Unique Invariant Point Theorems for Random operators In Hilbert Space

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Abstract

We find unique common random fixed point of two random operators in closed subset of a separable Hilbert space by considering a sequence of measurable functions satisfying Theorem 1.1 and Theorem 1.2.

Keywords: Separable Hilbert space, random operators, common random fixed point.

Definition 1.1: A function $f: \Omega \times C \rightarrow C$ is Said to be random operator, if

F (. , x) : $\Omega \rightarrow C$ is measurable for all $x \epsilon C$.

Definition 1.2: A function $f: \Omega \times C \rightarrow C$ is Said to be measurable, if

$f'(B \cap C) \in \Sigma$ for every Borel subset B of H.

Definition 1.3: A function $f: \Omega \times C \rightarrow C$ is Said to be continuous, if

For fixed $\epsilon \Omega$, f (t , .) : C $\rightarrow C$ is Continuous.

Definition 1.4: A measurable function $g: \Omega \rightarrow C$ is Said to be random fixed point of the random operator $f: \Omega \times C \rightarrow C$, is Continuous.

Main Result

Theorem 1.1: Let C be a non empty subset of Hilbert Space H. Let R and S be continuous random operations defined on C such that for $\xi \epsilon \Omega$,

$T(\xi)$:C$\rightarrow C$ satisfying the condition :

$$\| Rx - Sy \|^2 \leq \alpha \max [\| x - y \|^2, \{\| x - Rx \|^2 + \| y - Sy \|^2\}, \frac{1}{2}$$

$$\{\| x - Sy \|^2 + \| y - Rx \|^2, \frac{\| x - y \|^2 + \| x - Sy \|^2 + \| x - Rx \|^2}{1 + \| x - y \| \| x - Sy \| \| x - Rx \|}]$$

For $0 \leq \alpha \leq \frac{1}{2}$ and.

Then the sequence $\{g_n\}$ converges to the unique common random fixed point of R and S.

Proof: Let $\{g_n\}$ be a sequence of function defined as :

$$g_{2n+1}(\xi) = R\left(\xi, g_{2n}(\xi)\right), g_{2n+2}(\xi) = S\left(\xi, g_{2n+1}(\xi)\right),$$

For $\xi \epsilon\ \Omega$ and n = 0,1,2,3......

$$\| g_{2n+1}(\xi) - g_{2n}(\xi) \|^2 = \| R\left(\xi, g_{2n}(\xi)\right) - S(\xi, g_{2n-1}(\xi) \|^2$$

$$\leq \alpha \max [\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{\| g_{2n}(\xi) - R(\xi, g_{2n}(\xi) \|^2 + \| g_{2n-1}(\xi) - S(\xi, g_{2n-1}(\xi) \|^2\},$$

$$\frac{1}{2} \| g_{2n}(\xi) - S(\xi, g_{2n-1}(\xi) \|^2 + \| g_{2n-1}(\xi) - R(\xi, g_{2n}(\xi) \|^2\},$$

$$\frac{\|g_{2n}(\xi) - g_{2n-1}(\xi)\|^2 + \|g_{2n}(\xi) - S(\xi, g_{2n-1}(\xi)\|^2 + \|g_{2n}(\xi) - R(\xi, g_{2n}(\xi)\|^2}{1 + \|g_{2n}(\xi) - g_{2n-1}(\xi)\| \|g_{2n}(\xi) - S(\xi, g_{2n-1}(\xi)\| \|g_{2n}(\xi) - R(\xi, g_{2n}(\xi)\|}]$$

$$= \alpha \max [\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2$$

$$+ \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2\}, \frac{1}{2}\{\| g_{2n}(\xi) - g_{2n}(\xi) \|^2\},$$

$$\{\| g_{2n-1}(\xi) - g_{2n+1}(\xi) \|^2\},$$

$$\frac{\|g_{2n}(\xi) - g_{2n-1}(\xi)\|^2 + \|g_{2n}(\xi) - g_{2n}(\xi)\|^2 + \|g_{2n}(\xi) - g_{2n+1}(\xi)\|}{1 + \|g_{2n}(\xi) - g_{2n-1}(\xi)\| \|g_{2n}(\xi) - g_{2n}(\xi)\| \|g_{2n}(\xi) - g_{2n+1}(\xi)\|}]$$

