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Effect of Multicolinearity and Autocorrelation on Predictive Ability of Some Estimators of Linear Regression Model

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Abstract

Violation of the assumptions of independent regressors and error terms in linear regression model has respectively resulted into the problems of multicollinearity and autocorrelation. Each of these problems separately has significant effect on parameters estimation of the model parameters and hence prediction. This paper therefore attempts to investigate the joint effect of the existence of multicollinerity and autocorrlation on Ordinary Least Square (OLS) estimator, Cochrane-Orcutt (COR) estimator, Maximum Likelihood (ML) estimator and the estimators based on Principal Component (PC) analysis on prediction of linear regression model through Monte Carlo studies using the adjusted coefficient of determination goodness of fit statistic of each estimator. With correlated normal variables as regressors, it further identifies the best estimator for prediction at various levels of sample sizes (n), multicollinearity (λ) and autocorrlation (ρ). Results reveal the pattern of performances of COR and ML at each level of multicollinearity over the levels of autocorrelation to be generally and evidently convex especially when $n \geq 30$ and $\lambda < 0$ while that of OLS and PC is generally concave. Moreover, the COR and ML estimators perform equivalently and better; and their performances become much better as multicollinearity increases. The COR estimator is generally the best estimator for prediction except at high level of multicollinearity and low levels of autocorrelation. At these instances, the PC estimator is either best or competes with the COR estimator. Moreover, when the sample size is small (n=10) and multicollinearity level is not high, the OLS estimator is best at low level of autocorrelation whereas the ML is best at moderate levels of autocorrelation.

.**Keywords:** Prediction, Estimators, Linear Regression Model, Multicollinearity, Autocorrelation.

1.0 Introduction

In linear regression analysis modeling of business, economic and social sciences data, the dependence of regressors often leads to the problem of multcollinearity. For instance, the independent variables such as family income and assets or store sales and number of employees or age and years of experience would tend to be highly correlated. With strongly interrelated regressors, the regression coefficients provided by the OLS estimator are no longer stable even though they are still unbiased as long as multicollinearity is not perfect. Furthermore, the regression coefficients may have large sampling errors which affect both the inference and forecasting that is based on the model (Chartterjee et al., 2000). Various other estimation methods have been developed to tackle this problem. These estimators include Ridge Regression estimator developed by Hoerl (1962) and Hoerl and Kennard (1970), Estimator based on Principal Component Regression suggested by Massy (1965), Marquardt (1970) and Bock, Yancey and Judge (1973), Naes and Marten (1988), and method of Partial Least Squares developed by Hermon Wold in the 1960s (Helland, 1988, Helland, 1990, Phatak and Jony 1997).

The dependence of error terms, as often being found in time series data, leads to be problem of autocorrelated error terms of regression model. Several authors have worked on this problem especially in terms of the parameter estimation of the linear regression model when the error term follows autoregressive of orders one. The OLS estimator is inefficient even though unbiased. Its predicted values are also inefficient and the sampling variances of the autocorrelated error terms are known to be underestimated causing the t and the F tests to be invalid (Johnston, 1984; Fomby et al., 1984; Chartterjee, 2000; Maddala, 2002). To compensate for the lost of efficiency, several feasible generalized least squares (GLS) estimators have been developed. These estimators include those provided by Cochrane and Orcutt (1949), Paris and Winstern (1954), Hildreth and Lu (1960), Durbin (1960), Theil (1971), the maximum likelihood and the maximum likelihood grid (Beach and Mackinnon, 1978), and Thornton (1982). Chipman (1979), Kramer (1980), Kleiber (2001), Iyaniwura and Nwabueze (2004), Nwabueze (2005a, b,c), Ayinde and Ipinyomi (2007) and many other authors have not only observed the asymptotic equivalence of some of these estimators but have also noted that that their performances and efficiency depend on the structure of the regressor used. Rao and Griliches (1969) did one of the earliest Monte-Carlo investigations on the small sample properties of several two-stage regression methods in the context of autocorrelation error. Other recent works done on these estimators include that of Iyaniwura and Olaomi (2006), Ayinde and Oyejola (2007), Ayinde (2007a, b), Ayinde and Olaomi (2008), Ayinde (2008), and Ayinde and Iyaniwura (2008).

In spite of these several works on these estimators, none has actually studied these estimators especially in term of their predictive ability when both multicollinearity and autocorrelation together. More so, situations where the two problems exist together in a data set are not uncommon. Therefore, this paper attempts to investigate the predictive ability / potential of some of these estimators under the joint existence of these problems through Monte Carlo studies.

