

Another Look at the Consumer Price Index - A wavelet Approach

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Abstract

A wavelet approach was applied to a consumer price index (CPI) series to address the draw backs of some periodic models. The method requires no assumption of the data generating process but involves the spliting of a given signal into several components with each component reflecting the evolution trough of the signal at a particular time. The multi-level stationary Haar wavelet decomposition was applied to the series which gave rise to a dyadic sequence of 2^8 , and the series was decomposed accordingly using a computer program written for the purpose. Multi-resolution wavelet method was then used to reconstruct the series and the significant details $(d_{j,t})$ that captured the season were added to the trend $(s_{j,t})$ component for the estimatation of the series $\{Y_t\}$. The resulting wavelet model was subjected to diagnostic checks and were found to be adequate. Comparative study was carried out with some hilighted CPI models built by some researchers. It was discovered that the wavelet models performs better.

Keywords: Mother wavelets, Haar wavelet decomposition, Multi-resolution, Auto-correlation and Partial auto-correlation function.

1.0 Introduction/Review

Consumer price index (CPI) is an economic indicator that gives a comprehensive measure used for the estimation of price changes in a basket of goods and services representative of consumption expenditure in an economy. It measures changes in the price level of a market basket of consumer goods and services purchased by households. Statistically, it is an estimate constructed using the prices of a sample of representative items whose prices are collected periodically. The percentage change in the CPI over a period of time gives the amount of inflation over that specific period. Thus, the CPI provides a measure of inflation.

In recent years, inflation has become one of the major economic focus of most countries of the world, especially those in Africa and Asia. Due to its impact on the nation's economy, the control of inflation has become imperative for any nation. To control inflation in the future, there is need to relate the past and the present effect. A body of techniques that can be used for such predictive purposes is time series.

According to Abraham (2014), Consumer price index (CPI) measures changes in the price level of market basket of consumer goods and services purchased by households over a period of time. Abraham (2014) modelled the CPI using Fourier series approach. The approach identified the period to be 12 with a frequency of 0.02678. The Fourier series model was subjected to some diagnostic checks and was found to be adequate. However, the root mean square error was found to be moderately high with a value of 7.2587.

In particular, Akpanta and Okorie (2015) modeled the Nigerian CPI in the time-domain using the Seasonal Autoregressive Integrated Moving Average model (SARIMA). In this case, the seasonal component of the series was assumed to be stochastic and correlated with non-seasonal components. Despite the correlation structure, the model was still found to be adequate with a root means square error of 4.2345.

Taking into consideration the periodic variation found in the data, Omekara *et al* (2013), Nachane and clavel (2014) modeled inflation rate in the frequency domain using the Mixed Fourier series and the ARMA Model with Fourier coefficients respectively. The work showed that the residuals followed a white noise process, indicating a good fit of the model.

In Nigeria, the CPI is calculated by the National Bureau of Statistics and assisted by the Central Bank of Nigeria. It is one of the most frequently used statistics for identifying periods of inflation and deflation and can be used to index the real values of wages and salaries. However, since most financial data like the CPI are usually defective in terms of irregular characteristics, the data is usually smoothened by log transformation, differencing or filtering before analysis is carried out. According to Al Wadi *et al* (2010), one of such filtering approaches in the frequency domain is wavelet analysis.

A Wavelet is a function which enables us to split the given signal into several frequency components, each reflecting the evolution trough time of the signal at a particular frequency. Wavelet as its name suggests, is a small wave. In this context, the term "small" essentially means that the wave grows and decays in a limited time frame.

Masset (2008) considered wavelet as a very potent method in studying financial data or variable that exhibit a cyclical behaviour and/or affected by a seasonal effects. He applied the wavelet method in the analysis of several seasonal data and it was discovered that wavelet methods produced reliable results than the linear models.

The spread in the acceptability of Wavelet analysis is seen in its adoption by Wall Street analysts as a veritable mathematical tools for analyszing financial data (Manahanda *et* al, 2007). The range of the application of wavelet in the financial data is potentially used in denoising and seasonal filtering, identification of regimes shift and jumps.

According to Mallet (2001), Gencay *et al* (2002) and Crowdly (2005), Wavelet analysis takes its root from Filter and Fourier analysis and is able to overcome most of the limitations of Fourier series analysis. This is because, they can combine information from both time-domain and frequency-domain, and do not require assumptions concerning the data generating process.

Because of the drawbacks of Fourier or Spectral Analysis, Masset (2008) presented a set of tools which allows gathering information about the frequency components. This method was able to address the problem of the drawbacks of spectral analysis temporarily.

Yogo (2003) in his paper, pointed out that Multiresolution wavelet analysis is a natural way of decomposing economic time series into components of various frequencies which are long-run trend, business-cycle component and high frequency noise. The paper was applied to the real Gross National Product and inflation and was found to address the limitations of the Fourier models.

Renaud *et al* (2004) took a critical look at the Wavelet-Based method for time series. The work was based on multiple decomposition of signal using a redundant (a trous) wavelets transform which has the advantage of being shift-invariant. The result was a decomposition of the signal into range of frequency scales which explicitly showed that the method works well and adapts itself to studies involving financial data. It was also discovered that in a series whose dynamics is made of Autoregressive integrated moving average (ARIMA) model and *s* cyclical components, the wavelet analysis can be used to remove the impact of trend, noise and the seasonality.

In the same vein, Mehala and Dahiya (2013) revealed that the Wavelet transform are capable of revealing detailed aspects of data such as trends, breakdown points, discontinuities in higher derivatives and self-similarity which cannot be adduced using Fourier transform.

Perhaps it was such findings which encouraged Mukhopadhyay *et al* (2013) to adopt Wavelet transform in the study of wind speed data. The study used continuous Wavelet transform (MCWT) like Morlet to check the periodicity of wind speed. It was shown that wavelet transform provided more information about signal constituents of the dynamic speckle.

As spelt out in the review; except the wavelet approach, several time series techniques have been applied in modelling the CPI series. Non of this techniques actually reflected the evolution trough time of the signal at a particular frequency. This work therefore seeks to address the CPI series in another dimension using the wavelet platform.

2.0 Methodology

2.1 *Wavelets*

A Wavelet is a function which enables us to split a given signal into several components, each reflecting the evolution trough time of the signal at a particular time.The essence of wavelet analysis consists of projecting the time series of interest $[Y_t] = 0,1,2, \dots, (\mathrm{N}-1)$ onto a discrete wavelet filter often called the mother wavelet. The mother wavelet is represented as:

$$[h_l] = (h_0, h_1, \dots, h_{L-1}, 0, \dots, 0)$$

The discrete wavelet filter satisfies the properties:

1. $\sum_{l=0}^{L-1} h_l = 0$ (1)

2. $\sum_{l=0}^{L-1} h_l^{\,2} = 0$ (2)

3. $\sum_{l=0}^{L-1} h_l h_{l+2n} = 0 \ \forall$ non − zero integers n (3)

where L is a suitably chosen positive integer and $L < N$ and padded with zeros at the end so that it has the same dimension N as $[Y_t]$.

