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Counting Dropout Rate of Consumers: A Case Study of Durable Goods.

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Abstract

Customer buying behavior research has been a special research activity in industry due to its economic, social and developmental relevance. However, summary measures such as the total buying behavior per month provides little insight about individual level of shopping behavior. This research work attempts to predict the percentage of customers' churn of less than 10% using the data of some packaged goods that were collected within a specific period. The computational experience provides a string statistical evidence (p-value = 0.9928) for the use of proposed beta-binominal models as an effective statistical tools.

It was estimated that at the end of thirty sixth months, only 25% of the customers is expected to achieve the targeted drop-out rate of less than 10%. In view of the industrial and commercial goal of reducing the rate of customers churn to less than 5%, the predicted percentage seems to be high as regards the packaged goods that were examined.

Keywords: Beta-Binominal Model, Maximum Likelihood Estimation, Dropout -Rate, Consumer Buying -Behavior.

1.0 Introduction

The common objective of science and by business research is to model the rate at which customers drop out during transaction (consumer's churn rate). For example, in marketing research applications, the goal of many concept and product tests is to understand the buying behavior of population of consumers so as to determine whether or not to introduce a product to the market and if so to which market segments. In such concept and product texts, we cannot observe the actual buying of individual data used as proxy variables.

Over the last fifty years, several studies have examined the relationship between buying behavior and the customer's rate of drop out. Nigerian government has been conducting studies and experiment concerning buying behavior data. In many of these studies, significant relationship between buying behavior of durable goods and subsequent purchases were found to be significant in various economic and statistical models used on the data (Akomolafe & Amahia,2009). Based on the studies carried out by these researchers, it was noted that intenders' purchase rates are higher than those that drop-out of the interaction. In fact, for most of the durable goods, majority of households report that consumers do not actually show their intention carry out most of the actual purchases. Several other studies have examined the theory and practical application of their developed models towards the tracking and forecasting of future buying- behavior of consumers .This may be responsible for incorporating the instability in the consumer buying-behavior, drop-out effect as well as measuring the effect of heterogeneity on the purchase behavior (Akomolafe et al. 2010; Peter,2008; Schmittlem & Petterson 1994; Metheringgham,1988; Davis,1989; Ishii & Hayakawa, 1990; Colombo & Weima,1990).

One of the key economic objectives here is to increase consumer buying behavior of goods with average consumer drop out of less than 10%. For this purpose, a Beta-Binomial Model is being employed in this study because of its capability of capturing buying behavior pattern of consumers. The computed predictions are also compared with those obtained on the basis of conventional binomial model.

2.0 Materials and Method

2.1 Data Description

The result reported in this research work is based on secondary data available from wholesales standard outlets of beverages outfit for one hundred and six weeks (three years). These reports have been generated with the help of sales record available in the outlet which showcases the identity of customers as they patronize the shop and eventually buy the products in large quantity.

The customers drop out rate (d_g^1) for customers, g at a point of time t is defined as;

$$(d_g^1) = \frac{number\ of\ customers\ dropping\ out\ from\ the\ transaction, g\ from\ week, t}{total\ number\ of\ customers\ in\ primary, (g)\ in\ week, t} \quad \ldots\ldots(1)$$

The average dropout rate, $\bar{d}$ for d customers level, with g= 6 and at appointed time (t) is given by

$$\bar{d} = \frac{1}{3}\sum_{g=1}^{6}\begin{pmatrix} d & 1 \\ & g \end{pmatrix} \quad \ldots\ldots(2)$$

The average drop-out rates from the four selected beverages (yoghurt, *can* coke, bottled coke and vanilla fan ice) selected for this research are presented in Tables 1 -4

Table 1 : (**YOGHUR**T)

AGENT CODE	2003-2004	2004 -2005	2005 - 2006
A	9	22.3	26.1
B	5.5	11	18.5
C	7.4	2.9	7.3
D	20.2	5.7	0.9
E	30.4	6.9	4.3
F	5.4	13.5	0.1

TABLE 2: (CAN COKE)

