The Moments of the Optimal Average Run Length of the Multivariate Exponentially Weighted Moving Average Control Chart For Equally Correlated Variables

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ABSTRACT
The Hotelling's T^2 is a well-known statistic for detecting a shift in the mean vector of a multivariate normal distribution. Control charts based on T^2 have been used in statistical process control for monitoring a multivariate process. Although it is a powerful tool, the T^2 statistic is deficient when the shift to be detected in the mean vector of a multivariate process is small and consistent. The Multivariate Exponentially Weighted Moving Average (MEWMA) control chart is one of the control statistics used to overcome the drawback of the Hotelling's T^2 statistic. In this paper, the distribution of the Average Run Length (ARL) of the (MEWMA) control chart when the quality characteristics exhibit substantial cross correlation and when the process is in control and out-of-control was derived using the Markov Chain algorithm. The derivation of the probability functions and the moments of the run length distributions were also obtained and they were consistent with some existing results for the in – control and out –of –control situation. By simulation process, the procedure identified a class of ARL for the MEWMA control chart when the process is in – control and out- of – control. From our study it was observed that the MEWMA scheme is quite adequate for detecting a small shift and a good way to improve the quality of goods and services in a multivariate situation. It was also observed that as the in-control average run length ARL_0 and the number of variables (p) increases, the optimum value of the ARL_{opt} increases asymptotically and as the magnitude of the shift σ increases, the optimal ARL_{opt} decreases. Finally we use examples from the literature to illustrate our method and demonstrate its efficiency.

Keywords: Moments, Average Run Length, multivariate exponentially weighted moving average, Markov Chain, optimal smoothing parameter.

1.0 Introduction
In recent years, the importance of quality has become increasingly apparent. Stiffer competition, tougher environment and safer regulations and rapidly changing economic conditions have been key factors in tightening products and services quality. Thus to attain uniformity in the production of goods and services practitioner employed a technique called Statistical Process Control (SPC) to monitor, detect and eliminate the substandard goods and services. By monitoring various steps in the process using SPC methods, abnormal trends can be identified and problems solved before they get out of hand thereby reducing cost of production. The multivariate control charts are important tools of SPC for monitoring and improving the quality of products. Recently, the relevant of multivariate control charts has increased because more quality characteristics are measured in mass production than ever before. These quality characteristics often exhibit substantial cross correlation. In most production situations, it would be more efficient to monitor the quality of goods and services by a multivariate control chart than several univariate control charts because it is possible that individual control charts might not detect assignable causes when quality characteristics are

dependent. Several multivariate quality control charts have been proposed to monitor the mean vector of the quality characteristics. The three most common multivariate control charts are the multivariate Shewhart control chart otherwise known as the Hotelling's χ^2 control chart proposed by Hotelling (1947),the multivariate cumulative sum (MCUSUM) control chart proposed by Woodall and Ncube (1985) and later by Crosier (1988), the multivariate EWMA proposed by Lowry et al (1992). When several quality characteristics are involved, the conventional multivariate Shewhart control chart loose efficiency with respect to shift detection (Montegomery, 2005), hence the MEWMA control charts was develop to take care of this deficiency of the multivariate Shewhart control chart.

For simplicity, suppose X_t is P-dimensional random vector distributed normally with a known variance – covariance matrix σ_0 and a known p–dimensional mean vector μ_0, the multivariate version of the univarate EWMA control chart proposed by Lowry et al (1992) is therefore defined by

$$Z_t = \lambda X_t + (1-\lambda)z_{t-1}, 0 < \lambda \leq 1 \quad (1.1)$$

Z_0= O_p and λ is the smoothing parameter, the MEWMA control chart gives an out-of-control signal when

$$T_t^2 = Z_t^1 S_{z_t}^{-1} Z_t > h \quad (1.2)$$

Where S_{z_t} is the variance – covariance matrix of Z_t and it is given as

$$S_{z_t} = \left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2t}]S_z \quad (1.3)$$

$$\text{Where } S_z = \begin{bmatrix} \sigma_{11} Cor(1,2), \ldots, Cor(1,P) \\ Cor(2,1), \sigma_{22}, \ldots, Cor(2,P) \\ . \\ . \\ . \\ Cor\ (P,1) Cor\ (P,2) \ldots\ \sigma_{pp} \end{bmatrix} \quad (1.4)$$

