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SOME PROPERTIES OF SOFT b-I-COMPACT AND SOFT GENERALIZED b-I-COMPACT TOPOLOGICAL SPACE

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Abstract:- The main object of this paper is to study the relation between some types of weak soft compactness and soft ideals. We initiate new types of soft compactness modulo an soft ideal that generalize the concepts soft b-I-compact, we study some of their properties and characterizations. This paper, not only can form the theoretical basis for further applications of topology on soft sets, but also lead to the development of information systems.

Keyword: Soft set, Soft topological space, Soft interior, Soft closure, Soft b-I-Compact space, Soft b-closed, Soft $b^{\#}$-closed, Soft generalized b-I-Compact space

1. Introduction

In 1999 Molodtsov [10] was firstly introduced the concept of soft set , as a general mathematical tool for dealing with uncertain objects .He successfully applied the soft set theory into several direction such as Smoothness of functions , Game theory, Riemann integration, Perron integration, Theory of measurement, probability, and operations research. After that an increasing numbers of paper have been written about the properties and application of soft sets theory and it is applications in varios fileds [1, 7, 8, 9]. In 2011 Shabir and Naz [12] introduced the notion of soft topological space, are used the concept of soft set to define a topology ,which are defined over an initial universe with a fixed set of parameters. Later, Zorlutuna[13], and Hussain and Ahmad [4] are continued to study the properties of soft topological space. They got many important results in soft topological space .That leads to a new world in general topology. In 2012 Zorlutuna and Akdag[13] are firstly introduced the compactness for soft topological space. And in 2014 Akdag and Ozkan[2] studied the concept of b-open sets in soft settings. In 2014 Kandil [6] presented the notion of soft ideal ,these concept is discussed with a view to find new soft set from the original one , is said to be soft topological space with soft ideal (X,T,E,I).

In this paper we present new types of soft compactness modulo on soft ideal that generalize the concepts soft b-I-compact space, we study some of their properties and characterizations. Also we present the concept of soft generalized b-I-compact space.

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2. *Preliminaries*

In this section, we present the basic definitions and results of soft set theory which will be needed in this paper.

Definition(1-2):- [1]

Let X be an initial univeres set and E a set of parameters. A pair (F,E) where F is a map from E to P(X), is said to be soft set over X.

In what follows by SS(X,E) we denoted the family of all soft sets (F,E)over X.

Definition(2-2):-[8]

Let (F,A),(G,B)$\in$SS(X), then (F,E)is said to be a soft subset of (G,B) if F(p)$\subseteq$G(p), for every p$\in$A. symbolically, we write (F,A)$\subseteq$(G,B).

Also ,we say that the pairs (F,A),(G,B) are soft equal if (F,A)$\subseteq$(G,B) and (G,B)$\subseteq$(A,F). Symbolically, we write (F,A)=(G,B).

Definition(3-2):- [1]

Let (F,E)$\in$SS(X,E). The soft complement of (F,E), denoted by $(F,E)^c$, is a soft set of the form $(F,E)^c$ ={X-F(e), $\forall$ e$\in$E}.

Definition(4-2):-[8]

Let (F,E)$\in$SS(X,E). Where F(p)=Φ for every p$\in$E is said to be null soft set of SS(X,E) and denoted by Φ or Φ_E .

Definition(5-2):-[8]

Let (F,E)$\in$SS(X,E) ,Where F(p)=X for every p$\in$X is said to be absolute set of SS(X,E) and denoted by X_E .

Definition(6-2):-[8]

Let Λ be an arbitrary index set and $\{(F_i,E):i\in\Lambda\} \subseteq$ SS(X,E). The soft union of these soft sets is the soft set(F,E)$\in$SS(X,E),Where the map F:E$\rightarrow$P(X) defined as follows:

$F(p)=\cup \{F_i(p) :i\in\Lambda\}$, for every p$\in$E ,Symbolically, we write $(F,E)= \cup\{(F_i,E):i\in\Lambda\}$

Definition(7-2):-[8]

Let Λ be an arbitrary index set and $\{(F_i,E):i\in\Lambda\} \subseteq$ SS(X,E). The soft intersection of these soft sets is the soft set(F,E)$\in$SS(X,E),Where the map F:E$\rightarrow$P(X) defined as follows:

$F(p)=\cap\{F_i(p) :i\in\Lambda\}$, for every p$\in$E ,Symbolically, we write $(F,E)= \cap\{(F_i,E):i\in\Lambda\}$.