By virtue of (1), $[h_l]$ is a high-pass filter. Associated with $[h_l]$ is a scaling filter (or father wavelet) which is a low-pass filter, recoverable from $[h_l]$ via the relationship

$$g_l = (-1)^{l+1} h_{L-1-l} \quad ; \quad l = 0,1,\dots,L-1 \qquad (4)$$

Following Daubechies (1992), Db1 wavelet fliter which is equivalent to Haar wavelets filter can be represented as:

$$\psi(y) = 1, \quad \text{if } \ y \in [0\,, 0.5]$$

$$\psi(y) = -1, \quad \text{if } \ y \in [0.5\,, 1]$$

$$\psi(y) = 1, \quad \text{if } \ y \notin [0\,, 0.5]$$

$$\psi(y) = 1, \quad \text{if } \ y \in [0\,, 1]$$

$$\psi(y) = 0, \quad \text{if } \ y \notin [0\,, 1]$$

The Haar wavelet is the first and the simplest. Haar wavelet is discontinuous and resembles a step function. For prediction purposes, we use the stationary discrete wavelet transform introduced by Masset (2008). The coefficients can be obtained via a pyramid algorithm and the wavelet coefficients at each level j comprise N elements.

The algorithm yields the $N-$ dimensional vector of wavelet coefficients

$$w_t = \left(w_t^{(1)}, w_t^{(2)}, \dots, w_t^{(j)}, v_t^{j}\right)^T \qquad (5) \ ;$$

where the $N/2^j$ vector $\left\{w_t^{(j)}\right\}$ can be interpreted as the vector of wavelet coefficients associated with the dynamics of the series $\{Y_t\}$ on a scale of length $\lambda^j = 2^{j-1}$, (with increasing scales corresponding to lower frequencies) and $\left\{v_t^{(j)}\right\}$ represents the averages on the scale of length 2^j.

2.2 *White Noise Process*

A process $\{\varepsilon_t\}$ is said to be a white noise process with mean 0 and variance σ_ε^2 written $\{\varepsilon_t\} \sim WN(0, \sigma_\varepsilon^2)$, if it is a sequence of uncorrelated random variables from a fixed distribution.

2.3 *Multi-resolution*

Multi-resolution represents a convenient way of decomposing a given series $\{Y_t\}$ into changes attributable at different scales.

Let the filter coefficients be expressed in reverse order as:

$$q_1 = (h_N, h_{N-1}, \dots, h_1, h_0)^{\mathrm{T}}$$

Let q_j denote the zero-padded scale j wavelet filter coefficients obtained by j convolutions of q_1 with itself and let φ_j represent the $\mathrm{N}/_{2^{\mathrm{j}}} \times \mathrm{N}$ matrix of "circularly shifted" coefficients of q_1 (by a factor of 2^j).

We can now write the $N \times N$ matrix φ as

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \cdots \\ \vdots \\ \cdots \\ \varphi_j \\ \vartheta_j \end{bmatrix} = \varphi$$

where,

ϑ_j is $N \times N$ vector with each term equal to $1/\sqrt{N}$.

The multi-resolution scale defines the j th level wavelet detail $d_{j,t}$ as:

$$d_{j,t} = \varphi_j^T w_t^{(j)}, \quad j = 1,2, \dots, J \qquad (6)$$

where $w_t^{(j)}$ are the wavelet coefficients at the jth scale defined in (5).

The wavelet smooth is defined as:

$$s_{j,t} = \vartheta_j^T v_t^{(j)} \qquad (7)$$

Hence, the multi-resolution Wavelet can now be expressed by the relationship:

$$\mathrm{Y}_t = \sum_{\mathrm{j}=1}^{\mathrm{J}} d_{j,t} + s_{j,t} + \varepsilon_{\mathrm{t}} \qquad (8)$$

where ε_{t} is a white noise process.

That is, each observation in the series is additively decomposed into the J wavelet details and the wavelet smooth.

2.4 *Diagnostic Check of the Model*

The diagnostic check is based on the behaviour of the residuals obtained from fitting the model. For model adequacy, the residuals are expected to be uncorrelated at the various lags. These non correlated random varables can be confirmed if the Autocorrelation Function (ACF) plot and Partial Autocorrelation Function (PACF) plot does not show any spike above or below the 95% confidence interval.

3.0 Data Analysis

The data was obtained from the Central bank of Nigerian official web site (www.cbn.gov.ng). The analysis was done using Minitab and Matlab softwares.

3.1 *The Wavelet Model*

The raw data plot (figure 1) shows clearly that the series $\{Y_t\}$ is non-stationary and contains trend. The behaviour of the Autocorrelation and Partial Autocorrelation functions (figures 2 and 3) suggest an $ARIMA(1,0,0)$ model for $\{Y_t\}$. Also, the Autocorrelation function (figure 2) exhibit significant spikes at lag 12, 24, 36,… . This shows that the series is seasonal and since the series is a monthly data; the season $s = 12$. Hence, the series $\{Y_t\}$ contains trend, noise and seasonality. According to Renaud *et al* (2004), for a series $\{Y_t\}$ whose dynamics is made of $ARIMA(1,0,0)$ and $s = 12$ cyclical components, the wavelet analysis can be used to remove these irregularities.

The Matlab script in Appendix A was used to decompose the series $\{Yt\}$ into trend, seasonal and the error component. The series $\{Y_t\}$ contains 256 data points which give rise to a dyadic sequence $\{2^J;\ \text{i.e.}\ 2^8\}$. This means that we can decompose the data set until level 8. Nevertheless, it was found that level 3 and upward had similar results. Therefore the series was decomposed until level 3 as suggested by Daubechies (1992). The multi-level stationary Haar wavelet decomposition was applied to the data set. The multiresolution wavelet analysis was then used to reconstruct the series and the significant details $(d_{j,t})$ that captured the seasonal period (see figure 4 and 5) were added to the smooth or trend $(s_{j,t})$ so as to estimate $\{Y_t\}$.

At scale j, the wavelet detail d_j captures frequencies ${}^1/_{2^{j+1}} \leq f \leq {}^1/_{2^j}$ and the wavelet smooth s_j captures frequencies $f < {}^1/_{2^j}$. The level three multi-resolution captures the components of the time series which have a frequency $f < {}^{1}/_{2^3}$. This means that the smooth s_3 takes into account changes in Y_t that are associated with a period length of at least 8 units of time. Therefore, s_3 keeps the $ARIMA(1, 0, 0)$ dynamics of Y_t while removing it seasonal behaviour and noise. The coefficient at detail one from the periodogram in figure 6 depicts a high frequency noise, while the coefficients at detail three and two captured seasonal variation of period length 4-16 as seen in fiqure 4 and 5. The coefficients of detail three and two were added to the coefficients of the smooth series so as to obtain the estimate of $\{Y_t\}$. This result given in Appendix B is a decomposition of the signal into a range of frequency scales. The series needed no additional decomposition at this stage because the residual after reconstruction was found to be random as shown by the ACF in figure 7.