$$= \alpha \max [\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2$$

$$+ \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2\}, \frac{1}{2}\{\| g_{2n-1}(\xi) - g_{2n+1}(\xi) \|^2\},$$

$$\{\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2 + \| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2\}]$$

$$=\alpha \max [\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{ \| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2\},$$

$$\frac{1}{2}\{\| g_{2n-1}(\xi) - g_{2n}(\xi) + g_{2n}(\xi) - g_{2n+1}(\xi) \|^2\},$$

$$\{\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2 + \| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2\}]$$

$$=\alpha \max [\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{ \| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2\},$$

$$\frac{1}{2}\{2 \| g_{2n-1}(\xi) - g_{2n}(\xi) + 2 \| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 - \| g_{2n-1}(\xi) - g_{2n}(\xi) - g_{2n}(\xi) + g_{2n+1}(\xi) \|^2\}, \{\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2 + \| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2\}]$$

$\because (\| x+y \|^2 + \| x-y \|^2 = 2 \| x \|^2 + 2 \| y \|^2)$ (By parallelogram)

$$\leq \alpha[\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2$$

$$\therefore \| g_{2n+1}(\xi) - g_{2n}(\xi) \|^2 \leq \frac{\alpha}{1-\alpha} \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2$$

Similarly, $\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2 \leq \frac{\alpha}{1-\alpha} \| g_{2n-2}(\xi) - g_{2n-1}(\xi) \|^2$

Hence, Proceeding in the same manner , we get

$$\| g_n(\xi) - g_{n+1}(\xi) \|^2 \leq \frac{\alpha}{1-\alpha} \| g_{n-1}(\xi) - g_n(\xi) \|^2$$

Since $0 \leq \frac{\alpha}{1-\alpha} \leq 1$, therefore $\{g_n(\xi)\}$ be a Cauchy sequence and hence $\{g_n\}$ converges in H.

As $n \to \infty, g_n(\xi) \to g(\xi)$

Since C is closed and $g: C \to C$ be a function.

For all $\xi \epsilon\ \Omega$.

$\| g(\xi) - S(\xi, g(\xi)) \|^2 = \| g(\xi) - g_{2n+1}(\xi) + g_{2n+1}(\xi) - S(\xi, g(\xi)) \|^2$

$= 2\| g(\xi) - g_{2n+1}(\xi) \|^2 + 2\| g_{2n+1}(\xi) - S(\xi, g(\xi)) \|^2 -$

$$\| g(\xi) - g_{2n+1}(\xi) - g_{2n+1}(\xi) + S(\xi, g(\xi)) \|^2$$

$$\leq 2 \| g(\xi) - g_{2n+1}(\xi) \|^2 + 2 \| g_{2n+1}(\xi) - S(\xi, g(\xi)) \|^2$$

$= 2\| g(\xi) - g_{2n+1}(\xi) \|^2 + 2 \| R(\xi, g_{2n}(\xi)) - S(\xi, g(\xi)) \|^2$

$= 2\| g(\xi) - g_{2n+1}(\xi) \|^2 + 2\alpha\, max\, [\| g_{2n}(\xi) - g(\xi) \|^2,$

$\{ \| g_{2n}(\xi) - R(\xi, g_{2n}(\xi) \|^2 + \| g(\xi) - S(\xi, g(\xi)) \|^2\},$

$\frac{1}{2}\{\| g_{2n}(\xi) - S(\xi, g(\xi) \|^2 + \| g(\xi) - R(\xi, g_{2n}(\xi) \|^2\},$

$$\frac{\| g_{2n}(\xi) - g(\xi) \|^2 + \| g_{2n}(\xi) - S(\xi, g(\xi)) \|^2 + \| g_{2n}(\xi) - R(\xi, g_{2n}(\xi) \|^2}{1 + \| g_{2n}(\xi) - g(\xi) \| \| g_{2n}(\xi) - S(\xi, g(\xi)) \| \| g_{2n}(\xi) - R(\xi, g_{2n}(\xi) \|}]$$

$= 2\| g(\xi) - g_{2n+1}(\xi) \|^2 + 2\alpha\, max\, [\| g_{2n}(\xi) - g(\xi) \|^2,$

