzzy entropy. The main aim of these results is that it generalizes some well-known fuzzy information measures already existing in the literature of fuzzy information theory.

4. ***Noiseless Coding theorems***

Define a two parametric new generalized measure of entropy as:

$$H_\alpha^\beta(P) = \frac{\beta}{\beta-\alpha}\sum_{i=1}^{n} p_i^{\alpha\beta}, \qquad (2.1)$$

Where $0 < \alpha < 1, 0 < \beta \le 1, \beta > \alpha.$ $p_i \ge 0\ \forall\ i = 1,2,\dots,n, \sum_{i=1}^{n} p_i = 1$

Further we define a two parametric new generalized code-word length corresponding to (2.1) and is given by

$$L_\alpha^\beta = \frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n} p_i^\beta D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^\alpha, 0 < \alpha < 1, 0 < \beta \le 1, \beta > \alpha. \qquad (2.2)$$

Where *D* is the size of code alphabet.

Corresponding to (2.1) we propose the following measure of fuzzy entropy as

$$H_\alpha^\beta(A) = \frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n}\left(\mu_A^{\alpha\beta}(x_i) + \left(1-\mu_A(x_i)\right)^{\alpha\beta}\right)\right], 0 < \alpha < 1, 0 < \beta \le 1, \beta > \alpha. (2.3)$$

And the generalized fuzzy average codeword length corresponding to (2.3) as

$$L_\alpha^\beta(A) = \frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n}\left(\mu_A^{\alpha\beta}(x_i) + \left(1-\mu_A(x_i)\right)^{\alpha\beta}\right) D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^\alpha \qquad (2.4)$$

Remarks for (2.1)

I. When $\beta = 1$, (2.1) reduces to entropy,

i.e., $H_\alpha(P) = \frac{1}{1-\alpha}\sum_{i=1}^{n} p_i^\alpha, 0 < \alpha < 1$

II. When $\beta = 1, and\ \alpha \to 1$, (2.1) reduces to Shannon's [20] entropy,

i.e., $H(P) = -\sum_{i=1}^{n} p_i log p_i$

III. When $\alpha \to 1$ (2.1) reduces to the entropy of β-power distribution,

i.e., $H^\beta(P) = -\beta\sum_{i=1}^{n} p_i^\beta\ log p_i^\beta$

Remarks for (2.2)

I. For $\beta = 1$ (2.2) reduces to code-word length,

$$\text{i.e., } L_\alpha = \frac{1}{1-\alpha}\left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^\alpha$$

II. For $\beta = 1, and\ \alpha \to 1$, (2.2) reduces to optimal code-word length corresponding to Shannon [20] entropy

$$\text{i.e., } L = \sum_{i=1}^{n} p_i l_i$$

Now we found the bounds of (2.4) in terms of (2.3) under the condition

$$\sum_{i=1}^{n}\left(\mu_A^{\alpha\beta}(x_i) + \left(1-\mu_A(x_i)\right)^{\alpha\beta}\right) D^{-l_i} \le 1 \tag{2.5}$$

Or we can write

$$\sum_{i=1}^{n}\left[f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right] D^{-l_i} \le 1 \tag{2.6}$$

Where

$$f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right) = \left(\mu_A^{\alpha\beta}(x_i) + \left(1-\mu_A(x_i)\right)^{\alpha\beta}\right)$$

Which is generalized fuzzy Kraft [9] inequality, where D is the size of code alphabet, it is easy to see that for $\beta = 1, \alpha \to 1$ the inequality (2.6) reduces to Kraft [9] inequality.

Theorem 4.1: For all integers $(D > 1)$ the code word lengths $l_1, l_2, \dots, l_n$ satisfies the condition (2.6) then the generalized fuzzy code-word length (2.4) satisfies the inequality

$$L_\alpha^\beta(A) \ge H_\alpha^\beta(A) \quad \text{Where } 0 < \alpha < 1, 0 < \beta \le 1, \beta > \alpha \tag{2.7}$$

Where equality holds good iff

$$l_i = -\log_D\left[\frac{1}{\sum_{i=1}^{n}\left[f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right]}\right] \tag{2.8}$$

Where

$$f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right) = \left(\mu_A^{\alpha\beta}(x_i) + \left(1-\mu_A(x_i)\right)^{\alpha\beta}\right)$$

Proof: By Holder's inequality we have

$$\sum_{i=1}^{n} x_i y_i \ge \left(\sum_{i=1}^{n} x_i^{\,p}\right)^{\frac{1}{p}} \left(\sum_{i=1}^{n} y_i^{\,q}\right)^{\frac{1}{q}} \tag{2.9}$$

For all $x_i, y_i > 0$, $i = 1, 2, 3, \dots, n$ and $\frac{1}{p} + \frac{1}{q} = 1$, $p < 1 (\neq 0)$, $q < 0$ or $q < 1(\neq 0)$, $p < 0$.