Hence from (8), the model that reconstructs the series is

$$\hat{Y}_t = \sum_{j=1}^{3} \sum_{t=0}^{203} d_{j,t} + S_{j,t} \qquad (9)$$

3.2 *Diagonistic Checks*

The diagnostic check based on the residuals do not raise any alarm on the validity and adequacy of the fitted model since the residual ACF plot (figure 7) does not show any significant spike above or below the 95 percent confidence interval. This means that the residuals are consistent with the white noise process; confirming the adequacy of the wavelet model. Also, the root means square error (RMSE) obtained in fitting the wavelet model is calculated to be 0.15262. This shows that the strength of the discrepancies between real values and those estimated by the model is rather very small; indicating a good fit of the model.

In addition, the actual values of the series $\{Y_t\}$ and the values estimated by the wavelet model (9) are strongly positively correlated (see Y_t and Fits in appendix B). This is also confirmed by visual inspection of the actual and estimate plot (figure 8) in which the two superimposed plots strongly agree and move in the same direction. This further confirms the adequacy of the model.

4.0 Discussion and Conclusion

As noted in the review, several approaches have been made in the modelling of the CPI and some good results have been achieved. However, even though some of the fitted models were found to be adequare; they still suffer some draw backs in taking care of the trend (smooth) and splitting of the given signal into components that can reflect the evolution trough time of the signal at a particular time. Besides, the obtained root mean square errors (7.2587 and 4.2345) in Abraham (2014), and Akpanta and Okorie (2015) seem to be moderately high and should not be considered as the best fit for the CPI series. Contrary to these approaches, the Wavelet approach has decomposed the series into smooth (trend) ($S_{j,t}$) and details ($d_{j,t}$) by using Haar stationary Wavelet technique. The reconstruction of the ($S_{j,t}$) at Multi-resolution three gave the smooth S_3 (figure 9). The residual analysis discussed in section 3.2 has shown clearly that the wavelet model is adequate and by comparing its root means square error (0.15262) with others; the wavelet model fits the CPI series better than the Fourier and SARIMA aproaches noted in the review.