See Edokpa et al. (2009).
and h is a specific control limit obtained by simulation to achieve a specified in – Control ARL.
Lowry et al (1992) asserted that the p-dimensional random vector X_t can therefore be transformed to have mean zero and identity covariance matrix. T_t in equation (1.2) can be written as

$$T_t^2 = Z_t' \left[\left(\frac{\lambda}{2-\lambda}\right) S_{z_t}\right]^{-1} Z_t \quad (1.5)$$

$$= \left[\frac{2-\lambda}{\lambda}\right] Z_t' S_{z_t}^{-1} Z_t \quad (1.6)$$

$$= \left[\frac{2-\lambda}{\lambda}\right] ||Z_t|| \quad (1.7)$$

Then the out-of-control situation given in equation 2, T_t^2 > h implies$\| Z_t \| > \left(\frac{\lambda}{2-\lambda}\right)^{\frac{1}{2}}$.

2.0 REVIEW OF THE ARL OF MEWMA CHART

Control chart performance can be majorly measured by the ARL. The ARL is the average number of points that must be plotted before an out - of - control condition is indicted. When a process in control, it is expected that the ARL is large, when the process is out – of – control, the ARL should be small (Pham, 2006). Lowry et al 1992 studied the ARL of the MEWMA and stated that the performance of MEWMA procedures depends only on μ_0 and σ_0 through the value of the noncentallity parameter. Rigdon (1995a, 1995b) respectively gave an integral and a double integral equation for calculating the in-control and out –of – control ARLs respectively. Bodden and Rigdon (1999) developed a computer program for approximating the in-control ARL of the MEWMA. Runger and Prabhu(1996) used a markov chain approximation to determine the run length performance of the MEWMA.

The ARL of the MEWMA Scheme can be computed using the markov chain approach (see Runger and Prabhu (1996). For the in-control, ARL analysis can be considered as a one-dimensional markov chain, to approximate $K_t = ||Z_t||$, the control limits region [0,UCL] is partitioned into m + 1 transient states such that m of them have the same length g and one of them has the length $m/2$, therefore

$$\frac{g}{2} + mg = UCL \tag{2.1}$$

$$g = \left[\frac{2UCL}{2m+1}\right] \text{width} \tag{2.2}$$

When the process is in-control, that is, without loss of generality $\mu = 0$, the in-control distance of $K_t = ||Z_t||$, can be approximated by the markov chain

$$P(i.j) = P(K_t \; in\; state\; j \,|K_{t-1} \; in \; state \; i) \tag{2.3}$$

$$= P\{(j - 0.5)g < ||\lambda X_t + (1-\lambda)Z_{t-1}|| < (j+0.5)g|k_{t-1} = g_i\} \tag{2.4}$$

$$= P\left\{\frac{(j-0.5)g}{\lambda} < \left\|\frac{X_t + (1-\lambda)igu}{\lambda}\right\| < \frac{(j+0.5)g}{\lambda}\right\} \tag{2.5}$$

Since $S(\lambda)$ is a P–dimensional sphere of radius $\lambda > 0$ and Z_t has a spherical distribution, then the conditional distance of Z_t given $\|Z_t\|$ is the same as $\|Z_t\|U$, where U is uniform random variable on the P- dimensional sphere with radius 1(Eaton, 1983).

From equation (2.5), $X_t \sim N_p(0,1)$ and U is a spherical distributed with radius 1 and $\|X_t + (1-\lambda)igu|\lambda\|^2$ is chi–square distributed with parameters (p, c) where c is the non-centrality parameter defined as

$$c = \left(\frac{1-\lambda}{\lambda}igu\right)' I\left(\frac{1-\lambda}{\lambda}igu\right) = \left(\frac{1-\lambda}{\lambda}ig\right)^2 \mu'\mu = \left(\frac{1-\lambda}{\lambda}igu\right)\|\mu\|^2 = \left(\frac{(1-\lambda)}{\lambda}ig\right)^2 \tag{2.6}$$