Proposition(8-2):- [3]

Let $(E,F) \in SS(X,E)$. The following statements are true:

1. $(F,E) \cap (F,E) = (F,E)$
2. $(F,E) \cup (F,E) = (F,E)$
3. $(F,E) \cap \Phi_E = \Phi_E$
4. $(F,E) \cup \Phi_E = (F,E)$
5. $(F,E) \cap X_E = (F,E)$
6. $(F,E) \cup X_E = X_E$
7. $(F,E) \cap (F,E)^c = \Phi_E$
8. $(F,E) \cup (F,E)^c = X_E$
9. $(\Phi_E)^c = X_E$
10. $(X_E)^c = \Phi_E$
11. $((F,E)^c)^c = (F,E)$
12. $\Phi_E \subseteq (F,E) \subseteq X_E$

Definition(9-2):-[12]

Let X be an initial univeres set, E is a fixed set of parameters and T be a collection of soft sets over X , $T \subseteq SS(X,E)$ is called soft topology on X if the following axioms are true:

1. Φ_E , $X_E \in T$
2. If (G,E) , (H,E) $\in T$,Then $(G,E) \cap (H,E) \in T$
3. If $(G_i ,E) \in T$,$\forall i \in \Lambda$, then $\cup \{(G_i ,E): i \in \Lambda\} \in T$

The triplet (X,T,E) is said to be soft topological space over X, or soft space. The members of T are called soft open sets in X and we denoted by SO(X) or SO(X,T,E).

Also , a soft set(F,E) is called soft closed if the complement $(F,E)^c$ is soft open ($(F,E)^c \in T$). The family of soft closed sets is denoted by SC(X) or SC(X,T,E).

Definition(10-2):- [13]

Let (F,E)be a soft subset of a soft topological space(X,T,E) .The soft interior of (F,E) denoted by int(F,E) is the union of all soft open sets of (F,E), Clearly int(F,E) is the largest soft open set contained in(F,E).

i.e. int(F,E)=$\{\cup (O,E)$,(O,E) is soft open set and $(O,E) \subseteq (F,E)\}$.

Definition(11-2):- [12]

Let (F,E)be a soft subset of a soft topological space(X,T,E) .The soft closure of (F,E) denoted by cl(F,E) is the intersection of all soft closed sets of (F,E), Clearly, cl(F,E) is the smallest soft closed set over X which contains (F,E).

i.e. cl(F,E)=$\{\cap (G,E)$,(G,E) is soft closed set such that $(F,E) \subseteq (G,E)\}$.

Definition(12-2):- [4]

Let SS(X,A)and SS(Y,B)be a families of soft sets. $u:X\rightarrow Y$ and $p:A\rightarrow B$ be mappings. Let $f_{pu}:SS(X,A)\rightarrow SS(Y,B)$,Then :

1. If $(F,A)\in SS(X,A)$,Then the image of (F,A)under f_{pu} written as $f_{pu}=(f_{pu}(F), p(A))$ is a soft set in SS(Y,B) ,such that
 $f_{pu}(F)_{(b)}=\cup \{u(F(a)): a\in p^{-1}(b)\cap A\}$ if $p^{-1}(b)\cap A \neq Ø$.
And $f_{pu}(F)_{(b)}=\{Ø\}$ otherwise for all $b \in B$.

2. If $(G,B)\in SS(Y,B)$, Then the inverse image of (G,B) under f_{pu} ,written as $f^{-1}_{pu}(G, B) = (f^{-1}_{pu}(G), p^{-1}(B))$, is a soft set in SS(X,A) ,such that
$f^{-1}_{pu}(G)_{(a)}= u^{-1}(G(p(a)))$ if $p(a)\in B$ .And $f^{-1}_{pu}(G)_{(a)}=\{Ø\}$ otherwise for all $a\in A$. The soft function f_{pu} is called surjective if p and u are surjective. And f_{pu} is called injective if p and u are injective.

Definition(13-2):-[2]
Let (X,T_1,A) and (Y,T_2,B) be soft topological spaces and
$f_{pu} : SS(X,A) \rightarrow SS(Y,B)$ be a function. Then The function f_{pu} is called

(1) soft b−continuous if the inverse image of each soft open set in Y is a soft b−open set in X.