2.0 Materials and Methods

Consider the linear regression model is of the form:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t \quad (1)$$

Where $u_t = \rho u_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2)$, t = 1, 2, 3,...n and $X_i \sim N(0,1)$) i = 1, 2, 3 are fixed and correlated.

For Monte-Carlo simulation study, the parameters of equation (1) were specified and fixed as $\beta_0 = 4$, $\beta_1 = 2.5$, $\beta_2 = 1.8$ and $\beta_3 = 0.6$. The levels of multicollinearity among the independent variables were sixteen (16) and specified as: $\lambda(x_{12}) = \lambda(x_{13}) = \lambda(x_{23}) = -0.49, -0.4, -0.3, \ldots, 0.8, 0.9, 0.99$. The levels of autocorrelation is twenty-one (21) and are specified as: $\rho = -0.99, -0.9, -0.8, \ldots, 0.8, 0.9, 0.99$. Furthermore, the experiment was replicated in 1000 times (R =1000) under Six (6) levels of sample sizes (n =10, 15, 20, 30, 50, 100). The correlated normal regressors were generated by using the equations provided by Ayinde (2007) and Ayinde and Adegboye (2010) to generate normally distributed random variables with specified intercorrelation. With P= 3, the equations give:

$$X_1 = \mu_1 + \sigma_1 Z_1$$

$$X_2 = \mu_2 + \rho_{12}\sigma_2 Z_1 + \sqrt{m_{22}}\, Z_2 \quad (2)$$

$$X_3 = \mu_3 + \rho_{13}\sigma_3 Z_1 + \frac{m_{23}}{\sqrt{m_{22}}} Z_2 + \sqrt{n_{33}}\, Z_{3}$$

Where $m_{22} = \sigma_2^2(1-\rho_{12}^2)$, $m_{23} = \sigma_2\sigma_3(\rho_{23} - \rho_{12}\rho_{13})$ and $n_{33} = m_{33} - \frac{m_{23}^2}{m_{22}}$; and $Z_i \sim N(0, 1)$ i = 1, 2, 3. (The inter-correlation matrix has to be positive definite and hence, the correlations among the independent variable were taken as prescribed earlier). In the study, we assumed $X_i \sim N(0, 1)$, i = 1, 2, 3 as earlier mentioned.

The error terms were generated using one of the distributional properties of the autocorrelated error terms ($u_t \sim N(0, \frac{\sigma_\varepsilon^2}{1-\rho^2})$) and the AR(1) equation as follows:

$$u_1 = \frac{\varepsilon_1}{\sqrt{1-\rho^2}} \tag{3}$$

$$u_t = \rho u_{t-1} + \varepsilon_t \qquad t = 2,3,4,\ldots n \tag{4}$$

Since some of these estimators have now been incorporated into the Time Series Processor (TSP 5.0, 2005) software, a computer program was written using the software to estimate the Adjusted Coefficient of Determination of the model ($\bar{R}^2$) the Ordinary Least Square (OLS) estimator, Cochrane orcutt (COR) estimator, Maximum Likelihood estimator and the estimator based on Principal Component Analysis (PRN). The Adjusted Coefficient of Determination of the model was averaged over the numbers of replications. i.e.

$$\bar{\bar{R}} = \frac{1}{R}\sum_{i=1}^{R} \bar{R}_i^2 \tag{5}$$

The two possible PCs (PC1 and PC2) of the Principal Component Analysis were used. Each provides its separate Adjusted Coefficient of Determination. An estimator is best if its Adjusted Coefficient of Determination is closest to unity.

3.0 Results and Discussion

The full summary of the simulated results of each estimator at different level of sample size, muticollinearity, and autocorrelation is contained in the work of Alao (2011). The graphical representations of the results when n=10, 15, 20, 30, 50 and 100 are respectively presented in Figure 1, 2, 3, 4, 5 and 6.