Since U is Spherical distributed with radius 1 and p is the degree of freedom, equation (2.5) can therefore be written as

$$P(i,j) = \int_{s(1)} \cdots \int f(u)\, P\{(j-0.5)^2 g|\lambda^2 < \chi^2(p,c) < (j+0.5)^2 g^2|\lambda^2\} \tag{2.7}$$

$$= P\{(j-0.5)^2 g|\lambda^2 < \chi^2(p,c) < (j+0.5)^2|\lambda^2 \tag{2.8}$$

Since $\int_{s(1)} \cdots \int f(u)\, du = 1$

Hence, for $i = 1,2, \ldots.. m,$ the transitional probability from $i \; to \; state \; j$ is given as

$$P(i \cdot j) = \{p(j-0.5)^2 g^2|\lambda^2 < \chi^2(p,c) < (j+0.5)^2 g^2|\lambda^2\} \; if \; j \neq 0,$$

$$\{p(\chi^2(p,c) < 0.5^2 g^2|\lambda^2\} \; if \; j = 0 \tag{2.9}$$

Where $\chi^2(p,c)$ is a non-central chi- squared random variable with Probability density function p and $c = (1-\lambda)ig/\lambda$ while $g = 2ucl/(2m+1)$. Hence from Runger and

Prabhu(1996), π_0 is the $(m+1) \times (m+1)$ transition matrix and the in-control average rule length ARL is given by

$$\text{ARL}_0 = \lim_{M\to\infty} a'(1-\pi_0)^{-1} 1 \quad (2.10)$$

Where a is the starting probability vector and 1 denotes a vector of 1s of the dimension $m+1$.

When the process is out of control, the region Z_t is divided into Z_{t1} and Z_{t2}, Z_{t1} is one-dimensional with mean $\mu \neq 0$ and Z_{t2} is $p-1$ dimensional. Hence

$$k_t = \|Z_t\| = \sqrt{Z_{t1}^2 + Z_{t2}' Z_{t2}} \quad (2.11)$$

Using the same procedure as in the in-control state, the transition probability of Z_{t1} from stated j to i denoted by $p(j.i)$ is given as

$$P(j,i) = p\left(Z_{1t}\ in\ state\ i \middle| Z_{t-1,1} is\ state\ j\right) \quad (2.12)$$

$$= p\{(-ucL + (i-1)g_1 < \lambda\chi_{t1} + (1-\lambda)Z_{t-1,1} < -ucL + ig_i | Z_{t-1,1} = C_j\} \quad (2.13)$$

where $C_j = -ucL + (j-0.5)g_1$ and the transition probability of Z_{t2} from j to k can be denoted by $V(j,k)$

$$V(j,k) = P\left[\frac{(k-0.5)^2 g_2^2}{\lambda^2} - \chi^2(p-1,c) < \frac{(k+0.5)^2 g_2^2}{\lambda^2}\right] \quad (2.14)$$

when $C = \left((1-\lambda) i g_2 \middle| r\right)^2$ and when k = 0

$$V(j,k) = P\left[\chi^2(p-1,c) < \frac{(0.5)^2 g_2^2}{\lambda^2}\right] \quad (2.15)$$

The distribution of control chart for in-control and out –of – control situation in practical cases using the Markov chain model was studied in this paper, the process mean is assumed to be in-control until time t-1 with a transition matrix π_0 and change at the transition time t with a new transition matrix π_1 afterward.

The moments of the MEWMA control charts and the procedure of identifying the optimal smoothing parameter values for the out – of – control situations were also given.

3.0 USING AN EWMA CHART IN THE MONITORING OF MULTIVARIATE OBSERAVATIONS

Given that X_t follow an independent identically normally distributed multivariate random variable with mean μ_0 and variance σ_0, Tracy el at (1992) showed that the statistic

$$T_t^2 = (X_t - \mu)' \sigma_0^{-1} (X_t - \mu) \quad (3.1)$$

follows a chi-square distribution with p degrees of freedom. If the population parameters, μ_0 and σ_0 are both unknown and are estimated from the sample, then the statistic T_t^2 is defined as

$$T_t^2 = (X_t - \bar{X}_m)' S_m^{-1} (X_t - \bar{X}_m) \quad (3.2)$$

where the estimates $\bar{X}_m$ and S_m are sample mean and the sample variance of the process respectively and its exact distribution is given as

$$T_t^2 \sim \frac{p(m+1)(m-1)}{m(m-p)} F_{p,m-p} \quad (3.3)$$

m is the number of observations in a preliminary data set assumed to represent a stable and present situation. X_t are the p-dimensional vector of future observations on the p quality characteristics.