(2) soft b−open if the image of each soft open set in X is soft b -open set in Y.

(3) soft b−irresolute if the inverse image of each soft b-open set in Y is a soft b−open set in X.

(4) soft b-irresolute open function if the image of each soft b-open set in X is soft b-open set in Y.

Definition(14-2):- [5]

A non-empty collection I of subsets of a set X is called an ideal on X, if it satisfies the following conditions;

1. $A \in I$ and $B \in I \Rightarrow A \cup B \in I$,
2. $A \in I$ and $B \subseteq A \Rightarrow B \in I$.

i. e. I is closed under finite unions and subsets.

Definition(15-2):- [6]

Let I be a non-null collection of soft sets over a universe X with a fixed set of parameters E, then $I\subseteq SS(X,E)$ is called a soft ideal on X with a fixed set E if it satisfies the following conditions;

1.$(F,E)\in I$ and $(G,E)\in I \rightarrow (F,E) \cup (G,E) \in I$
2. $(F,E)\in I$ and $(G,E)\subseteq (F,E) \rightarrow (G,E)\in I$
i.e. I is closed under finite soft unions and soft subsets.

Examples(16-2):- [6]

Let X be a universe set. Then each of the following families is a soft ideal over X with the same set of parameters E,

1. I={Φ}
2. I = SS(X,E) = {(F,E) : (F,E), is a soft set over X with the fixed set of parameters E}.
3. I_f= {(F,E) ∈ SS(X,E) : (F,E) is finite}, called soft ideal of finite soft sets.
4. I_c ={(F,E) ∈ SS(X,E) : (F,E) is countable}, called soft ideal of countable soft sets.
5. $I_{(F,E)}$ = {(G,E) ∈ SS(X,E) : (G,E)⊆(F,E)}.
6. I_n = {(G,E) ∈ SS(X,E) : int(cl(G,E)) =Φ}, called soft ideal of nowhere dense soft sets in (X,T ,E).

Remarke(17-2):- [6]

The following proposition gives the sufficient condition for the inverse image of a soft ideal to be a soft ideal.

Theorem(18-2):- [6]

Let (X_1,T_1,A), (X_2,T_2,B) be soft topological spaces, f_{pu} : (X_1,T_1,A) → (X_2,T_2,B) be a soft injective function and I be a soft ideal on X_2. Then $f^{-1}{}_{pu}$ (I) is a soft ideal on X_1.

Proof**:** Obvious.

Definition(19-2):- [13]

Let Ω be a family of a soft set is called a soft cover of a soft set (F,E) if (F,E)⊂ ∪ {(F_i,E) : (F_i,E) ∈Ω ,i∈Λ} it is a soft open cover if each member of Ω is a soft open set. A subfamily of Ω is called soft subcover, which is also a soft cover of (F,E).

Definition(20-2):- [13]

A soft topological space (X,T,E) is called soft compact space if each soft open cover of X has a finite soft subcover.

Definition(21-2):-[10]

A soft topological space (X,T,E) is called a soft b-compact space if for each soft b-open cover of X has a finite soft subcover.

3. Soft b-I-compact Space

Definition(1-3) :-

A soft subset (F,E) of the space (X,T,E,I) is said to be soft b-I-compact if for every soft b-open cover {(G_α,E): α∈Λ} of (F,E) . There exists a finite subset Λ_0 of Λ such that (F,E)-∪ (G_α,E)∈ I.

The space (X,T,E,I) is said to be soft b-I-compact if X is soft b-I-compact as a soft subset.

Proposition(2-3) :-

1. Every soft b-compact soft topological space (X,T,E) is soft b-I-compact for any soft ideal I on X.
2. If I={Φ} ,then (X,T,E) is soft b-compact if and only if it is soft b-I-compact.

Theorem(3-3):-

A soft topological space (X,T,E) is soft b-compact if and only if (X,T,E,I_f) is soft b-I_f – compact, Where I_f is the soft ideal of finite soft subset on X.

Proof:

Let (X,T ,E) be a soft b-compact topological space and let $\{(G_\alpha ,E) : \alpha\in \Lambda\}$ be a soft b-open cover of X . Then there exists a finite subset Λ_0 of Λ such that $X = \cup_{\alpha\in\Lambda 0} (G\alpha ,E)$. It follows that $X- \cup_{\alpha\in\Lambda 0}(G_\alpha,E)=\Phi \in I_f$. Hence (X,T,E, I_f) is soft b-I_f -compact.