We see the equality holds iff there exists a positive constant c such that

$$x_i^p = c y_i^q \tag{2.10}$$

Making the substitution

$$x_i = \left[f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right]^{\frac{\alpha}{\alpha-1}} D^{-l_i}, \qquad y_i = \left[f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right]^{\frac{1}{1-\alpha}}$$

$$p = \frac{\alpha-1}{\alpha} \qquad \text{and} \qquad q = 1-\alpha$$

Using these values in (2.9) and after suitable simplification we get

$$\sum_{i=1}^{n}\left[f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right] D^{-l_i} \geq$$

$$\left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right\} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}} \left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right\}\right]^{\frac{1}{1-\alpha}}$$
(2.11)

Now using the inequality (2.6) we get

$$\left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right\} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}} \left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right\}\right]^{\frac{1}{1-\alpha}} \leq 1$$
(2.12)

Or equation (2.12) can be written as

$$\left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right\} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}} \leq \left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right\}\right]^{\frac{1}{\alpha-1}}$$
(2.13)

Here following cases arise

Case 1:

As $0 < \alpha < 1$, then $(\alpha - 1) < 0$, raising both sides to the power $(\alpha - 1) < 0$, to equation (2.13), we get

$$\left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right\} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} \geq \left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right\}\right]$$
(2.14)

As $0 < \alpha < 1, 0 < \beta \leq 1, \beta > \alpha$ then $(\beta - \alpha) > 0$ and $\frac{\beta}{(\beta-\alpha)} > 0$, multiply equation (2.14) both sides by $\frac{\beta}{\beta-\alpha} > 0$, we get

$$\frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right\} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} \geq \frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right\}\right]$$
(2.15)

Taking $f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right) = \left(\mu_A^{\alpha\beta}(x_i) + \left(1 - \mu_A(x_i)\right)^{\alpha\beta}\right)$, we get

$\frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n}\left(\mu_A^{\alpha\beta}(x_i)+\left(1-\mu_A(x_i)\right)^{\alpha\beta}\right)D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} \geq \frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n}\left(\mu_A^{\alpha\beta}(x_i)+\left(1-\right.\right.\right.$
$\left.\left.\left.\mu_A(x_i)\right)^{\alpha\beta}\right)\right]$

Or equivalently we can write

$$L_\alpha^\beta \geq H_\alpha^\beta(P), \text{ Hence the result for } 0<\alpha<1, 0<\beta\leq 1, \beta>\alpha.$$

Case 2:

From equation (2.8) we have

$$l_i = -\log_D\left[\frac{1}{\sum_{i=1}^{n}\left[f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right]}\right] \tag{2.16}$$

Or equivalently we can write equation (2.16) as

$$D^{-l_i} = \frac{1}{\sum_{i=1}^{n}\left[f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right]} \tag{2.17}$$

Raising both sides to the power $\left(\frac{\alpha-1}{\alpha}\right)$, to equation (2.17) and after suitable simplification we get

$$D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} = \left[\sum_{i=1}^{n}\left[f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right]\right]^{\frac{1-\alpha}{\alpha}} \tag{2.18}$$

Multiply equation (2.18) both sides by $f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)$ and then summing over $i=1,2,\ldots,n$, both sides to the resultant expression and after suitable simplification, we get

$$\left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right\}D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right] = \left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right\}\right]^{\frac{1}{\alpha}}$$
(2.19)

Raising both sides to the power α to equation (2.19), then multiply both sides by $\frac{\beta}{\beta-\alpha}$, we get

$$\frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right\}D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} = \frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n}\left\{f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right\}\right]$$

Or equivalently we can write

$$L_\alpha^\beta(A) = H_\alpha^\beta(A), \text{ Hence the result}$$

Where

$$f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right) = \left(\mu_A^{\alpha\beta}(x_i)+\left(1-\mu_A(x_i)\right)^{\alpha\beta}\right)$$

Theorem 4.2**:** For every code with lengths $l_1, l_2, \ldots, l_n$ satisfies the condition (2.6), $L_\beta^\alpha(A)$ can be made to satisfy the inequality,

$$L_\alpha^\beta(A) < H_\alpha^\beta(P)D^{(1-\alpha)}, \text{ Where } 0<\alpha<1, 0<\beta\le 1, \beta>\alpha. \quad (2.20)$$

Proof: From the theorem (2.1) we have,

$$L_\alpha^\beta(A) = H_\alpha^\beta(A).$$

Holds if and only if

$$D^{-l_i} = \frac{1}{\sum_{i=1}^n\left[f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right]}, 0<\alpha<1, 0<\beta\le 1, \beta>\alpha.$$

Or equivalently we can write

$$l_i = \log_D\left[\sum_{i=1}^n\left[f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right]\right]$$

We choose the code-word lengths $l_i, i = 1,2,\dots,n$ in such a way that they satisfy the inequality,

$$\log_D\left[\sum_{i=1}^n\left[f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right]\right] \le l_i < \log_D\left[\sum_{i=1}^n\left[f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right]\right] + 1 (2.21)$$

Consider the interval

$$\delta_i = \left[\log_D\left[\sum_{i=1}^n\left(f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right)\right], \log_D\left[\sum_{i=1}^n\left(f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right)\right]+1\right]$$

of length unity. In every δ_i, there lies exactly one positive integer l_i, such that,

$$0 < \log_D\left[\sum_{i=1}^n\left(f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right)\right] \le l_i < \log_D\left[\sum_{i=1}^n\left(f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right)\right] + 1 (2.22)$$

Now we will first show that the sequence $l_1, l_2, \dots, l_n$, thus defined satisfies the inequality (2.6) which is generalized fuzzy Kraft [9] inequality.