$\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g(\xi) - S(\xi, g(\xi)) \|^2\}$

$\frac{1}{2}\{ \| g_{2n}(\xi) - S(\xi, g(\xi)) \|^2 + \| g(\xi) - g_{2n+1}(\xi) \|^2\},$

$$\frac{\|g_{2n}(\xi) - g(\xi)\|^2 + \|g_{2n}(\xi) - S(\xi, g(\xi))\|^2 + \|g_{2n}(\xi) - g_{2n+1}(\xi)\|^2}{1 + \|g_{2n}(\xi) - g(\xi)\| \|g_{2n}(\xi) - S(\xi, g(\xi))\| \|g_{2n}(\xi) - g_{2n+1}(\xi)\|}]$$

Making $n \to \infty$, we have

$$\| g(\xi) - S(\xi, g(\xi)) \|^2 \leq 2\alpha \| g(\xi) - S(\xi, g(\xi)) \|^2$$

Since $0 \leq \alpha \leq \frac{1}{2}$, so $\forall\, \xi \epsilon\, \Omega$

We say that $S(\xi, g(\xi)) = g(\xi)$

Similarly, we can prove that $R(\xi, g(\xi)) = g(\xi)$.

Again, if $f: \Omega \times C \to C$ is a continuous random operation on a non empty subset C of a separated Hilbert space H, then for any measurable function $F: \Omega \to C$, the function $F(\xi, g(\xi)) = g(\xi)$ is also measurable .

Therefore the sequence of measurable function $\{g_n\}$ converges to measurable function with $R\big(\xi, g(\xi)\big) = g(\xi) = S\big(\xi, g(\xi)\big)$ which shows that $g: \Omega \to C$ is common random fixed point of R and S.

Uniqueness: Let h: $\Omega \to C$ be another common random fixed point of R and S. Then:

$$\| g(\xi) - \mathrm{h}(\xi) \|^2 = \| R(\xi, g(\xi)) - S\big(\xi, \mathrm{h}(\xi)\big) \|^2$$

$$\leq \alpha\, max[\| g(\xi) - \mathrm{h}(\xi) \|^2, \| g(\xi) - R\big(\xi, g(\xi)\big) \|^2$$

$$+\| h(\xi) - S\big(\xi, \mathrm{h}(\xi)\big) \|^2\}, \tfrac{1}{2}\{ \| g(\xi) - S\big(\xi, \mathrm{h}(\xi)\big) \|^2 + \| \mathrm{h}(\xi) - R\big(\xi, g(\xi)\big) \|^2\}$$

$$, \frac{\|g(\xi)-\mathrm{h}(\xi)\|^2 + \|g(\xi)-S\big(\xi,\mathrm{h}(\xi)\big)\|^2 + \|g(\xi)-R\big(\xi,g(\xi)\big)\|^2}{1+\|g(\xi)-\mathrm{h}(\xi)\|\|g(\xi)-S\big(\xi,\mathrm{h}(\xi)\big)\|\|g(\xi)-R\big(\xi,g(\xi)\big)\|}]$$

$$= \alpha\, max\, [\| g(\xi) - \mathrm{h}(\xi) \|^2, \| g(\xi) - \mathrm{h}(\xi) \|^2, 2 \| g(\xi) - \mathrm{h}(\xi) \|^2]$$

$$= 2\, \alpha \| g(\xi) - \mathrm{h}(\xi) \|^2$$

Which is a contradiction because :

$$\alpha \leq \frac{1}{2}$$

This shows that:

$$g(\xi) = \mathrm{h}(\xi)$$

Hence R and S have a unique common random fixed Point.

Theorem 1.2: Let C be a non-empty subset of Hilbert space H. Let R and S be continuous random operations defined on C such that for

$\xi \in \Omega$,$T\ (\xi)$:C→ C satisfying the condition.