Conversely, let (X,T,E, I_f) be a soft b-I_f -compact and let $\{(G_\alpha,E) : \alpha\in \Lambda\}$ be a b-open soft cover of X ,Then there exists a finite subset Λ_0 of Λ such that $X-\cup_{\alpha\in\Lambda 0} (G_\alpha,E)\in I_f$,Thus $X =\cup_{\alpha\in\Lambda 0} (G_\alpha,E)$.

Hence (X,T,E) is soft b- compact.

Theorem(4-3):-

Let (X,T,E, I) be a soft b-I-compact space and J be a soft ideal on X with the same set of parameters E such that $I \subseteq J$. Then (X,T,E,J) is a soft b-J-compact.

Proof:

Immediate.

Definition(5-3) :-

A soft topological space (X,T,E) is said to be soft b-closed if for every soft b-open cover $\{(G_\alpha,E) : \alpha\in \Lambda\}$ of X has a finite subcover $\Lambda_0\subseteq\Lambda$ whose soft closure covers X such that $X =\cup_{\alpha\in\Lambda 0} cl(G_\alpha,E)$.

Definition(6-3) :-

A soft subset (F,E) in a soft topological space (X,T,E) is said to be soft b-closed relative to X if and only if for every soft b-open cover $\{(G_\alpha,E) : \alpha\in \Lambda\}$ of (F,E) has a finite subcover $\Lambda_0\subseteq\Lambda$ whose soft closure covers (F,E) such that $(F,E) =\cup_{\alpha\in\Lambda 0} cl(G_\alpha,E)$.

Definition(7-3) :-

A soft subset (F,E) in a soft topological space (X,T,E) is said to be soft $b^{\#}$-closed if for every soft b-open cover $\{(G_\alpha,E) : \alpha\in \Lambda\}$ of (F,E) has a finite soft b-open subcover $\Lambda_0\subseteq\Lambda$ whose soft b-closure covers (F,E) such that $(F,E) =\cup_{\alpha\in\Lambda 0} Sbcl(G_\alpha,E)$.

Theorem(8-3):-

Let (X,T,E, I) be a soft topological space with soft ideal. If $I_n \subseteq I$ and (X,T,E) is soft b-closed, then (X,T,E) is soft b-I-compact.

Proof. Let $\{(G_\alpha,E): \alpha\in\Lambda\}$ be a soft b-open cover of X ,then there exist a finite subset $\Lambda_0 \subseteq\Lambda$ such that

$X= \cup_{\alpha\in\Lambda} cl(G_\alpha,E)\subseteq \cup_{\alpha\in\Lambda} cl(int(G_\alpha,E))$, where (G_α,E) is b-open set, it follows that

$X= \cup\, cl(int(G_\alpha,E))$

$X-\cup_{\alpha\in\Lambda} cl(int(G_\alpha,E))$

$\subseteq X\cap int(cl(\cup_{\alpha\in\Lambda} (G_\alpha,E)^c))$

$\subseteq int(cl(X-\cup_{\alpha\in\Lambda} cl(G_\alpha,E)))$

$=\Phi$

Thus $X- \cup_{\alpha\in\Lambda}cl(G_\alpha ,E)) \in I_n \subseteq I$,Therefore (X,T,E) is soft b-I-compact space.

Theorem(9-3) :-

If the space (X,T,E,I_f) is soft b-I_f -compact, then (X,T,E) is soft b-closed (resp. soft $b^{\#}$-closed).

Proof:

Let $\{(G_\alpha,E) : \alpha\in \Lambda\}$ be a soft b-open cover of X. Then there exists a finite soft b-open subcover Λ_0 of Λ such that $X- \cup_{\alpha\in\Lambda}(G_\alpha,E) \in I_f$.

$\therefore X= \cup_{\alpha\in\Lambda 0} (G_\alpha,E) \subseteq \cup_{\alpha\in\Lambda 0}cl(G_\alpha,E)$. Therefore (X,T,E) is soft b-closed.

The rest of the proof is similar.

Theorem(10-3) :-[6]

Let (X_1,T_1,A,I) be a soft topological space with soft ideal, (X_2,T_2,B) be a soft topological space and $f_{pu} : (X_1,T_1,A,I) \rightarrow (X_2,T_2,B)$ be a soft function. Then

$f_{pu}(I) = \{ f_{pu}((F,A)) : (F,A) \in I\}$ is a soft ideal on X_2.