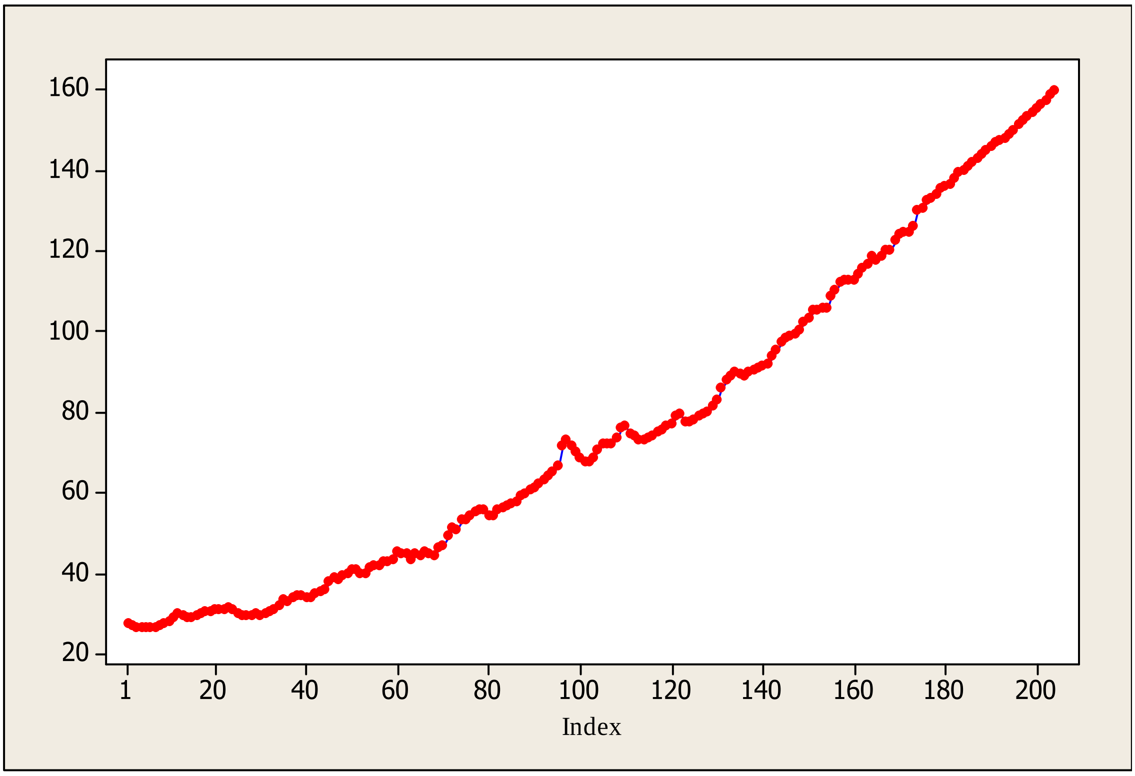

figure 1: Raw data plot of $\{Y_t\}$

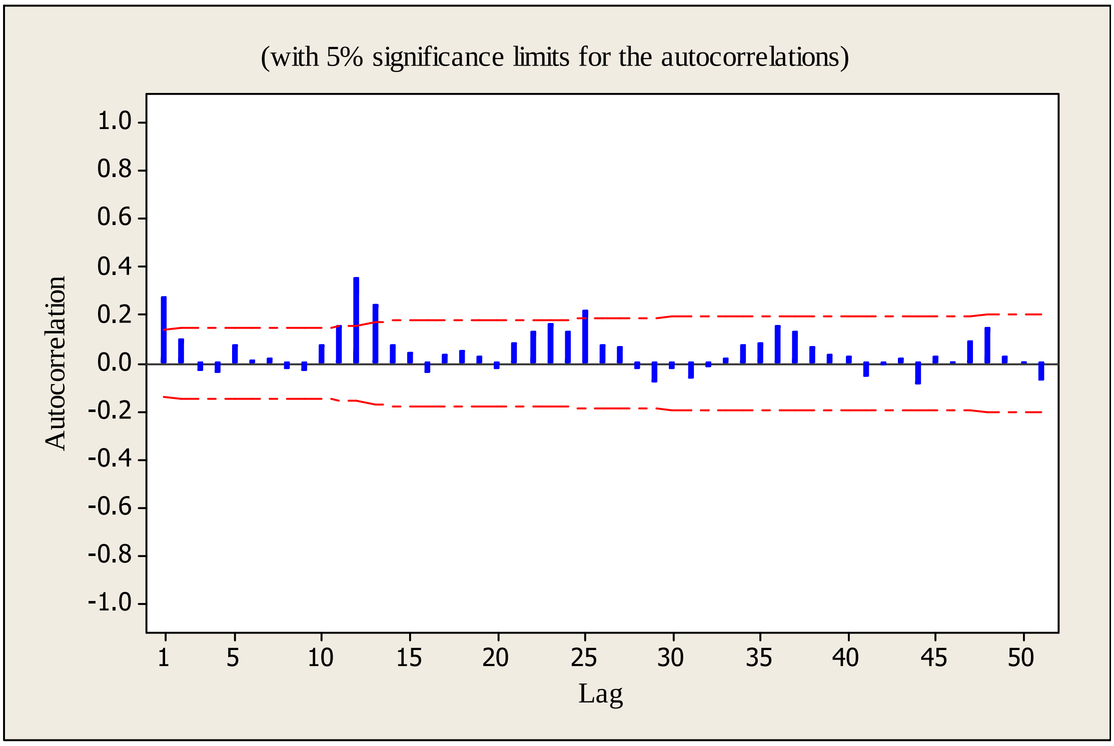

figure 2 : Autocorelation plot of second difference of $\{Y_t\}$

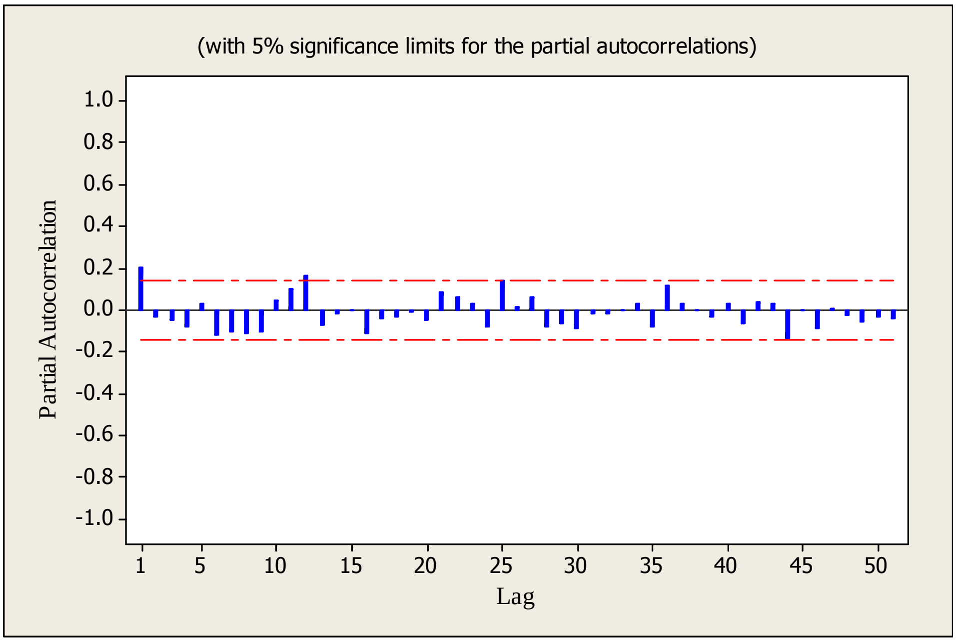

figure 3 : Partial Autocorelation plot of second difference of $\{Y_t\}$

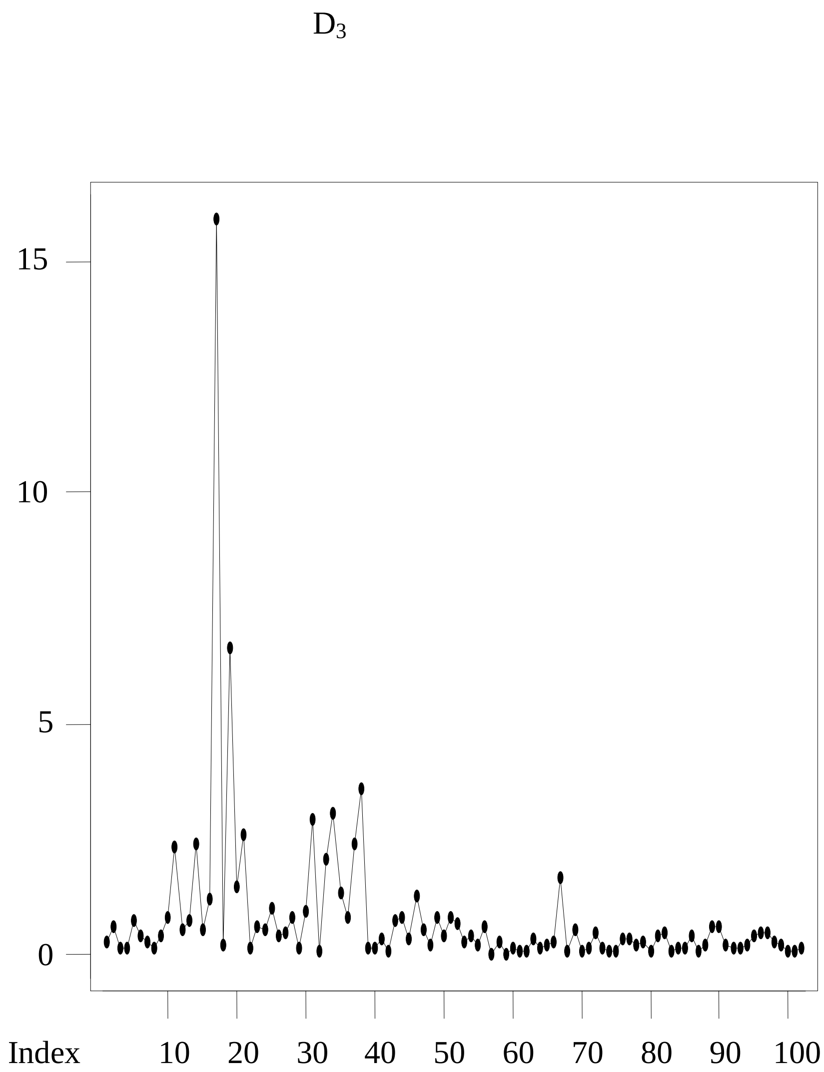

figure 4: Periodogram plot for D_3

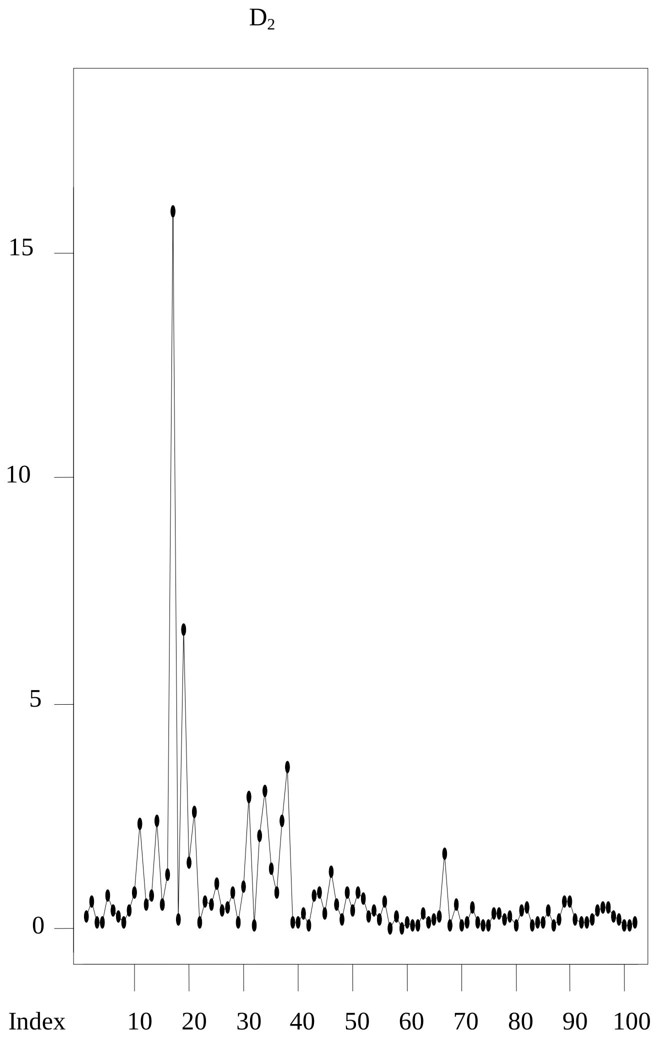

figure 5: Periodogram plot for D_2

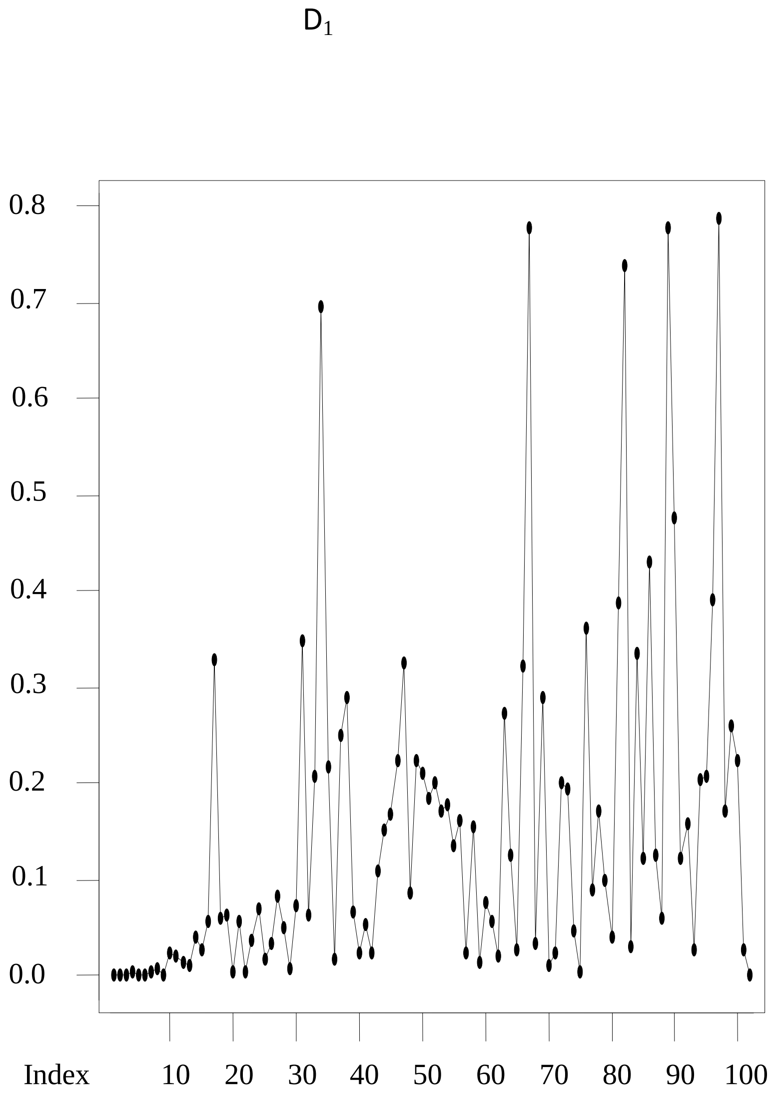

figure 6: Periodogram plot for D_1

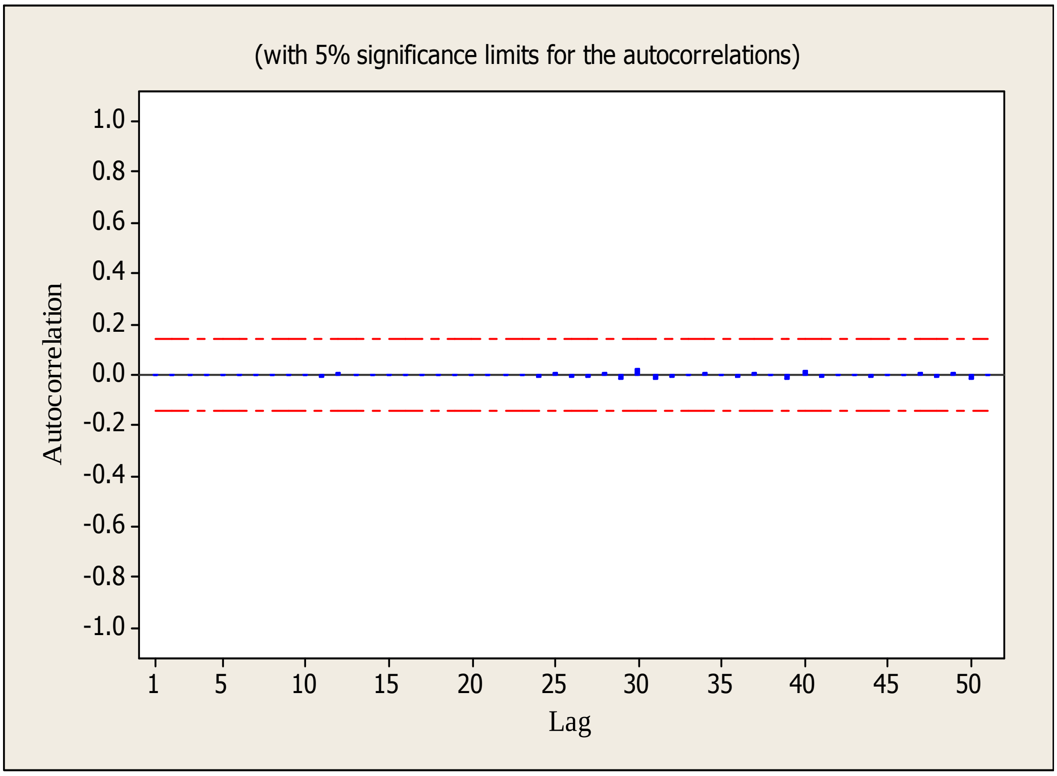

figure 7: Residual autocorrelation function plot of the wavelet analysis

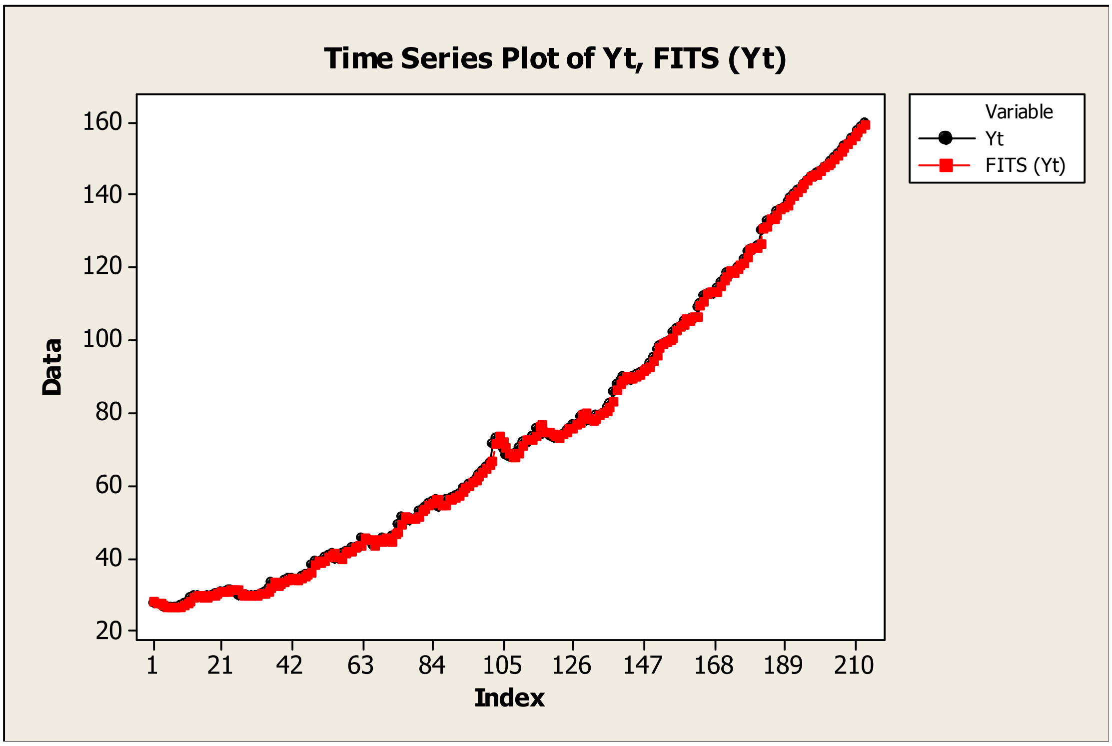

figure 8: Actual and wavelet estimate plot of the CPI series