AGENT CODE	2003-2004	2004 -2005	2005 -2006
A	6	8.5	2.4
B	6.7	5.7	17.9
C	5.2	8.9	7.4
D	4.7	8.7	1.5
E	13.9	8.8	6.8
F	4	1	1
G	8.3	19.5	0.7
H	2.6	15.7	10.8
I	9.3	13.7	7
J	8.2	12.8	10.6
K	11.2	16	21.3
L	8	18.5	9.6
M	2	3.3	0
N	12.2	10.4	10.6
O	6.8	0.6	6.8
P	26.9	3.6	9.4

TABLE 3: (BOTTLED **COKE**)

AGENT CODE	2003-2004	2004-2005	2005 - 2006
A	20.1	3.7	22.5
B	21	18.9	16.2
C	23.3	9.1	12
D	9.6	11.6	12.6
E	12.9	8.7	11.5
F	26.3	19.9	13.1
G	20.2	19.4	4.8
H	9.4	9.1	7.7
I	28.7	17.4	22.4
J	15.5	9.4	7.8
K	13.7	18.7	7.8
L	8.4	7.1	5.4
M	10	5.6	9.9

N	11.2	5.6	13.3
O	2	11.7	9.8
P	24.8	12.5	25.4
Q	11.6	14.8	13.7
R	9.3	1.6	11
S	21.8	1.6	16.2
T	31.6	9.3	7.8
U	26.2	11.1	7.8
V	8.5	5.2	17.7
W	12.8	1.6	4.1

TABLE 4: (**VANNILA FAN ICE**)

AGENT CODE	2003-2004	2004 -2005	2005 - 2006
A	8	8.9	8
B	8.7	9.9	9.3
C	11.6	9.5	10.9
D	15	13.9	12.4
E	32.1	6	5.3
F	1.2	6.8	7.5
G	9.8	10	9.9
H	14.4	13	13.1
I	6.5	6.2	4.2
J	7.9	14.5	9.8
K	9	13.1	12.3
L	11.2	11.5	10.1
M	11.4	6.4	9.1
N	10.2	9.9	9.3
O	7.6	9.7	10
P	1.4	10.3	13
Q	32.4	16.6	13.2
R	12.3	10.6	10.4
S	24.4	21.3	21.3

SOURCE: *Sales records of the wholesales standard outlets of the beverages marketing company.*

2.2 Beta-Binomial Model

Let p(x) be the proportion of customers which attain dropout rate of less than 10% in exactly x out of n years,(x=1,2,3,…,n). Assuming a Bernoulli local process with a constant probability P of "success" the value of p(x) can be obtained from the conventional binomial distribution.

$p(x) = {}^{n}C_{x}\, P^{x} q^{n-x}$, x=1,2,3,…,n , q = 1-p ……………………………………………………………………………………………….(3)

where"success " is defined as given level of customers dropout rate of less than 10% in a particular time set for carrying out the study. The assumption of the constant value of P however does not seem to be reasonable in the present investigation as it varies from one customer to the other.

Another way of addressing this issue is to consider some form of compound binominal distribution to compute p(x) in equation (3).

An attempt is therefore being made to compound binominal distribution with a beta distribution for P (Ishii & Hayakawa ,1980). The beta distribution is given by

$f(p) = \frac{P^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha,\beta)}$,
$0 \le P \le 1$ ……………………………………………………………………………….(4)

where α, β *are positive constants and* $B(\alpha,\beta) = \int_0^1 y^{\alpha-1}(1-y)^{\beta-1}dy$ *is a beta function*

Thus, the distribution of the number of weeks in which a customer drop-out rate of less than 10% can be obtained by compounding binominal distribution with beta distribution. Compounding these two distributions leads to beta-binominal distribution (BBD) model.

Using BBD model , the proportion p(x) expressed in (3) can be computed as follows.

$p(x) = \int_0^1 {}^nC_x$
$p^x(1-p)^{n-x} f(p)dp$ ……………………………………………………………………….(5)

By simplifying, we have:

$p(x) = {}^nC_x \frac{B(\alpha+x,\beta+n-x)}{B(\alpha,\beta)}$,
x=0,1,2,3… ……………………………………………………………… (6)

The model (6) is referred to as Beta- Binominal Distribution (BBD) model with parameter (α>0) and (β>0). The shape of the BBD model depends on the value of α and β.