Theorem(11-3):- [6]

Let (X_1,T_1,A), (X_2,T_2,B) be soft topological spaces, $f_{pu}:(X_1,T_1,A) \rightarrow (X_2,T_2,B)$ be an injective soft function and I be a soft ideal on X_2. Then $f^{-1}_{pu}(I)$ is a soft ideal on X_1.

Theorem(12-3):-

Let (X_1,T_1,A,I) be a soft topological space with soft ideal, (X_2,T_2,B) be a soft topological space and $f_{pu} : (X_1,T_1,A, I) \rightarrow (X_2,T_2,B)$ be a soft b-irresolute surjective function. If (X_1,T_1,A) is soft b-I-compact, then (X_2,T_2,B) is soft b- $f_{pu}(I)$-compact.

Proof:

Let $\{(G_\alpha,B) : \alpha\in \Lambda\}$ be a soft b-open cover of X_2. Since f_{pu} is a soft b-irresolute function, then

$\{ f^{-1}_{pu}(G_\alpha,B):\alpha\in\Lambda\}$ is a soft b-open cover of X_1.

Since(X_1,T_1,A) is soft b-I-compact , then there exists a finite subset Λ_0 of Λ such that $X_1 - \cup \{ f^{-1}_{pu}(G_\alpha,E):\alpha\in\Lambda_0\} \in I$.

This implies that $f_{pu} [X_1 - \cup \{ f^{-1}_{pu}(G_\alpha,E):\alpha\in\Lambda_0\}]= X_2 - \cup_{\alpha\in\Lambda 0} (G_\alpha,E) \in f_{pu}(I)$. Therefore (X_2,T_2,B) is soft b- $f_{pu}(I)$-compact.

Theorem(13-3):-

Let (X_1,T_1,A) be a soft topological space, (X_2,T_2,B, I) be a soft topological space with soft ideal and $f_{pu} : (X_1,T_1,A) \rightarrow (X_2,T_2,B, I)$ be a bijection b-irresolute open soft function. If (X_2,T_2,B) is soft b-I-compact, then (X_1,T_1,A) is soft b-$f^{-1}_{pu}(I)$-compact.

Proof:

Let $\{(G_\alpha,A) : \alpha\in \Lambda\}$ be a soft b-open soft cover of X_1. Since f_{pu} is a bijection b-irresolute soft open function, then $\{ f_{pu}(G_\alpha,A) : \alpha\in \Lambda\}$ is a soft b-open cover of X_2, since $(X_2,T_2,B)$is soft b-I-compact, then there exists a finite subset Λ_0 of Λ such that $X_2 - \cup \{ f_{pu}(G_\alpha,A) : \alpha\in \Lambda_0\}\in I$.

$\therefore f^{-1}_{pu} [X_2 - \cup \{ f_{pu}(G_\alpha,A) : \alpha\in \Lambda_0\}] = X_1 - \cup_{\alpha\in\Lambda 0} (G\alpha,A) \in f^{-1}_{pu}(I)$.

Then (X_1,T_1,A) is soft b-$f^{-1}_{pu}(I)$-compact.

Theorem(14-3):-

Let (X_1,T_1,E, I_1) , (X_2,T_2,E, I_2) , (X_3,T_3,E,I_3) are soft topological spaces, and let $f_{pu} : (X_1,T_1,E, I_1) \rightarrow (X_2,T_2,E,I_2)$, $g_{pu} : (X_2,T_2,E, I_2) \rightarrow (X_3,T_3,E,I_3)$ are soft b-irresolute surjective function. If (X_1,T_1,E,I_1) is soft b-I-compact, then (X_3,T_3,E,I_3) is soft b-$(gof)_{pu}(I_3)$-compact.

Proof:-

Let $\{(G_\alpha,B) : \alpha\in \Lambda\}$ be a soft b-open cover of X_3. Since g_{pu} is a soft b-irresolute surjective function, then $\{ g^{-1}_{pu}(G_\alpha,B):\alpha\in\Lambda\}$ is a soft b-open cover of X_2.And since f_{pu} is a soft b-irresolute surjective function, then $\{ f^{-1}(g^{-1}_{pu}(G_\alpha,B)):\alpha\in\Lambda\}$ is a soft b-open cover of X_1.By hypothesis there exists a finite subset Λ_0 of Λ such that $X_1 - \cup \{ (gof)^{-1}_{pu}(G_\alpha,E):\alpha\in\Lambda_0\} \in I_1$.