From the left inequality of (2.22), we have

$$\log_D\left[\sum_{i=1}^n\left(f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right)\right] \le l_i$$

Or equivalently we can write

$$D^{-l_i} \le \frac{1}{\left[\sum_{i=1}^n\left(f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right)\right]} \quad (2.23)$$

Multiply equation (2.23) both sides by $f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)$ and then summing over $i = 1,2,\dots,n$, on both sides to the result that we obtain we get the required result i.e., (2.6), which is generalized fuzzy Kraft [9] inequality.

Now the last inequality of (2.22) gives

$$l_i < \log_D\left[\sum_{i=1}^n\left(f\left(\mu_A(x_i),\mu_{A^c}(x_i)\right)\right)\right] + 1$$

Or equivalently we can write

$$D^{l_i} < \left[\sum_{i=1}^{n}\left(f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right)\right] D$$
(2.24)

As $0 < \alpha < 1$, then $(1-\alpha) > 0$, and $\left(\frac{1-\alpha}{\alpha}\right) > 0$, raising both sides to the power $\left(\frac{1-\alpha}{\alpha}\right) > 0$, to equation (2.24), we get

$$D^{l_i\left(\frac{1-\alpha}{\alpha}\right)} < \left[\sum_{i=1}^{n}\left(f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right)\right]^{\left(\frac{1-\alpha}{\alpha}\right)} D^{\frac{1-\alpha}{\alpha}} \qquad (2.25)$$

Or we can write the equation (2.25) as

$$D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} < \left[\sum_{i=1}^{n}\left(f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right)\right]^{\left(\frac{1-\alpha}{\alpha}\right)} D^{\frac{1-\alpha}{\alpha}} \qquad (2.26)$$

Multiply equation (2.26) both sides by $f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)$ and then summing over $i = 1,2,\ldots,n$, both sides to the resulted expression, and after making suitable operations, we get

$$\sum_{i=1}^{n}\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\}D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} < \left[\sum_{i=1}^{n}\left(f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\right)\right]^{\frac{1}{\alpha}} D^{\frac{1-\alpha}{\alpha}} \quad (2.27)$$

As $0 < \alpha < 1$, raising both sides to the power α to equation (2.27) we get

$$\left[\sum_{i=1}^{n}\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\}D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} < \left[\sum_{i=1}^{n}\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\}\right]D^{1-\alpha} \quad (2.28)$$

As $0 < \alpha < 1, 0 < \beta \leq 1, \beta > \alpha$ then $(\beta-\alpha) > 0$ and $\frac{\beta}{\beta-\alpha} > 0$, multiply equation (2.28) both sides by $\frac{\beta}{\beta-\alpha} > 0$, we get

$$\frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n}\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\}D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} < \frac{\beta}{\beta-\alpha}\left[\sum_{i=1}^{n}\{f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right)\}\right]D^{1-\alpha}$$

Or equivalently we can write

$$L_\alpha^\beta(A) < H_\alpha^\beta(P)D^{(1-\alpha)}, \text{ Where } 0 < \alpha < 1, 0 < \beta \leq 1, \beta > \alpha.$$

Where

$$f\left(\mu_A(x_i), \mu_{A^c}(x_i)\right) = \left(\mu_A^{\alpha\beta}(x_i) + \left(1-\mu_A(x_i)\right)^{\alpha\beta}\right),$$

Thus from the above two coding theorems we have shown that

$$H_\alpha^\beta(P) \leq L_\alpha^\beta(A) < H_\alpha^\beta(P)D^{(1-\alpha)}.$$

Where $0 < \alpha < 1, 0 < \beta \leq 1, \beta > \alpha.$ (2.29)

5. ***Conclusion:***

In this paper we define a two parametric new generalized fuzzy entropy measure. This measure also generalizes some well-known fuzzy information measures already existing in the literature of fuzzy information theory. Also two parametric new generalized fuzzy code-word mean length is considered and bounds have been obtained in terms of two parametric new generalized fuzzy entropy measure and show that

$$H_\alpha^\beta(P) \leq L_\alpha^\beta(A) < H_\alpha^\beta(P) D^{(1-\alpha)}. \text{ Where } 0 < \alpha < 1, 0 < \beta \leq 1, \beta > \alpha.$$

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