$$\| Rx - Sy \|^2 = \alpha \| x - \mathrm{y} \|^2 + \beta \{\| x - Rx \|^2 + \| y - Sy \|^2\} +$$

$$\frac{\gamma}{2} \| x - Sy \|^2 + \| y - Rx \|^2\} +$$

$$\delta\{\frac{\|x-y\|^2+\|x-Sy\|^2+\|x-Rx\|^2}{1+\|x-\mathrm{y}\|\|x-Sy\|\|x-R\mathrm{x}\|}\} + \frac{\eta}{2}\{\frac{\|x-Rx\|^2+\|y-Sy\|^2+\|x-Sy\|^2}{1+\|y-R\mathrm{x}\|\|y-Sy\|\|x-Sy\|}\}$$

$$+\rho\{\frac{\|x-Rx\|^2+\|y-Rx\|^2+\|y-Sy\|\|x-Sy\|}{1+\|x-Rx\|\|x-Sy\|+\|y-Rx\|\|x-Sy\|}\}$$

For

$$\alpha + 2(\beta+\gamma+\delta) + 5(\frac{\eta}{2}+ \rho) < 1$$

And $\alpha,\beta,\gamma,\delta,\eta,\rho > 0$

Then $\{g_n\}$ $c$onverges to the unique common random fixed point R and S.

Proof: Let $\{g_n\}$ be a sequence of the function defined for $\xi \in \Omega$ and n=0,1,2,3.........

$g_{2n+1}(\xi) = R(\xi, g_{2n}(\xi))$,

$g_{2n+2}(\xi) = S(\xi, g_{2n+1}(\xi))$

$$\| g_{2n+1}(\xi) - g_{2n}(\xi) \|^2 = \| R(\xi, g_{2n}(\xi)) - S(\xi, g_{2n-1}(\xi)) \|^2$$

$$\leq \alpha\, [\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2 + \beta \| g_{2n}(\xi) - Rg_{2n}(\xi) \|^2$$

$$+\| g_{2n-1}(\xi) - Sg_{2n-1}(\xi) \|^2 + \frac{\gamma}{2}\{\| g_{2n}(\xi) - Sg_{2n-1}(\xi) \|^2$$

$$+\| g_{2n-1}(\xi) - Rg_{2n}(\xi) \|^2\}$$

$$+\delta\{\frac{\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2+\|g_{2n}(\xi)-Sg_{2n-1}(\xi)\|^2+\|g_{2n}(\xi)-Rg_{2n}(\xi)\|^2}{1+\|g_{2n}(\xi)-g_{2n-1}(\xi)\|+\|g_{2n}(\xi)-Sg_{2n-1}(\xi)\|\|g_{2n}(\xi)-Rg_{2n}(\xi)\|}\}$$

$$+\frac{\eta}{2}\{\frac{\|g_{2n-1}(\xi)-Rg_{2n}(\xi)\|^2+\|g_{2n-1}(\xi)-Sg_{2n-1}(\xi)\|^2+\|g_{2n}(\xi)-Sg_{2n-1}(\xi)\|^2}{1+\|g_{2n-1}(\xi)-Rg_{2n}(\xi)\|\|g_{2n-1}(\xi)-Sg_{2n-1}(\xi)\|\|g_{2n}(\xi)-Sg_{2n-1}(\xi)\|}\}+$$

$$\rho\{\frac{\|g_{2n}(\xi)-Rg_{2n}(\xi)\|^2+\|g_{2n-1}(\xi)-Rg_{2n}(\xi)\|^2+\|g_{2n-1}(\xi)-Sg_{2n-1}(\xi)\|^2\|g_{2n}(\xi)-Sg_{2n-1}(\xi)\|^2}{1+\|g_{2n}(\xi)-Rg_{2n}(\xi)\|+\|g_{2n}(\xi)-Sg_{2n-1}(\xi)\|+\|g_{2n-1}(\xi)-Rg_{2n}(\xi)\|\|g_{2n}(\xi)-Sg_{2n-1}(\xi)\|}\}$$

$$=\alpha \| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2 + \beta\{\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2\} + \frac{\gamma}{2}\{\| g_{2n}(\xi) - g_{2n}(\xi) \|^2 + \| g_{2n-1}(\xi) g_{2n+1}(\xi) \|^2$$

$$+\delta\{\frac{\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2+\|g_{2n}(\xi)-g_{2n}(\xi)\|^2+\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2}{1+\|g_{2n}(\xi)-g_{2n-1}(\xi)\|\|g_{2n}(\xi)-g_{2n}(\xi)\|\|g_{2n}(\xi)-g_{2n+1}(\xi)\|}\}$$

$$+\frac{\eta}{2}\{\frac{\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|^2+\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2+\|g_{2n}(\xi)-g_{2n}(\xi)\|^2}{1+\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|+\|g_{2n-1}(\xi)-g_{2n}(\xi)\|\|g_{2n}(\xi)-g_{2n}(\xi)\|}\}$$