2.3 Application of Beta Binomial Distribution Model (BBD).

Some interesting applications of BBD model include those related to analysis of leadership halut, consumer purchasing behavior and analysis of household distribution of the total number of cases of a disease (Chatfield & Gardhardt , 1998; Akomolafe et al., 2010)

Using maximum likelihood estimation of α and β in (6), let f(x) be the observed number of customers who drop out at the rate of less than 10% in exactly x' and of 'n' weeks, n=0,1,2,3… and for a total number of N customers $\sum f_x(x)$= N. The likelihood function (L) for the BBD model (6) is expressed as

$L = \frac{n}{\Lambda_{x=0}} [P(x)]^{f(x)}$

Hence, Log L = $\sum_{x=0}^{n} f(x)\log[p(x)]$

where p(x) is given by (1) for the binominal model, that is

Log L = $\sum_{x=0}^{n} f(x)\{\log({}^{n}C_{x}) + xlogP + (n-n)\log(1-p)\}$ ……………………………………………….(7)

Maximum likelihood estimates p for the parameter p of binomial model is obtained by equating derivatives of $logL$ to zero and solving for p in BBD model to have:

Log L = $\sum_{x=0}^{n} f(x)\{\log({}^{n}C_{x}) + xlogB(\alpha+x, \beta-x)\} - Nlog\ B(\alpha,\beta)$ ……………………………………(8)

Maximum likelihood estimates $\hat{\alpha}$ and $\hat{\beta}$ for the parameters of BBD model (8) are computed by equating partial derivatives of log L to zero and solving for α and β. That is α and β are the solutions of the equations;

$\frac{\delta L}{\delta \alpha}=0$ and $\frac{\delta L}{\delta \beta}=0$

Standard errors for $\hat{\alpha}$ and $\hat{\beta}$ are given by square root of the principal diagonal elements of the matrix evaluated and α = $\hat{\alpha}$ and β=$\hat{\beta}$.

The value of maximum likelihood estimates $\hat{\alpha}$ and $\hat{\beta}$ *and* their standard errors were computed by using the MATLAB statistical software in order to ease our computation.

3.0 Data Analysis and Results

Analysis of the average drop-out rate data for the weeks under consideration for each of the four beverages is presented in beow:

Table 5

Year	Observed number with average drop-out rate < 10%	Binomial model	Beta-binominal
0	13	6.82	13.00(20.3%)
1	17	22.69	16.97(26.6%)
2	18	25.18	18.03(28.1%)
3	16	9.31	16.00(25.0%)
Total	64	64	64

χ^2	13.885	< 1	1
Degree of freedom		2	1
p-value		< 1	0.9928
Maximum likelihood estimates		$\hat{p}$ =0.5260	$\hat{\alpha} = 1.4306, \hat{\beta} = 1.2889$
Standard Error (S.E.)		S.E.($\hat{p}$)=0.0361	S.E.($\hat{\alpha}$)= 0.6185 , S.E.($\hat{P}$)=0.0361

3.1 Discussion of Results

Table 5 shows that expected frequencies based on binomial model are not satisfactory (p-value is 0.000966 in goodness of fit test).Thus, binomial model does not fit the drop-out adequately. The fit based on BBD model however appears to be reasonably satisfactory (p-value, is 0.9928 in χ^2 goodness of fit test). It can also been seen from Table 5 that percentage of customers expected to achieve an average drop-out rate of less than 10% is close to the observed percentage of customers. Interestingly, the expected percentage computed from BBD model revealed that in the next fifty two weeks, 20% of the customers are expected to have drop-down rate of more than 10% whereas only 25% of the customers are likely to achieve the targeted drop-out of less than 10%.

4.0 Conclusion

The Investigation based on the above analysis coupled with the organizational goal of achieving drop-out rate less than 10% showed that a model based on beta-binominal distribution contributes significantly in capturing customer's dropout rate effectively without concealing the richness of the data. It is surprising to note that despite extensive effort made by the organizations on education, a chunk of customers are still expected to have even more than double(>20%) the customers drop-out target rate of less than 10%. There seems to be immediate need to retrospect policy initiative and reformulate appropriate strategies to ensure that more number of customers have less than 10% drop out rate in order to sustain the existence of the manufacturing company as well as its work force.

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