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An Application of New Transform " Mahgoub Transform" to Partial Differential Equations

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Abstract
In this paper .The Mahgoub Transform of partial derivatives is derived, and its applicability demonstrated using five different partial differential equations. In this paper we find the particular solutions of these equations. The result reveals that the proposed method is very efficient, simple and can be applied to linear and nonlinear problems.
Keywords: Mahgoub Transform-Partial Differential Equations- wave equation- heat equation

Introduction

The theory and application of partial differential equations (PDEs) play an important role in the mathematical modeling of many fields: physical phenomena, biological models, chemical kinetics and engineering sciences in which it is necessary to take into account the effect of the real world problems. therefore, there are so many different techniques In order to solve differential equations, several integral transforms were extensively used and applied in theory and application such as the Laplace, Fourier, Mellin, Hankel , Sumudu transforms, Elzaki transform , and Aboodh Transform . Recently, Mohand Mahgoub (2016) introduced a new integral transform, named the Mahgoub Transform, and further applied it to the solution of ordinary and partial differential equations. The definition, properties and applications of the Mahgoub transform to ordinary differential equations are described in [1]

In this paper we derive the formulate for the Mahgoub transform of partial derivatives and apply them in Solving five types of initial value problems. Our purpose here is to show the applicability of this interesting new transform and its effecting in solving such problems.

Definition and Derivations the Mahgoub Transform of Derivatives

The Mahgoub transform of the function $f(t)$is defined as

$$M[f(t)] = H(v) = v\int_0^\infty f(t)e^{-vt}dt\ .\quad t \ge 0 \quad , \ k_1 \le v \le k_2 \tag{1}$$

To obtain the Mahgoub transform of partial derivatives we use integration by parts as follows:

$$\begin{aligned} M\left[\frac{\partial f}{\partial t}(x\,,t)\right] &= \int_0^\infty v\frac{\partial f}{\partial t}e^{-vt}\,dt = \lim_{p\to\infty}\int_0^p v\frac{\partial f}{\partial t}e^{-vt}\,dt \\ &= \lim_{p\to\infty}\ \left\{[ve^{-vt}f(x\,,t)]_0^p + \int_0^p ve^{-vt}\,f(x\,,t)dt\right\} \\ &= vH(x\,,v) - vf(x\,,0) \end{aligned} \tag{2}$$

We assume that f is piecewise continuous and is of exponential order.

Now

$$M\left[\frac{\partial f}{\partial x}\right] = \int_0^\infty ve^{-vt}\frac{\partial f(x\,,t)}{\partial x}dt$$

$$= \frac{\partial}{\partial x}\int_0^\infty ve^{-vt}\,f(x\,,t)dt \qquad \text{(Using the Leibnitz rule)}$$

$$= \frac{\partial}{\partial x}[H(x\,,v]$$

And $$M\left[\frac{\partial f}{\partial x}\right] = \frac{d}{dx}[H(x\,,v)] \tag{3}$$

Also we can find:

$$M\left[\frac{\partial^2 f}{\partial x^2}\right] = \frac{d^2}{dx^2}[H(x\,,v)] \tag{4}$$

To find $M\left[\frac{\partial^2 f}{\partial t^2}(x\,,t)\right]$

$$M\left[\frac{\partial^2 f}{\partial t^2}(x\,,t)\right] = \int_0^\infty ve^{-vt}\frac{\partial^2 f(x\,,t)}{\partial t^2}dt$$

Let

$$\frac{\partial^2 f(x\,,t)}{\partial t^2} = d\tau \to \tau = \frac{\partial f(x\,,t)}{\partial x}$$

Then

$$M\left[\frac{\partial^2 f}{\partial t^2}(x,t)\right] = \lim_{p\to\infty}\left\{\left[ve^{-vt}\frac{\partial f(x,t)}{\partial x}\right]_0^p + \int_0^p ve^{-vt}\frac{\partial f(x,t)}{\partial x}dt\right\}$$
$$= -v\, f'(x,0) + v[v\, H(x,v) - vf(x,0)]$$
$$= v^2\, H(x,v) - v^2 f(x,0) - v\, f'(x,0) \qquad (5)$$

We can easily extend this result to the nth partial derivative by using mathematical induction.

Solution of Partial Differential Equations

In this section we solve first order Partial differential Equations and the Second order partial differential equation, wave equation, heat equation Laplace's and Telegraphers equation which are known as four. Fundamental equations in mathematical physics and occur in many branches of physics, in applied mathematics as well as in engineering.

Example 1:

Find the solution of the first order initial value problem:

$$\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial t} + y \quad , \quad y(x,0) = 6e^{-3x} \qquad (6)$$

And y is bounded for $x > 0$, $t > 0$.