$+\rho\{\frac{\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2+\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|^2+\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2\|g_{2n}(\xi)-g_{2n}(\xi)\|^2}{1+\|g_{2n}(\xi)-g_{2n+1}(\xi)\|\|g_{2n}(\xi)-g_{2n}(\xi)\|+\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|\|g_{2n}(\xi)-g_{2n}(\xi)\|}\}$

$=\alpha \| g_{2n}(\xi)-g_{2n-1}(\xi) \|^2+\beta\{\| g_{2n}(\xi)-g_{2n+1}(\xi) \|^2+\| g_{2n-1}(\xi)-g_{2n}(\xi) \|^2\}+\frac{\gamma}{2}\{\| g_{2n-1}(\xi)-g_{2n+1}(\xi) \|^2+\delta \| g_{2n}(\xi)-g_{2n-1}(\xi) \|^2$

$+\| g_{2n}(\xi)-g_{2n+1}(\xi) \|^2\}+\frac{\eta}{2}\{\| g_{2n-1}(\xi)-g_{2n+1}(\xi) \|^2$

$+\| g_{2n-1}(\xi)-g_{2n}(\xi) \|^2\}+\rho \| g_{2n}(\xi)-g_{2n+1}(\xi) \|^2+\| g_{2n-1}(\xi)-g_{2n+1}(\xi) \|^2\}$

$=(\alpha+\beta+\delta+\frac{\eta}{2}) \| g_{2n}(\xi)-g_{2n-1}(\xi) \|^2+(\beta+\delta+\rho)$

$$\| g_{2n}(\xi)-g_{2n+1}(\xi) \|^2+(\frac{\gamma}{2}+\frac{\eta}{2}+\rho) \| g_{2n-1}(\xi)-g_{2n+1}(\xi) \|^2$$

NOW,

$\| g_{2n-1}(\xi)-g_{2n+1}(\xi) \|^2=[\| g_{2n-1}(\xi)-g_{2n}(\xi) \|^2]+$

$[\| g_{2n}(\xi)-g_{2n+1}(\xi) \|^2]$ ....(1.2.1)

By using parallelogram, we have **:**

$\| g_{2n-1}(\xi)-g_{2n+1}(\xi) \|^2=2\| g_{2n-1}(\xi)-g_{2n}(\xi) \|^2+$

$2\| g_{2n}(\xi)-g_{2n+1}(\xi) \|^2 -\| [g_{2n-1}(\xi)-g_{2n}(\xi)]-[g_{2n}(\xi)-g_{2n+1}(\xi)] \|^2$

$$\leq 2 \| g_{2n-1}(\xi)-g_{2n}(\xi) \|^2+2 \| g_{2n}(\xi)-g_{2n+1}(\xi) \|^2$$

Therefore by equation (1.2.1), we have:

$\| g_{2n+1}(\xi)-g_{2n}(\xi) \|^2\leq(\alpha+\beta+\delta+\frac{\eta}{2}) \| g_{2n}(\xi)-g_{2n-1}(\xi) \|^2+$

$$(\beta+\delta+\rho) \| g_{2n}(\xi)-g_{2n+1}(\xi) \|^2$$

$+(\frac{\alpha}{2}+\frac{\eta}{2}+\rho)[2 \| g_{2n-1}(\xi)-g_{2n}(\xi) \|^2+2 \| g_{2n}(\xi)-g_{2n+1}(\xi) \|^2]$

$$\| g_{2n+1}(\xi)-g_{2n}(\xi) \|^2\leq K \| g_{2n}(\xi)-g_{2n-1}(\xi) \|^2$$

Where:

$K=\frac{\alpha+\beta+\gamma+\delta+3\frac{\eta}{2}+2\rho}{1-(\beta+\gamma+\delta+\eta+3\delta)}<1,$

Similarly, $\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2 \leq \| g_{2n-1}(\xi) - g_{2n-2}(\xi) \|^2$

Proceeding in the same manner, we have:

$\| g_n(\xi) - g_{n+1}(\xi) \|^2 \leq K \ \| g_{n-1}(\xi) - g_n(\xi) \|^2 , \forall \xi \in \Omega$

Therefore $g_n(\xi)$ is a Cauchy sequence and hence it is convergent in the Hilbert spaces H.