Shiryayev (1963) proved that if T_t^2 statistic in equation (3.1) follows a chi-square distribution with p degrees of freedom, then $H_p(T_t^2)$ has a uniform distribution on the unit interval, where $H_p(.)$ is the chi-square distribution function with p degree of freedom. Secondly If $\Phi^{-1}(.)$ denotes the inverse of the standard normal distribution function, then

$$V_t = \Phi^{-1}\left[H_p\left(T_t^2\right)\right], t = 1,2,..., \quad (3.4)$$

are standard normal distributed random variables. Thus, for equation (3.2), where the parameters μ_0 and σ_0 are both unknown and are estimated from the sample,

$$V_t = \Phi^{-1}\left[F_{p,m-p}\left\{\frac{m(m-p)}{p(m+1)(m-1)} T_t^2\right\}\right] t = 1,2,..., \quad (3.5)$$

are also standard normal distributed random variables and $F_{p,m-p}(.)$ represents the Snedecor F distribution function with $(p, m-p)$ degree of freedom.

The monitoring of a multivariate process using a EWMA chart can now be done easily since the V_t statistics in equations (3.4) and (3.5) are all standard normal variables. Michael Khoo (2004) asserted that this procedure can be used to monitor the V_t statistics for out-of-control signals since a shift in a multivariate mean vector from the target value, μ_0, will cause the V_t statistics to shift. In a multivariate process monitoring, the performance of the control charts such as the Hotelling or MEWMA charts is determined mainly by the distance of the off-target mean vector from the on-target mean vector and not by the particular direction of the shift. Here, the distance of the shift is measured by the square-root of the non-centrality parameter given below in equation (3.6).

$$\delta^2 = (\mu_0 - \mu_1)' \sigma_0^{-1} (\mu_0 - \mu_1) \quad (3.6)$$

where μ_0 and μ_1 represent the on-target and off-target mean vectors respectively, See Edokpa and Iduseri (2011). Due to the directional invariance property of the T_t^2 statistics in equations (3.1) and (3.2), the new EWMA chart has only an upper control limit since we are actually monitoring the significance of the magnitude of the shift from μ_0 to μ_1.

4.0 THE MOMENTS FOR IN – CONTROL AND OUT – OF – CONTROL ARL OF MEWMA CHART.

Let assume that E(x) = μ_0 (In - control mean of the process with a transition matrix π_0 and $E(X) = \mu_1$ (out of control mean of the process with a transition matrix π_1) where π_0 and π_1 are defined as $(m_1 + 1) \times (m_1 + 1)$ transition matrix of the markov chain when the process is In-control and when the process is out–of–control respectively. Runger and Prabhu (1996) stated that the markov chain method for the MEWMA control chart lead to

$$P(X > x) = {}_{M1\to\infty}^{Lim}\ a^1 \pi_o^n 1, x = 0,1,2 ... \quad (4.1)$$

X is the run length (RL) of the scheme, **a** is the $m_1 \times 1$ starting probability vector with a one in the component that corresponds to the starting state and zero elsewhere, π_0 is the

$(m_1+1)\times(m_1+1)$ transition matrix for the markov chain and 1 is the vector of 1s of the dimension (m_1+1). The probability mass function (pmf) of the RL of X is given by

$$P(X=x)=P(X>x-1)-P(X>x) \quad (4.2)$$
$$=a^1\,\pi_0^{n-1}1-a^1\,\pi_0^n 1$$
$$=a^1\,\pi_0^{n-1}(1-\pi_0)1, x=1,2\ldots \quad (4.3)$$

Where $\pi_0^0=I$ and I is the $(m_1+1)\times(m_1+1)$ identity matrix. The In-Control ARL_0 of the MEWNA is given by

$$ARL_0 = E(X) = \sum_{n=1}^{\infty} xf(x)$$
$$= 1f(1) + 2f(2) + 3f(3) + \ldots$$
$$= f(1) + (2) + f(3) + \ldots] + f(2) + f(3) + f(4) + \ldots] + f(3) + f(4) + \ldots$$
$$= \sum_{x=1}^{\infty} f(n) + \sum_{x=2}^{\infty} f(x) + \sum_{x=1}^{\infty} f(x)$$
$$= P(X\geq 1) + P(X\geq 2 + P(X\geq 3) + \cdots \quad (4.4)$$