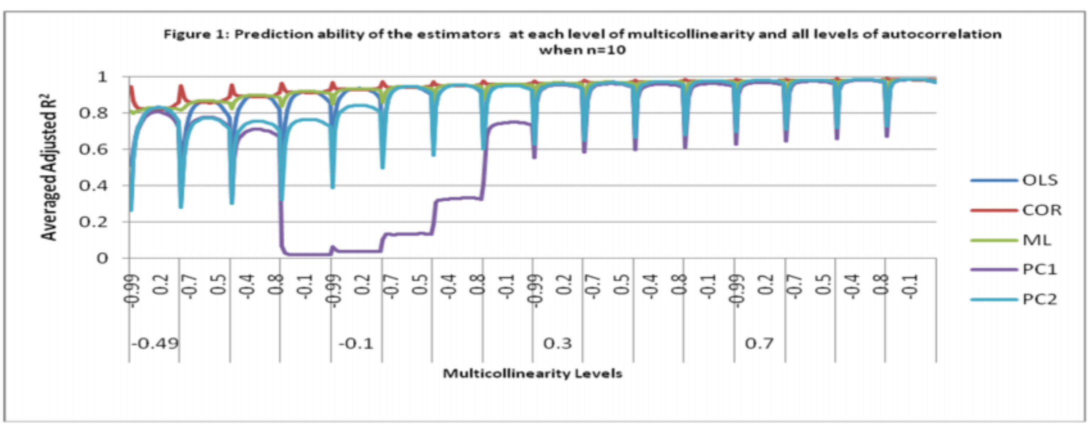


Figure 1: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=10

Figure 2: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=15

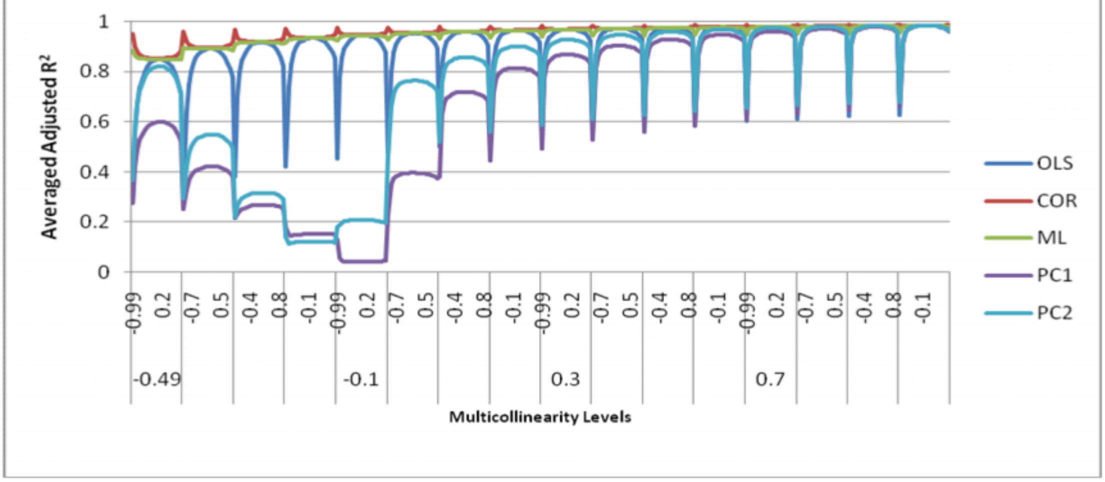


Figure 3: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=20

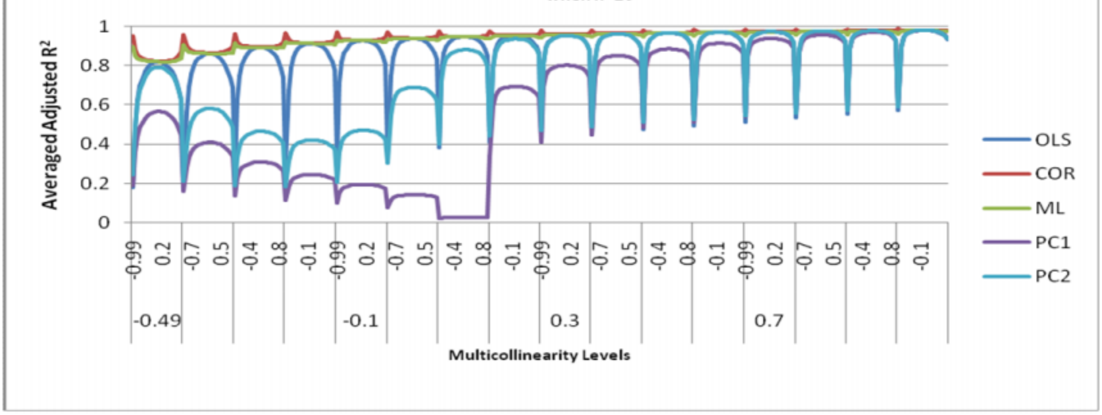
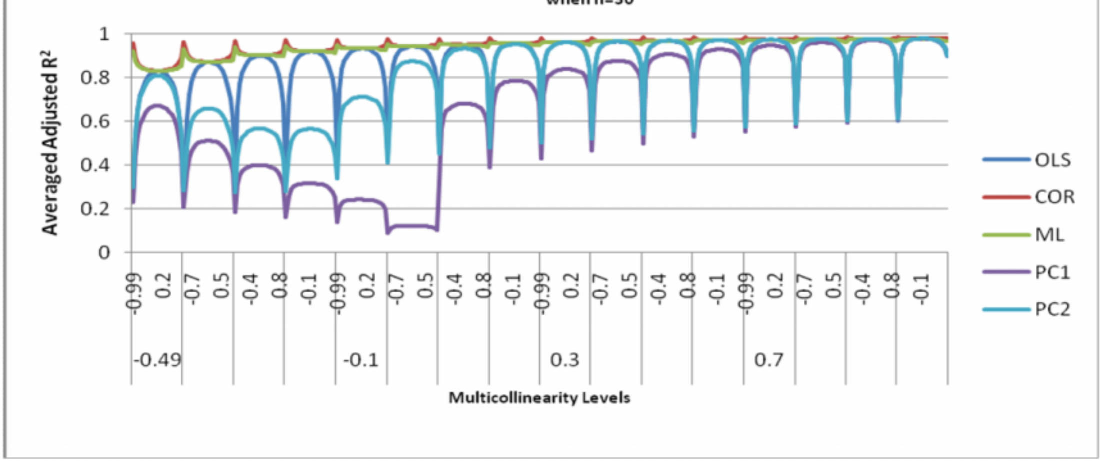