As $n \to \infty$, $g_n(\xi) \to g(\xi)$. Since C is closed and g:C→ C be a function for all $\xi \in \Omega$.

$$\| g(\xi) - S(\xi, g(\xi)) \|^2 = \| g(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n+1}(\xi) - S(\xi, g(\xi)) \|^2$$

$$\leq 2 \| g(\xi) - g_{2n+1}(\xi) \|^2 + 2 \| g_{2n+1}(\xi) - S(\xi, g(\xi)) \|^2, \quad \text{By parallelogram}$$

$$= 2\| g(\xi) - g_{2n+1}(\xi) \|^2 + 2 \| R(\xi, g_{2n}(\xi) - S(\xi, g(\xi)) \|^2$$

$$= 2 \| g(\xi) - g_{2n+1}(\xi) \|^2 + 2[\alpha \| g_{2n}(\xi) - g(\xi) \|^2 +$$

$$\beta\{\| g_{2n}(\xi) - Rg_{2n}(\xi) \|^2 + \| g(\xi) - Sg(\xi) \|^2\} + \frac{\gamma}{2}\{\| g_{2n}(\xi) - Sg(\xi) \|^2$$

$$+ \| g(\xi) - Rg_{2n}(\xi) \|^2\}$$

$$+\delta\{\frac{\|g_{2n}(\xi)-g(\xi)\|^2+\|g_{2n}(\xi)-Sg(\xi)\|^2+\|g_{2n}(\xi)-Rg_{2n}(\xi)\|^2}{1+\|g_{2n}(\xi)-g(\xi)\|\|g_{2n}(\xi)-Sg(\xi)\|\|g_{2n}(\xi)-Rg(\xi)\|}\}$$

$$+\frac{\eta}{2}\{\frac{\|g(\xi)-Rg_{2n}(\xi)\|^2+\|g(\xi)-Sg(\xi)\|^2+\|g_{2n}(\xi)-Sg(\xi)\|}{1+\|g(\xi)-Rg_{2n}(\xi)\|\|g(\xi)-Sg(\xi)\|\|g_{2n}(\xi)-Sg(\xi)\|}\}+$$

$$\rho\{\frac{\|g_{2n}(\xi)-Rg_{2n}(\xi)\|^2+\|g(\xi)-Rg_{2n}(\xi)\|^2+g(\xi)-Sg(\xi)\|g_{2n}(\xi)-Sg(\xi)\|}{1+\|g_{2n}(\xi)-Rg_{2n}(\xi)\|\|g_{2n}(\xi)-Sg(\xi)\|+\|g(\xi)-Rg_{2n}(\xi)\|\|g_{2n}(\xi)-Sg(\xi)\|}\}]$$

$$= 2\| g(\xi) - g_{2n+1}(\xi) \|^2 + 2[\alpha \| g_{2n}(\xi) - g(\xi) \|^2 +$$
$$\beta\{\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g(\xi) - Sg(\xi) \|^2\} + \frac{\gamma}{2}\{\| g_{2n}(\xi) - Sg(\xi) \|^2 +$$
$$\| g(\xi) - g_{2n+1}(\xi) \|^2\}$$

$$+ \delta\{\frac{\|g_{2n}(\xi)-g(\xi)\|^2+\|g_{2n}(\xi)-Sg(\xi)\|^2+\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2}{1+\|g_{2n}(\xi)-g(\xi)\|\|g_{2n}(\xi)-Sg(\xi)\|\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2}\}$$

$$+\frac{\eta}{2}\{\frac{\|g(\xi)-g_{2n+1}(\xi)\|^2+\|g(\xi)-Sg(\xi)\|^2+\|g_{2n}(\xi)-Sg(\xi)\|^2}{1+\|g(\xi)-g_{2n+1}(\xi)\|\|g(\xi)-Sg(\xi)\|\|g_{2n}(\xi)-Sg(\xi)\|}\}$$

$$+\rho\{\frac{\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2+\|g(\xi)-g_{2n+1}(\xi)\|^2+\|g(\xi)-Sg(\xi)\|^2\|g_{2n}(\xi)-Sg(\xi)\|^2}{1+\|g_{2n}(\xi)-g_{2n+1}(\xi)\|\|g_{2n}(\xi)-Sg(\xi)\|+\|g(\xi)-g_{2n+1}(\xi)\|\|g_{2n}(\xi)-Sg(\xi)\|}\}]$$