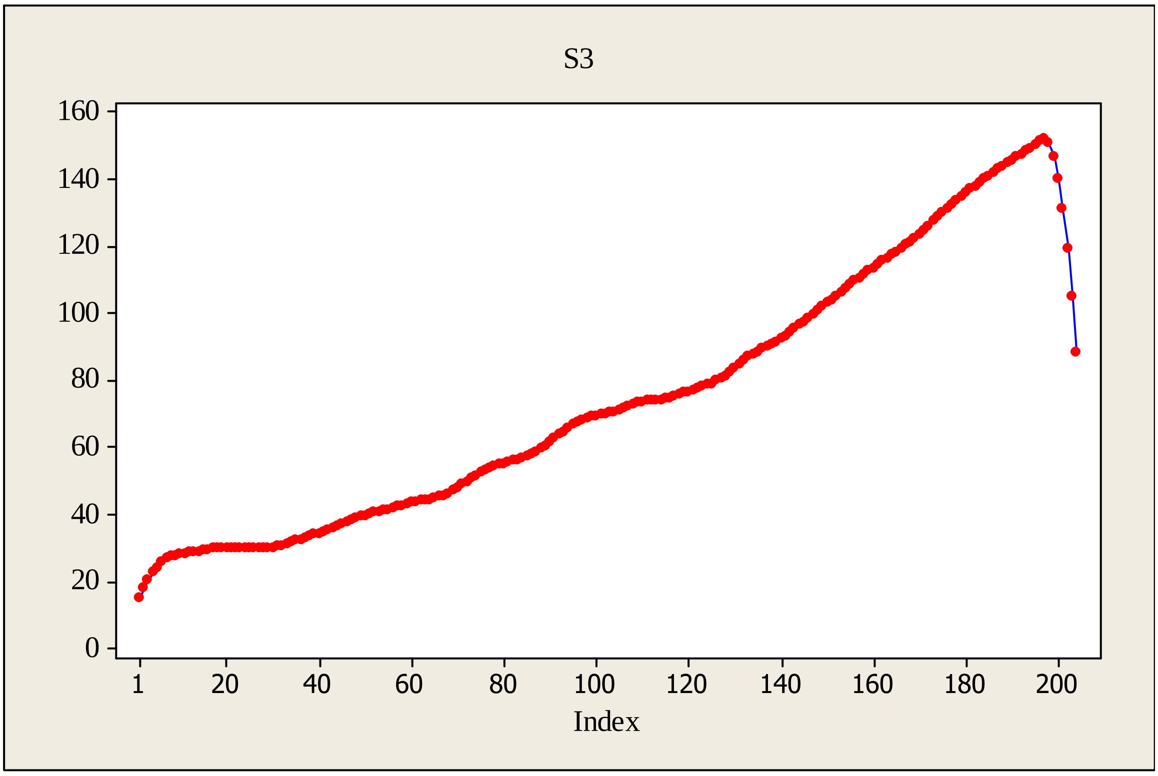

figure 9: Time Plot for Trend or Smooth S_3

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Appendix A

Wavelet Analysis Program Using Matlab

```
s=consumer price index(CPI)
(swa,swd)=swt(s,1,'db1');
[swa,swd]=swt(s,1,'db1');
whos
Subplot(1 2,1),plot(swa);title('Approximation cfs')
subplot(1,2,2),plot(swd);title('Detail cfs')
A0=iswt(swa,swd,'db1);
err=norm(S-A0)
nulcfs=zeros(size(swa));
A1=iswt(swa,nulcfs,'db1');
D1=iswt(nulcfs,swd,'db1');
subplot(1,2,1),plot(A1);title('Approximation A1')
subplot(1,2,2),plot(D1);title('Detail D1')
[swa,swd]=swt(s,3,'db1);
[swa,swd]= swt(s,3,'db1');
clear A0 A1 D1 err nulcfs
whos

Multilevel decomposition and reconstruction.
kp=0;
for i=1:3
subplot(3,2,kp+1), plot(swa(i,:));
title(['Approx. cfs level ',num2str(i)])
subplot(3,2,kp+2),plot(swd(i,:));
title(['Detail cfs level ',num2str(i)])
kp=kp+2;
end
mzero= zeros(size(swd));
A= mzero;
A(3,:)=iswt(swa,mzero,'db1');
D= mzero;
```

```
for i = 1:3
swcfs = mzero;
swcfs(i,:) = swd(i,:);
D(i,:) = iswt(mzero,swcfs,'db1');
end
A(2,:)=A(3,:) + D(3,:);
A(1,:)=A(2,:) + D(2,:);
kp = 0;
for i = 1:3
subplot(3,2,kp+1), plot(A(i,:));
title(['Approx. level ',num2str(i)])
subplot(3,2,kp+2),plot(D(i,:));
title(['Detail level ',num2str(i)])
kp = kp + 2;
end
[thr,sorh] = ddencmp('den','wv',s);
dswd = wthresh(swd,sorh,thr);
clean = iswt(swa,dswd,'db1');
subplot(2,1,1),plot(s);
title('original signal')
subplot(2,1,2), plot(clean);
title('De-noised signal')
err=norm(s-clean).
```