Therefore

$$\sum_{x=1}^{\infty} P\,(X\geq n)=\sum_{x=1}^{\infty} f(x)a^1\,\pi_0^{n-1} \quad (4.5)$$
$$=a^1\left(\sum_{i=0}^{n}\pi_0\right)1$$
$$=a^1(1+\pi_0+\ \pi_0^2+\cdots)1$$
$$=a^1(1-\pi_0)^{-1}1 \quad (4.6)$$

The variance of the run length (RL) when the process is running on target (In-control) denoted by VRLo is given as

$$VRLo = Var(X) = EX^2 - (EX)^2 \quad (4.7)$$

But $$EX^2= 2a^1\pi_0(1-\pi_0)^{-2}a^1(1-\pi) \quad (4.8)$$

Hence, $VRLo = 2a^1\pi_0\,(1-\pi_0)1+\ a^1(1-\pi_0)^{-1}\,1[1-a^1(1-\pi_0)^{-1}\,1]$ (4.9)

The out-of-control case is when the process goes out –of – control from μ_0 to μ_1 at the time t=r and the changes is sustained. That is

$$\mu=\begin{cases}\mu_{0,}\ t=1,2\ldots r-1\\ \pi_1,\ t=r,r+1,r+2\ldots\ldots\end{cases} \quad (4.10)$$

and $X_i\sim N_p(\mu_{0\ ,}\ S_z)$ for $i=1,2,3,\ldots r-1$ and $X_i\sim N_p(\mu_{1\ ,}S_z)$ for $i=r+1,r+2,\ldots$(see Schaffer and Vandenhul, 2005)

And the transition matrix π changes accordingly to

$$\pi=\begin{cases}\pi_{0,}\ t=1,2,\ldots,r-1\\ \pi_1,\ t=r,\ \ r+1,\ldots\ldots\ldots\end{cases} \quad (4.11)$$

In this case, the survivor distribution of the RLX is given as

$$P(X>x)=\begin{cases}a^1\ \ \pi_0^n\ 1, x=1,2,\ldots,r-1\\ a^1\pi_0^{r-1}\pi_1^{x-r+1}1, n=r,r+1\end{cases} \quad (4.12)$$

then the general form of the pmf of the run length of X is given by

$$P(X>x)=\begin{cases}a\pi_0^{n-1}\,(1-\pi_0)1\ x=1,2,\ldots,r-1\\ a^1\pi_0^{r-1}(1-\pi_1)1, x=r\\ a^1\pi_0^{n-1}\,(1-\pi_1 1, x=r+1,r+2\end{cases} \quad (4.13)$$

Proof From equation (4.2), the pmf for X for x= 1,2,…r-1 is given as

$$P(X=x)=P(X>x-1)-P(X>x)$$
$$=a^1\pi_0^{n-1}1-a^1\,\pi_0^n\,1$$
$$=a^1\pi_0^{n-1}(1-\pi_0)1 \quad (4.14)$$

For n=r

$$P(X=r)=P(X>r-1)-P(X>r)$$
$$=a^1\pi_0^{r-1}1,-a^1\,\pi_0^{r-1}\,\pi_1 1$$
$$=a^1\pi_0^{r-1}\,(1-\pi_1)1 \quad (4.15)$$

For n= $r+1, r+2\ldots$

$$P(X=x)=P(X>x-1)-P(X>x)$$
$$=a^1\pi_0^{r-1}\pi^{n-r}\,1,-a^1\,\pi_0^{r-1}\,\pi_1^{r-r+1}1$$
$$=a^1\pi_0^{r-1}\pi_0^{n-r}\,(1-\pi_1)1 \quad (4.16)$$

If $r=\infty$, P(X = x) = $a^1\pi_0^{x-1}(1-\pi_0)1$ (4.17)

and if $r=1$, P(X = x) = $a^1\pi_0^{x-1}(1-\pi_1)1$ (4.18)

Equations (4.15) - (4.18) are consistent with problem of Runger and Prabhu (1996).