Figure 4: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=30



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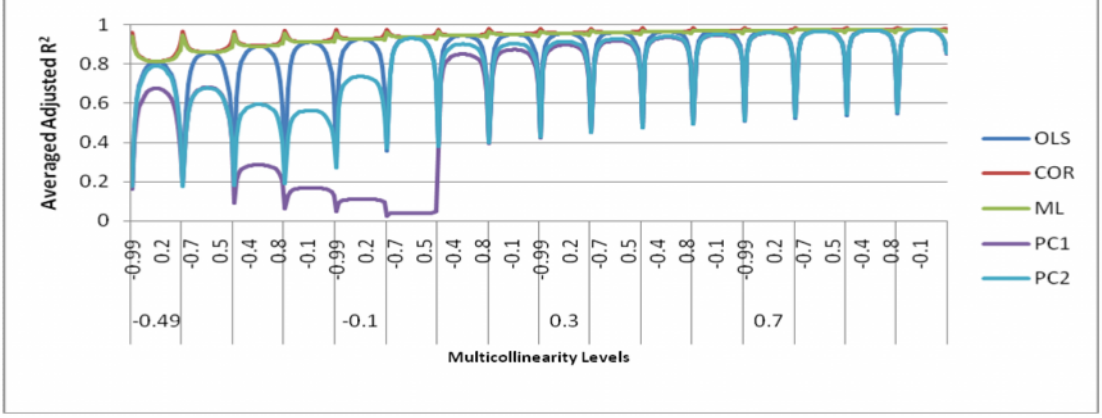