Appendix B

Estimates of Consumer Price Index using Wavelet analysis

Yt	S3	D3	D2	Fit	D_1
27.46	22.058	1.8197	3.5919	27.470	-0.010000
26.96	17.964	3.8386	5.1550	26.958	0.002500
26.45	20.461	4.5433	1.5681	26.572	-0.122500
26.43	22.581	3.9723	-0.1238	26.430	0.000000
26.41	24.338	2.1392	-0.0750	26.403	0.007500
26.36	25.730	0.8041	-0.1369	26.398	-0.037500
26.46	26.749	-0.0311	-0.1856	26.533	-0.072500
26.85	27.392	-0.3456	-0.1338	26.912	-0.062500
27.49	27.653	-0.1333	-0.1150	27.405	0.085000
27.79	27.947	0.1034	-0.0875	27.962	-0.172500
28.78	28.263	0.2928	0.1813	28.738	0.042500
29.6	28.567	0.3542	0.4413	29.363	0.237500
29.47	28.835	0.2606	0.2494	29.345	0.125000
28.84	29.074	0.0780	-0.1544	28.997	-0.157500
28.84	29.307	-0.0839	-0.2681	28.955	-0.115000
29.3	29.539	-0.1598	-0.1294	29.250	0.050000
29.56	29.756	-0.1275	0.0088	29.637	-0.077500
30.13	29.943	-0.0013	0.0781	30.020	0.110000

30.26	30.083	0.1423	0.0594	30.285	-0.025000
30.49	30.185	0.2922	0.0006	30.477	0.012500
30.67	30.253	0.4184	0.0231	30.695	-0.025000
30.95	30.287	0.4497	0.1956	30.933	0.017500
31.16	30.263	0.3953	0.3537	31.012	0.147500
30.78	30.181	0.2244	0.1994	30.605	0.175000
29.7	30.065	-0.0234	-0.1262	29.915	-0.215000
29.48	29.961	-0.2278	-0.2531	29.480	0.000000
29.26	29.901	-0.3902	-0.1881	29.323	-0.062500
29.29	29.915	-0.4777	-0.0694	29.368	-0.077500
29.63	30.002	-0.4980	-0.0219	29.483	0.147500
29.38	30.168	-0.5277	-0.1081	29.532	-0.152500
29.74	30.432	-0.5088	-0.1931	29.730	0.010000
30.06	30.788	-0.4095	-0.2487	30.130	-0.070000
30.66	31.214	-0.2548	-0.2094	30.750	-0.090000
31.62	31.676	-0.0175	0.0637	31.723	-0.102500
32.99	32.151	0.2198	0.2369	32.608	0.382500
32.83	32.603	0.3561	0.1013	33.060	-0.230000
33.59	33.041	0.3923	0.0838	33.518	0.072500
34.06	33.484	0.2631	0.2456	33.993	0.067500
34.26	33.928	0.0105	0.1537	34.092	0.167500
33.79	34.365	-0.2508	-0.1719	33.942	-0.152500
33.93	34.826	-0.4875	-0.2862	34.053	-0.122500
34.56	35.329	-0.5466	-0.2025	34.580	-0.020000
35.27	35.880	-0.4148	-0.3075	35.157	0.112500
35.53	36.483	-0.2097	-0.2456	36.028	-0.497500
37.78	37.108	0.0853	0.2988	37.493	0.287500
38.88	37.696	0.3084	0.4531	38.458	0.422500
38.29	38.241	0.4098	-0.0181	38.633	-0.342500
39.07	38.775	0.4961	-0.1956	39.075	-0.005000
39.87	39.267	0.4406	0.1375	39.845	0.025000
40.57	39.709	0.2834	0.4825	40.475	0.095000
40.89	40.093	0.1041	0.3106	40.508	0.382500
39.68	40.421	-0.1375	-0.3388	39.945	-0.265000
39.53	40.770	-0.3116	-0.5313	39.928	-0.397500
40.97	41.174	-0.3270	-0.0769	40.770	0.200000
41.61	41.602	-0.2748	0.1456	41.473	0.137500
41.7	42.001	-0.1537	0.0556	41.902	-0.202500
42.6	42.402	0.0539	-0.0212	42.435	0.165000
42.84	42.789	0.2208	-0.1969	42.813	0.027500
42.97	43.178	0.3783	-0.0738	43.482	-0.512500
45.15	43.561	0.4641	0.4850	44.510	0.640000

44.77	43.843	0.3953	0.5869	44.825	-0.055000
44.61	44.064	0.2248	-0.0213	44.267	0.342500
43.08	44.259	0.0139	-0.4525	43.820	-0.740000
44.51	44.523	-0.1756	-0.2369	44.110	0.400000
44.34	44.844	-0.4050	0.1863	44.625	-0.285000
45.31	45.232	-0.5628	0.2313	44.900	0.410000
44.64	45.698	-0.7648	-0.2481	44.685	-0.045000
44.15	46.273	-0.9016	-0.6063	44.765	-0.615000
46.12	47.017	-0.7575	-0.5219	45.737	0.382500
46.56	47.877	-0.5769	-0.2450	47.055	-0.495000
48.98	48.845	-0.1923	0.2244	48.877	0.102500
50.99	49.827	0.1745	0.3231	50.325	0.665000
50.34	50.756	0.3541	0.0100	51.120	-0.780000

52.81	51.651	0.5875	0.0637	52.303	0.507500
53.25	52.476	0.6669	0.1800	53.323	-0.072500
53.98	53.207	0.7278	0.0906	54.025	-0.045000
54.89	53.822	0.7222	0.2606	54.805	0.085000
55.46	54.332	0.4955	0.5575	55.385	0.075000
55.73	54.737	0.2277	0.2806	55.245	0.485000
54.06	55.086	-0.1308	-0.4406	54.515	-0.455000
54.21	55.472	-0.3816	-0.5275	54.563	-0.352500
55.77	55.893	-0.4584	-0.0269	55.407	0.362500
55.88	56.348	-0.4727	0.1144	55.990	-0.110000
56.43	56.856	-0.3556	-0.0925	56.407	0.022500
56.89	57.425	-0.2633	-0.2019	56.960	-0.070000
57.63	58.074	-0.1986	-0.0975	57.777	-0.147500
58.96	58.806	-0.1281	0.0600	58.738	0.222500
59.4	59.601	-0.1344	0.0706	59.538	-0.137500
60.39	60.498	-0.2106	-0.0119	60.275	0.115000
60.92	61.493	-0.3833	-0.0950	61.015	-0.095000
61.83	62.576	-0.5842	-0.1319	61.860	-0.030000
62.86	63.693	-0.7212	-0.0988	62.873	-0.012500
63.94	64.795	-0.5405	-0.2819	63.972	-0.032500
65.15	65.841	0.0375	-0.7531	65.125	0.025000
66.26	66.805	0.8384	-0.4306	67.213	-0.952500
71.18	67.690	1.6705	1.0294	70.390	0.790000
72.94	68.380	2.0039	1.7913	72.175	0.765000
71.64	68.858	1.6380	1.0413	71.538	0.102500
69.93	69.175	0.7655	0.0319	69.972	-0.042500
68.39	69.396	-0.3278	-0.5481	68.520	-0.130000
67.37	69.607	-1.1097	-0.8550	67.642	-0.272500
67.44	69.895	-1.3320	-0.8675	67.695	-0.255000