Making $n \to \infty, we\ get$

$$\| g(\xi) - S(\xi, g(\xi)) \|^2 \leq (2\beta + \gamma + 2\delta + 2\eta + 4\rho) \| g(\xi) - S(\xi, g(\xi)) \|^2$$

Since $(2\beta + \gamma + 2\delta + 2\eta + 4\rho) < 1$

Therefore for $\xi \epsilon \Omega$, we say that $S\big(\xi, g(\xi)\big) = g(\xi)$.

Similarly we can prove that $R\big(\xi, g(\xi)\big) = g(\xi)$

Again if F:$\Omega \to C \to C$ is a continuous random operations on a non empty sabset C of a separated Hilbert space H . Then for any measurable function

F:$\Omega \to C$ the function $F\big(\xi, g(\xi)\big) = g(\xi)$ is also measurable.

Therefore the sequence of measurable function $\{g_n\}$ converges to measurable

Function with $R\big(\xi, g(\xi)\big) = g(\xi) = S\big(\xi, g(\xi)\big)$,

Which shows that g:$\Omega \to C$ is common random fixed point of R and S.

Uniqueness: Let h: $\Omega \to C$ be another common random fixed point of R and S. Then:

$$\| g(\xi) - h(\xi) \|^2 = \| R(\xi, g(\xi)) - S(\xi, h(\xi)) \|^2$$

$$\leq \alpha \| g(\xi) - h(\xi) \|^2 + \beta\{\| g(\xi) - Rg(\xi) \|^2 + \| h(\xi) - Sh(\xi) \|^2\}$$

$$+\frac{\gamma}{2}\{\| g(\xi) - Sh(\xi) \|^2 + \| h(\xi) - Rg(\xi) \|^2\}$$

$$+\delta\{\frac{\|g(\xi)-h(\xi)\|^2+\|g(\xi)-Sh(\xi)\|^2+\|g(\xi)-Rg(\xi)\|^2}{1+\|g(\xi)-h(\xi)\|\|g(\xi)-Sh(\xi)\|\|g(\xi)-Rg(\xi)\|^2}\}+$$

$$\frac{\eta}{2}\{\frac{\|h(\xi)-Rg(\xi)\|^2+\|h(\xi)-Sh(\xi)\|^2+\|g(\xi)-Sh(\xi)\|^2}{1+\|h(\xi)-Rg(\xi)\|\|h(\xi)-Sh(\xi)\|\|g(\xi)-Sh(\xi)\|}\}$$

$$+\rho\{\frac{\|g(\xi)-Rg(\xi)\|^2+\|h(\xi)-Rg(\xi)\|^2+\|h(\xi)-Sh(\xi)\|\|g(\xi)-Sh(\xi)\|}{1+\|g(\xi)-Rg(\xi)\|\|g(\xi)-Sh(\xi)\|+\|h(\xi)-Rg(\xi)\|^2\|g(\xi)-Sh(\xi)\|}\}$$

$$=(\alpha + \gamma + 2\delta + \eta + \rho) \| g(\xi) - h(\xi) \|^2$$

Which is a contradiction because:

$\alpha + \gamma + 2\delta + \eta + \rho < 1.$

This shows that $g(\xi) = \mathrm{h}(\xi)$

Hence R and S have a unique common random fixed point.

Corollary: Let C be a non empty subset of Hilbert Space H. Let R and S be continuous random operations defined on C such that for

$\xi \epsilon \Omega$, $\mathrm{T}(\xi): C \to C$ satisfying the condition.

$\| \mathrm{x} - R\mathrm{x} \|^2 + \| \mathrm{y} - S\mathrm{y} \|^2 \leq \alpha \| \mathrm{x} - y \|^2$

For $1 \leq \alpha \leq 2.$

Then the sequence $\{g_n\}$ converges to the unique common random fixed point of R and S.

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