Which implies that $X_3 - \cup \{(G_\alpha,E):\alpha\in\Lambda_0\} \in (gof)_{pu}(I_3)$. Therefore (X_3,T_3,E,I_3) is soft b-$(gof)_{pu}(I_3)$-compact.

Theorem(15-3):-

Let (X_1,T_1,E, I_1) , (X_2,T_2,E, I_2) ,…, $(X_{n+1},T_{n+1},E,I_{n+1})$ are soft topological spaces, and let

$f_{1(pu)} : (X_1,T_1,E, I_1) \rightarrow (X_2,T_2,E,I_2)$

$f_{2(pu)} : (X_2,T_2,E, I_2) \rightarrow (X_3,T_3,E,I_3)$

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$f_{n(pu)} : (X_n,T_n,E, I_n) \rightarrow (X_{n+1},T_{n+1},E,I_{n+1})$
are soft b-irresolute surjective function. If (X_1,T_1,E,I_1) is soft b-I-compact, then $(X_{n+1},T_{n+1},E,I_{n+1})$ is soft b-$(f_n o f_1)_{pu}(I_n)$-compact.

Theorem(16-3):-

Let (X_1,T_1,E, I_1) , (X_2,T_2,E, I_2) , (X_3,T_3,E,I_3) are soft topological spaces, and let $f_{pu} : (X_1,T_1,E, I_1) \rightarrow (X_2,T_2,E,I_2)$, $g_{pu} : (X_2,T_2,E, I_2) \rightarrow (X_3,T_3,E,I_3)$ are soft b-irresolute bijection function. If (X_3,T_3,E,I_3) is soft b-I-compact, then (X_1,T_1,E,I_1) is soft b-$(fog)^{-1}{}_{pu}(I_1)$-compact.

Proof:-

Let $\{(G_\alpha,B) : \alpha\in \Lambda\}$ be a soft b-open cover of X_1. Since f_{pu} is a soft b-irresolute bijection function, then $\{ f_{pu}(G_\alpha,B):\alpha\in\Lambda\}$ is a soft b-open cover of X_2. And since g_{pu} is a soft b-irresolute bijection function, then $\{ g_{pu}(f_{pu}(G_\alpha,B)):\alpha\in\Lambda\}$ is a soft b-open cover of X_3.By hypothesis there exists a finite subset Λ_0 of Λ such that $X_3 - \cup \{ (fog)_{pu}(G_\alpha,E):\alpha\in\Lambda_0\} \in I_3$.
Which implies that $X_1 - \cup \{(G_\alpha,E):\alpha\in\Lambda_0\} \in (fog)^{-1}{}_{pu}(I_1)$. Therefore (X_1,T_1,E,I_1) is soft b-$(fog)^{-1}{}_{pu}(I_1)$-compact.

Theorem(17-3):-

Let (X_1,T_1,E, I_1) , (X_2,T_2,E, I_2) ,…, $(X_{n+1},T_{n+1},E,I_{n+1})$ are soft topological spaces, and let
$f_{1(pu)} : (X_1,T_1,E, I_1) \rightarrow (X_2,T_2,E,I_2)$
$f_{2(pu)} : (X_2,T_2,E, I_2) \rightarrow (X_3,T_3,E,I_3)$
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$f_{n(pu)} : (X_n,T_n,E, I_n) \rightarrow (X_{n+1},T_{n+1},E,I_{n+1})$
are soft b-irresolute bijection function. If $(X_{n+1},T_{n+1},E,I_{n+1})$ is soft b-I_{n+1}-compact, then (X_1,T_1,E,I_1) is soft b-$(f_1 o f_n)_{pu}(I_1)$-compact.

Theorem(18-3):-

Let (X,T,E,I) be a soft topological space with soft ideal I. Then (X,T,E) is soft b-I-compact if and only if for every family $\{(G_\alpha,E) :\alpha\in\Lambda \}$ of soft b-closed subsets of X for which $\cap\{(G_\alpha,E) : \alpha\in\Lambda\} = \phi$, there exists a finite $\Lambda_0 \subseteq \Lambda$ such that $\cap\{(G_\alpha ,E) : \alpha\in\Lambda_0\} \in I$.