It can be shown likewise that the out – of –control ARL_1 for the MEWMA is given by.

$$\{a^1(I-\pi_0^{r-1})(I-\pi_0)\,1+\pi_0^{r-1}\}1 \quad (4.19)$$

Proof Using the law of total probability

$$E(X)=\ E[(X|X<r).P(X<x)+E[(X|X\geq r).P(X\geq x) \quad (4.20)$$

But $P(X\geq r)=P(X>r-1)=a^1\pi_0^{r-1}$ (4.21)

$and\ P(X<r)+P(X\geq r)=1$ (4.22)

$which\ implies\ P(X<r)=1-a^1\pi_0^{r-1}1$ (4.23)

$P(X=x|X<r=\frac{P(X=x)}{P(X<r)},\quad x=1,2,\ldots r-1$ if and only if $P(X<r)\ is\ non\ negative$

Therefore $P(X=x|X<r)=\frac{P(X=x)}{P(X<r)}=\frac{a^1\left(I+(r-1)\pi_0^r-r\pi_0^{r-1}\right)(1-\pi_0)1}{1-a^1\pi_0^{r-1}\,1}$ (4.24)

Hence E(X/X<r). P(X/r) = $a^1[I+(r-1)\pi_0^r-\ r\,\pi_0^{r-1}](1-\pi_0)^{-1}1$ (4.25)

$$VRL_1 = Var(N)=E[N(N-1)]-E(N)-E(N)^2$$

$$=2a'\pi_1(1-\pi_1)^{-2}1+a'(1-\pi_1)^{-1}1-[a'(1-\pi_1)^{-1}1]^2 \quad (4.26)$$

5.0 The Computation of the Optimal Average Run Length

Lowry et. al (1992) asserted that when a shift has taken place in the process mean, it is very important to detect the occurrence of the change quickly. Hence in the MEWMA control charts, smaller values of λ are more effective in detecting small shifts in the mean. In this work, the optimal smoothing parameter λ is the minimum smoothing parameter associated with ARL for a given ARL_0. This smoothing parameter which corresponding to the minimum ARL_1 for a given ARL_0 was obtained by the modified markov chain algorithm and was compared on with the lck and Aguilera (2010) for selected values of ARL_0. In this study, we obtained the mid point CL_{mid} between the Lower Control Limit LCL and the Upper Control Limit UCL of the MEWMA control chart such that for a given ARL_0, $ARL_{LCL}\leq ARL_0$ and $ARL_{UCL}\geq ARL_0$. The ARL of the MEWMA chart can therefore be calculated using the Markov Chain algorithms. Once CL_{mid}, the mid-point of the two control limits is obtained. If the absolute difference between ARL_0 and the ARL_{mid} is sticky less than 0.001, the procedure terminates and ARL_{mid} is considered as the optimal otherwise the procedure continue until a sought pair is found. The values of the ARL_{mid} can be calculated for a given magnitude of the shift σ, the smoothing parameter λ. Table 1 shows the optimal ARL when p = 2, with the correlation coefficient $\rho = 0.5$ and the number of states $m_1 = m_2 = 25$, Table 2 shows the optimal ARL when p =4, with the correlation coefficient $\rho = 0.5$ and the number of states $m_1 = m_2 = 25$ and Table 3 shows the optimal ARL when p =10, with the correlation coefficient $\rho = 0.5$ and the number of states $m_1 = m_2 = 25$.

Table 1: Optimal ARL when p = 2 and the number of states $m_1 = m_2 = 25, \rho = 0.5$

σ	ARL_0	200	500	700	900
0.5	λ	0.05	0.04	0.04	0.04
	h	7.38	9.24	10.08	10.70
	ARL	26.75	35.07	38.24	40.79
	ARL_{opt}	26.65	34.95	27.25	40.55
1.0	λ	6.14	0.12	0.11	0.11
	h	9.16	11.05	11.70	12.27
	ARL	9.99	12.17	12.99	13.61
	ARL_{opt}	9.91	11.95	12.55	13.06
1.5	λ	0.25	0.22	0.20	0.20
	h	9.88	11.75	12.39	12.93
	ARL	5.44	6.43	6.79	7.07
	ARL_{opt}	5.31	6.41	6.72	6.95
2.0	λ	0.38	0.32	0.31	0.30
	h	10.25	12.06	12.75	12.25
	ARL	3.53	4.09	4.29	4.45
	ARL_{opt}	3.33	3.93	4.12	4.15
2.5	λ	0.53	0.45	0.43	0.41
	h	10.43	12.25	12.92	13.42
	ARL	2.52	2.89	3.03	3.13
	ARL_{opt}	2.26	2.67	2.95	3.10
3.0	λ	0.68	0.61	0.58	0.56
	h	10.52	12.34	13.01	13.51
	ARL	1.88	2.17	2.27	2.35
	ARL_{opt}	1.72	2.10	2.14	2.26