68.53	70.282	-1.1055	-0.4438	68.733	-0.202500
70.43	70.722	-0.6355	0.2531	70.340	0.090000
71.97	71.226	-0.1756	0.5475	71.597	0.372500
72.02	71.762	0.1947	0.0156	71.972	0.047500
71.88	72.305	0.5284	-0.5612	72.272	-0.392500
73.31	72.841	0.8061	-0.1050	73.543	-0.232500
75.67	73.313	0.9917	0.8875	75.193	0.477500
76.12	73.645	0.9273	0.9600	75.533	0.587500
74.22	73.830	0.5480	0.1850	74.563	-0.342500
73.69	73.961	0.0230	-0.3012	73.682	0.007500
73.13	74.091	-0.4975	-0.4006	73.192	-0.062500
72.82	74.292	-0.8131	-0.4413	73.037	-0.217500
73.38	74.581	-0.8517	-0.3044	73.425	-0.045000
74.12	74.913	-0.6847	-0.0706	74.158	-0.037500
75.01	75.285	-0.4134	0.0038	74.875	0.135000
75.36	75.713	-0.0750	-0.0775	75.560	-0.200000
76.51	76.210	0.2781	-0.1781	76.310	0.200000
76.86	76.715	0.4964	0.0613	77.272	-0.412500
78.86	77.218	0.5856	0.6544	78.457	0.402500
79.25	77.666	0.4302	0.6437	78.740	0.510000
77.6	78.068	0.0667	-0.1469	77.987	-0.387500
77.5	78.530	-0.3214	-0.5763	77.633	-0.132500
77.93	79.097	-0.7048	-0.2875	78.105	-0.175000
79.06	79.786	-0.9512	-0.0044	78.830	0.230000
79.27	80.580	-1.0667	-0.1412	79.372	-0.102500
79.89	81.485	-0.9930	-0.4469	80.045	-0.155000
81.13	82.484	-0.6828	-0.5963	81.205	-0.075000
82.67	83.581	-0.2036	-0.3300	83.047	-0.377500
85.72	84.742	0.4184	0.2625	85.422	0.297500
87.58	85.855	0.8919	0.6231	87.370	0.210000
88.6	86.870	1.0713	0.6487	88.590	0.010000
89.58	87.770	0.9652	0.4650	89.200	0.380000
89.04	88.568	0.5645	0.0300	89.163	-0.122500
88.99	89.312	0.1228	-0.2644	89.170	-0.180000
89.66	90.043	-0.2464	-0.1812	89.615	0.045000
90.15	90.777	-0.5450	-0.0344	90.198	-0.047500
90.83	91.531	-0.6838	-0.0544	90.792	0.037500
91.36	92.338	-0.7070	-0.2681	91.362	-0.002500
91.9	93.221	-0.5838	-0.4494	92.188	-0.287500
93.59	94.212	-0.3034	-0.3087	93.600	-0.010000

95.32	95.290	0.0061	0.0837	95.380	-0.060000
97.29	96.422	0.2331	0.4175	97.073	0.217500
98.39	97.543	0.2697	0.4250	98.238	0.152500
98.88	98.642	0.1575	0.0756	98.875	0.005000
99.35	99.711	0.0105	-0.3262	99.395	-0.045000
100	100.794	-0.0467	-0.3719	100.375	-0.375000
102.15	101.897	0.0127	-0.0519	101.857	0.292500
103.13	102.992	0.0684	0.3025	103.363	-0.232500
105.04	104.082	0.0225	0.4231	104.527	0.512500
104.9	105.140	-0.1147	0.1144	105.140	-0.240000
105.72	106.203	-0.2472	-0.4506	105.505	0.215000
105.68	107.284	-0.2367	-0.5869	106.460	-0.780000
108.76	108.413	-0.0155	-0.1125	108.285	0.475000
109.94	109.519	0.2438	0.3644	110.128	-0.187500
111.87	110.601	0.3895	0.5244	111.515	0.355000
112.38	111.621	0.3592	0.3569	112.338	0.042500
112.72	112.596	0.1561	-0.1050	112.647	0.072500
112.77	113.563	-0.0378	-0.4056	113.120	-0.350000
114.22	114.525	-0.0572	-0.2675	114.200	0.020000
115.59	115.479	-0.0014	0.0475	115.525	0.065000
116.7	116.410	0.0734	0.3394	116.823	-0.122500
118.3	117.340	0.0995	0.3006	117.740	0.560000
117.66	118.239	-0.0655	-0.0856	118.087	-0.427500
118.73	119.177	-0.1967	-0.2275	118.753	-0.022500
119.89	120.195	-0.2747	-0.2250	119.695	0.195000
120.27	121.267	-0.3122	-0.2800	120.675	-0.405000
122.27	122.416	-0.2750	0.0619	122.203	0.067500
124	123.603	-0.3334	0.4481	123.718	0.282500
124.6	124.812	-0.3561	0.0069	124.463	0.137500
124.65	126.065	-0.3456	-0.7519	124.968	-0.317500
125.97	127.391	-0.1830	-0.5131	126.695	-0.725000
130.19	128.753	0.1395	0.3325	129.225	0.965000
130.55	130.057	0.3586	0.5644	130.980	-0.430000
132.63	131.329	0.5370	0.2862	132.153	0.477500
132.8	132.524	0.4839	0.0000	133.008	-0.207500
133.8	133.683	0.2750	-0.0225	133.935	-0.135000
135.34	134.815	0.1258	0.0938	135.035	0.305000
135.66	135.900	-0.0353	-0.0575	135.808	-0.147500
136.57	136.944	-0.0725	-0.1838	136.688	-0.117500
137.95	137.950	-0.0280	-0.0125	137.910	0.040000
139.17	138.958	0.0053	0.1112	139.075	0.095000
140.01	139.949	0.0564	0.0575	140.063	-0.052500

141.06	140.940	0.0855	-0.0075	141.017	0.042500
141.94	141.907	0.0925	-0.0144	141.985	-0.045000
143	142.856	0.1092	0.0250	142.990	0.010000
144.02	143.794	0.1250	0.0456	143.965	0.055000
144.82	144.713	0.1239	0.0256	144.862	-0.042500
145.79	145.619	0.0869	0.0562	145.762	0.027500
146.65	146.520	0.0094	0.1031	146.633	0.017500
147.44	147.419	-0.0853	0.0013	147.335	0.105000
147.81	148.324	-0.1706	-0.1631	147.990	-0.180000
148.9	149.254	-0.1883	-0.1631	148.902	-0.002500
150	150.208	-0.1519	-0.0562	150.000	0.000000
151.1	151.188	-0.0983	0.0331	151.122	-0.022500
152.29	152.190	-0.0383	0.0831	152.235	0.055000
153.26	150.696	2.4817	0.0319	153.210	0.050000
154.03	146.691	7.4994	-0.0531	154.138	-0.107500
155.23	140.164	15.0741	-0.0681	155.170	0.060000
156.19	131.090	25.2141	-0.0513	156.253	-0.062500
157.4	119.444	27.9198	10.0388	157.403	-0.002500
158.62	105.206	23.1216	30.2450	158.573	0.047500
159.65	128.355	10.7358	20.3894	159.480	0.170000