Proof:

($\Rightarrow$) Assume that(X,T,E) be a soft b-I-compact space and $\{(G_\alpha,E):\alpha\in\Lambda\}$ be a family of soft b-closed sets of X such that $\cap\{(G_\alpha,E):\alpha\in\Lambda\}=\Phi$. Then$\{X - (G_\alpha,E):\alpha\in\Lambda\}$ is a family of soft b-open sets of X such that $X = \cup \{X -(G_\alpha,E):\alpha\in\Lambda\}$
Since (X,T,E) is soft b-I-compact space ,then(by definition of soft b-I-compact space) there exists a finite $\Lambda_0\subseteq\Lambda$ such that
$X-(\cup \{X -(G_\alpha,E):\alpha\in\Lambda_0\})\in I$
i.e.
$X- (X\cup (\cap_{\alpha\in\Lambda 0}(G_\alpha,E)^c = X - (X\cap(\cup_{\alpha\in\Lambda 0}(G_\alpha,E))^c = X- (X\cap (\cap_{\alpha\in\Lambda 0} (G_\alpha,E))^c) = X -(X (\cap_{\alpha\in\Lambda 0} (G_\alpha,E)) =\cap_{\alpha\in\Lambda 0} (G_\alpha,E) \in I$.
($\Leftarrow$) Suppose that $\{(G_\alpha ,E) :\alpha\in\Lambda \}$ be a family of
Soft b-open cover of X . Then $\{X-(G_\alpha,E) : \alpha\in\Lambda\}$
be a family of soft b-closed sets of X with $\cap_{\alpha\in\Lambda} (X -(G_\alpha,E)) = \Phi$, and there exists finite $\Lambda_0\subseteq\Lambda$ such
that $\cap_{\alpha\in\Lambda} (X -(G_\alpha ,E)) \in I$. Thus $\cap_{\alpha\in\Lambda} (X -(G_\alpha,E))=X-\cup_{\alpha\in\Lambda} (G_\alpha,E) \in I$.
Therefore, (X,T,E) is soft b-I-compact.

4. Soft generalized b-I-compact space

This section contains the concept of soft generalized b-compact space with soft ideal I.

Definition(1-4):-

A soft set (F,E)in a soft topological space (X,T,E) is soft generalized b-closed set with respect to an ideal I if Sbcl(F,E)-(G,B)$\in$I , whenever (F,E)$\subseteq$ (G,B) and (G,B) is soft b-open set in X.

Definition(2-4):-

A soft set (F,E)in a soft topological space (X,T,E) is soft generalized b-open set with respect to an ideal I if the complement $(F,E)^c$ is soft generalized b-closed in X.

Definition(3-4) :- [11]

A collection $\{(G_\alpha,E):\alpha\in\Lambda\}$ of soft generalized b-open sets in X is called soft generalized b-open cover of a soft set (F,B) in X if $(F,B)\subset \cup_{\alpha\in\Lambda}(G_\alpha,E)$.

Definition(4-4):-

A soft subset (F,E) of the soft space (X,T,E,I) is said to be soft generalized b-I-compact if for every soft generalized b-open cover $\{(G_\alpha,E): \alpha\in\Lambda\}$ of (F,E) . There exists a finite subset Λ_0 of Λ such that $(F,E)-\cup (G_\alpha,E)\in$ I.

The space (X,T,E,I) is said to be soft generalized b-I-compact if X is soft generalized b-I-compact as a soft subset.

Theorem(5-4) :-

Let (X,T,E,I)be soft generalized b-I-compact space, And (F,E) be soft generalized b-closed set in X. Then (F,E) is soft generalized b-I-compact

Proof:

Let (F,E) be a soft generalized b-closed subset of X and$\{(G_\alpha,E):\alpha\in\Lambda\}$be a soft generalized b-open cover of X.

$(F,E)^c$ is soft generalized b-open set , then $\{(G_\alpha,E):\alpha\in\Lambda\}\cup (F,E)^c$ is soft generalized b-open cover of X

Since X be soft generalized b-I-compact , then there exist a finite subset $\Lambda_0\subseteq\Lambda$ such that $X-[(G_\alpha,E) \cup (F,E)^c]\in I$, But $(F,E),(F,E)^c$ are disjoint.

Hence $(F,E)-\{(G_\alpha,E):\alpha\in\Lambda_0\}=\Phi \in I$

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