Table 2: Optimal ARL when p = 4 and the number of states $m_2 = m_2 = 25, \rho = 0.5$

σ	ARL_0	200	500	800	900
0.5	λ	0.05	0.04	0.03	0.03
	h	11.27	13.50	14.26	14.60
	ARL	32.44	42.63	48.16	49.48
	ARL_{opt}	32.30	41.96	47.93	49.27
1.0	λ	0.13	0.11	0.10	0.09
	h	13.20	15.36	16.43	16.58
	ARL	12.06	14.64	15.96	16.29
	ARL_{opt}	11.92	14.32	15.86	16.02
1.5	λ	0.22	0.19	0.18	0.17
	h	13.97	16.09	17.18	17.40
	ARL	6.51	7.64	8.23	8.37
	ARL_{opt}	6.42	7.59	8.01	8.21
2.0	λ	0.33	0.29	0.27	0.26
	h	14.39	16.49	17.54	17.82
	ARL	4.18	4.82	5.14	5.23
	ARL_{opt}	4.12	4.73	4.97	5.01
2.5	λ	0.45	0.39	0.37	0.37
	H	14.61	16.68	17.74	18.01
	ARL	2.97	3.38	3.59	3.64
	ARL_{opt}	2.92	3.19	3.49	3.59
3.0	λ	0.61	0.52	0.49	0.48
	h	14.74	16.79	17.84	18.10
	ARL	2.23	2.56	2.70	2.73
	ARL_{opt}	2.13	2.51	2.59	2.62

Table 3: Optimal ARL when p = 10 and the number of states $m_2 = m_2 = 25, \rho = 0.5$

σ	ARL_0	200	500	800	900
0.5	λ	0.04	0.03	0.03	0.03
	h	20.09	22.98	24.80	25.24
	ARL	42.62	56.65	64.23	66.15
	ARL_{opt}	42.40	56.46	63.90	65.54
1.0	λ	0.11	0.09	0.08	0.08
	h	22.88	25.59	26.90	27.28
	ARL	15.95	19.34	21.04	21.47
	ARL_{opt}	15.45	19.04	20.82	21.07
1.5	λ	0.19	0.16	0.15	0.14
	h	23.92	26.55	27.89	28.15
	ARL	8.54	9.99	10.72	10.90
	ARL_{opt}	8.10	9.50	10,40	10,50
2.0	λ	0.28	0.24	0.23	0.22
	h	24.45	27.04	28.37	28.67
	ARL	5.43	6.23	6.63	6.73
	ARL_{opt}	5.10	5.92	6.20	6.30
2.5	λ	0.28	0.33	0.31	0.31
	H	24.75	27.31	28.60	28.93
	ARL	3.82	4.33	4.58	4.65
	ARL_{opt}	3.52	4.19	4.30	4.49
3.0	λ	0.48	0.42	0.40	0.40
	h	24.90	27.45	28.74	29.05
	ARL	2.88	3.23	3.40	3.44
	ARL_{opt}	2.63	2.91	3.19	3.32

CONCLUSION

From our study, it was observed that the MEWMA scheme is quite adequate for detecting a small shift and a good way to improve the quality of goods and services in a multivariate situation. The derivation of the moments for the optimal run length distribution was obtained and the moments were consistent with the Runger and Prabhu (1996) for the in-control and out-of-control situation. The optimums ARL for the MEWMA control charts were obtained using simulated data. It was observed that as the in-control average run length ARL_0 or the number of variables (p) increases, the optimum value of the ARL_{0pt} increases slowly. Lastly as the magnitude of the shift σ increases, the optimum average run length ARL_{opt} decreases

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