Figure 5: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=50

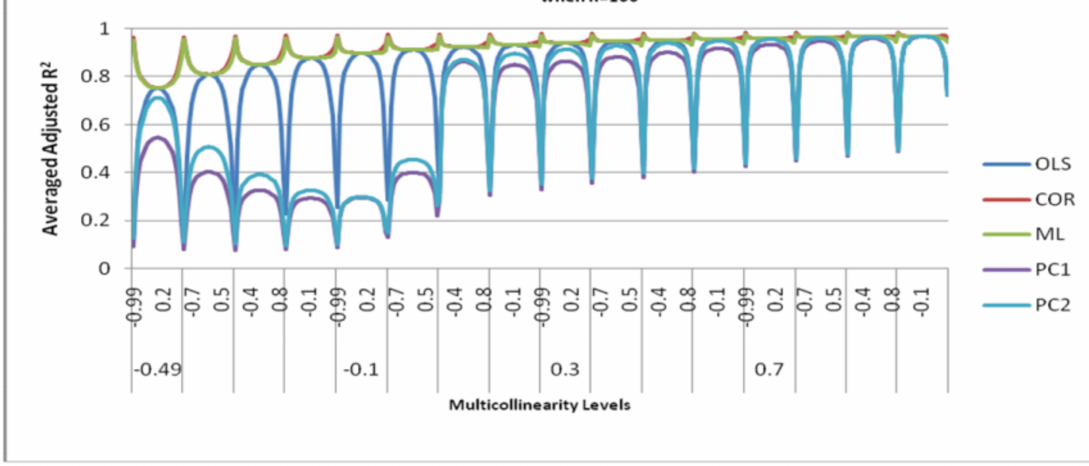

Figure 6: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=100

From these figures, results reveal the pattern of performances of COR and ML at each level of multicollinearity over the levels of autocorrelation to be generally and evidently convex especially when $n \geq 30$ and $\lambda < 0$ while that of OLS and PC is generally concave. Moreover, the COR and ML estimators perform equivalently and better in that the values of their averaged adjusted coefficient of determination are often greater than 0.8; and their performances become much better as multicollinearity increases. The COR estimator is generally the best estimator for prediction except at high level of multicollinearity and low levels of autocorrelation. At these instances, the PC estimator is either best or competes with the COR estimator. Moreover when the sample size is small (n=10) and multicollinearity level is not high, the OLS estimator is best at low level of autocorrelation whereas the ML is best at moderate levels of autocorrelation.

Very specifically in term of identification of the best estimator, Table 1, 2, 3, 4, 5 and 6 respectively summarize the best estimator for prediction at all the levels of autocorrelation and multicollinearity when the sample size is 10, 15, 20, 30, 50, 100.

Table 1: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=10.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	ML	ML	ML	ML	ML	ML	ML	ML	ML	COR	COR	COR	COR	COR	COR	COR
-0.3	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2
-0.2	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2
-0.1	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2	PC2
0	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	PC2	PC2	PC2	PC2	PC2
0.1	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	PC2	PC2	PC2	PC2	PC2
0.2	PC2	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	PC2	PC2	PC2	PC2	PC2
0.3	PC2	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2	PC2
0.4	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2
0.5	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2
0.6	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

From Table 1 when n = 10, the COR estimator is best except when $-0.4 \le \rho \le 0.5$. When $\lambda \ge 0.6$ at these instances, the PC2 estimator is most frequently best and when the COR or ML is best, the PC2 estimator competes very favorably. Furthermore when $\lambda < 0.5$, the ML estimator is best when $-0.4 \le \rho \le -0.1$ and $0.3 \le \rho \le 0.6$ while the OLS estimator is best when $0 \le \rho \le 0.3$. Moreover, the PC2 estimator is still best when $\lambda \le -0.49$ and $0.2 \le \rho \le 0.3$.

Table 2: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=15.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2
0	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2
0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2
0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2
0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

When n=15, the COR estimator is generally best except when $-0.1 \le \rho \le 0.3$ and $\lambda \to 1$. At these instances, the PC2 estimator is most frequently best and when the COR is best, the PC2 estimator competes very favorably.

Table 3: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=20.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2	PC2	PC2
0	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2	PC2	PC2	PC2
0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2	PC2	PC2	PC2

0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2	PC2	PC2	PC2
0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

When n=20, the COR estimator is generally best except when $-0.1 \le \rho \le 0.2$ and $\lambda \ge 0.6$. At these instances, the PC2 estimator is most frequently best and when the COR is best, the PC2 estimator competes very favorably.

Table 4: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=30.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2
0	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2	PC2
0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2
0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

When n=30, the COR estimator is generally best except when $0 \le \rho \le 0.1$ and $\lambda \ge 0.8$. At these instances, the PC2 estimator is most frequently best and when the COR is best, the PC2 estimator competes very favorably.

Table 5: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=50.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC1
0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

Table 6: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=100.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2
0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

When $n \geq 50$, the COR estimator is generally best except when there is no autocorrelation at all and $\lambda \rightarrow 1$. At these instances, the PC estimator is best.

4.0 Conclusions

The effect of two major problems, Multicollinearity and autocorrelation, on the predictive ability of the OLS, COR, ML and PC estimators of linear regression model has been jointly examined in this paper. Results reveal the pattern of performances of COR and ML at each level of multicollinearity over the levels of autocorrelation to be generally and evidently convex especially when $n \geq 30$ and $\lambda < 0$ while that of OLS and PC is generally concave. Moreover, the COR and ML estimators perform equivalently and better; and their performances become much better as multicollinearity increases. The COR estimator is generally the best estimator for prediction except at high level of multicollinearity and low levels of autocorrelation. At these instances, the PC estimator is either best or competes with the COR estimator. Moreover when the sample size is small (n=10) and multicollinearity level is not high, the OLS estimator is best. at low level of autocorrelation whereas the ML is best at moderate levels of autocorrelation.

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