Let Y be the Mahgoub transform of y. Then, taking the Mahgoub transform of (6) we have

$$\frac{dY(x,v)}{dx} = 2[vY(x,v) - vY(x,0)] + Y(x,v)$$
$$\frac{dY(x,v)}{dx} - (2v+1)Y(x,v) = -12ve^{-3x}$$

This is the linear ordinary differential equation

The integration factor is $p = e^{\int -(2v+1)x} = e^{-(2v+1)x}$

Therefore

$$Y(x,v) = \frac{6v}{v+2}e^{-3x} + ce^{(2v+1)x}$$

Since Y is bounded, c should be zero. Taking the inverse Mahgoub transform we have:

$$y(x,t) = 6e^{-2t-3x} = 6e^{-2t}e^{-3x}$$

Example (2):

Consider the Laplace equation:

$$u_{xx} + u_{tt} = 0 \quad , \quad u(x,0) = 0 \quad , \quad u_t(x,0) = \cos x \qquad (7)$$

Let H(v) be the Mahgoub transform of u. Then, taking the Mahgoub transform of equation (7) we have:

$$v^2 H(x,v) - v^2 u(x,0) - v u_t(x,0) + H''(x,v) = 0$$
$$H''(x,v) + v^2 H(x,v) = v\cos x$$

This is the second order differential equation have the particular, solution in the form

$$H(x,v) = \frac{v\cos x}{D^2+v^2} = \frac{v}{v^2-1}\cos x \qquad (8)$$

Where

$$D^2 = \frac{d^2}{dx^2}$$

If we take the inverse Mahgoub transform for Eq. (8), we obtain solution of Eq (7) in the form. $u(x,t) = \sinh t\cos x$

Example (3):

Solve the wave equation:

$$u_{tt} - 4u_{xx} = 0 \quad , \quad u(x,0) = \sin \pi x \quad , \quad u_t(x,0) = 0 \quad , \quad x,t > 0$$

Taking the Mahgoub transform for Eq (9) and making use of Conditions we obtain.

$$v^2 H(x,v) - v^2 u(x,0) - v u_t(x,0) - 4H''(x,v) = 0$$
$$v^2 H(x,v) - 4H''(x,v) = v^2 \sin \pi x$$
$$4H''(x,v) - v^2 H(x,v) = -v^2 \sin \pi x$$
$$H(x,v) = \frac{-v^2\sin \pi x}{4D^2 - v^2} = \frac{v^2}{v^2+(2\pi)^2}\sin \pi x \qquad (9)$$

Now we take the inverse Mahgoub transform to find the particular solution of (9) in the form

$$u(x,t) = \cos 2\pi t\sin \pi x$$

Example (4):

Consider the homogeneous heat equation in one dimension in a normalized form:

$$4u_t = u_{xx} \quad , \quad u(x,0) = \sin\frac{\pi}{2}x \quad , \quad x,t > 0 \qquad (10)$$

By using the Mahgoub transform for Eq (10) we can obtain

$$4v\, H(x,v) - 4v\, u(x,0) = H''(x,v)$$
$$H''(x,v) - 4v\, H(x,v) = -4v\sin\frac{\pi}{2}x$$

Solve for $H(x,v)$we find that the particular solution is

$$H(x\,,v)=\frac{-4v\sin\frac{\pi}{2}x}{D^2-4v}=\frac{4v\sin\frac{\pi}{2}x}{4v+\left(\frac{\pi}{2}\right)^2}=\frac{v}{v+\frac{\pi^2}{16}}\sin\frac{\pi}{2}x \qquad (11)$$

And similarly if we take the inverse Mahgoub transform for Eq (11), we obtain the Solution of (10) in the form.

$$u(x\,,t)=e^{-\frac{\pi^2}{16}t}\sin\frac{\pi}{2}x$$

Example (5)

Consider the telegraphers equation:

$$\frac{\partial^2 \mathrm{u}}{\partial x^2}=\frac{\partial^2 \mathrm{u}}{\partial x^2}+2\frac{\partial u}{\partial t}+\mathrm{u} \quad , \quad 0<\mathrm{x}<1, \quad \mathrm{t}>0 \qquad (12)$$

With the initial conditions:

$$\mathrm{u}(\mathrm{x}\,,0)=\mathrm{e}^{x} \quad , \quad \mathrm{u}_t(\mathrm{x}\,,0)=-2\mathrm{e}^{x} \qquad (13)$$

Solution:

Take Mahgoub transform of Eq (12) we get:

$$H''(x\,,v)=v^2H\,(x\,,v)-v^2\mathrm{u}(\mathrm{x}\,,0)-v\mathrm{u}_t(\mathrm{x}\,,0)+2v\,H(x\,,v)-2v\,u(x\,,0)+H\,(x\,,v) \qquad (14)$$

Substituting Eq (13) into Eq (14) we have:

$$H''(x\,,v)=v^2H\,(x\,,v)-v^2\mathrm{e}^x+2v\mathrm{e}^x+2v\,H(x\,,v)-2v\,\mathrm{e}^x+H\,(x\,,v)$$

$$H''(x\,,v)-(v^2+2v+1)\,H(\,x\,,v)=-v^2\mathrm{e}^x$$

$$H\,(x\,,v)=\frac{-v^2\mathrm{e}^x}{D^2-(v+1)^2}=\frac{v^2}{(v^2+2v)}\mathrm{e}^x=\frac{v}{v+2}\mathrm{e}^x \qquad (15)$$

Which is the particular solution of (12).

We take the inverse of Mahgoub transform for Eq (15) ,we find that

$$u(x\,,t)=H\,(v)=\mathrm{e}^xH^{-1}\left[\frac{v}{v+2}\right]=\mathrm{e}^xe^{-2t}$$

Conclusion

In this paper , We studied to obtain solutions of many partial differential equations by Mahgoub Transform. This new approach applied began to be used in the a lot of different area. As a consequently, we showed that analytical solutions of partial differential equations by Mahgoub Transform. This technique is useful to solve linear and nonlinear differential equations

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